Proceedings of the
Symposium on tidal instrumentation
and predictions of tides

Actes du Symposium
sur le matériel marégraphique
et la prédiction des marées

Paris, 3-7 mai 1965

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Adresse Présidentielle de
M. André GOUGENHEIM  
Ingénieur Hydrographie Général (C.R.)  
Membre de l'Institut, Ancien Directeur du Service Hydrographique de la Marine  
Président du Comité National Français de Géodésie et Géophysique

Mes Chers Collègues,

Il y a environ trois ans et demi, dans les derniers jours de novembre 1961, j'avais déjà eu, en tant que président de la Commission des marées (Committee on Tides) de l'Association internationale d'océanographie physique (A.I.O.P. - I.A.P.O.), l'honneur et l'agrément d'accueillir un certain nombre d'entre vous à Paris, dans le bel hôtel du Service Hydrographique de la Marine, riche des gloires du passé, pour un premier colloque consacré au traitement automatique des problèmes relatifs aux marées.

Les exposés présentés par les participants et les débats qui les suivaient firent apparaître les besoins et les tendances qui se manifestaient dans le domaine de l'observation des marées, dans les domaines conçus de l'analyse et de la prédiction du phénomène et enfin dans celui de l'étude théorique du régime des marées dans les mers et les océans.

En conclusion du colloque, des vœux furent émis pour souligner les directions dans lesquelles il paraissait essentiel de progresser, autant pour améliorer notre connaissance des phénomènes que pour harmoniser le plus possible les travaux poursuivis dans les divers pays, de manière à faciliter l'exécution d'études de synthèse.

Depuis lors, des recherches de plus en plus nombreuses ont été conduites, sur les problèmes relatifs aux marées, par un nombre croissant de spécialistes, cependant que des simplifications importantes étaient introduites parallèlement dans la mise en œuvre des calculatrices électroniques. Aussi l'A.I.O.P. considère-t-il qu'il y a déjà quelque temps qu'il était de nouveau nécessaire de faire le point dans le développement des travaux modernes sur les marées et il fut décidé de réunir à cet effet deux colloques jumelés dont l'un, consacré au matériel d'observation et d'enregistrement, serait organisé par le Dr. J. R. Rossiter, secrétaire de la Commission de l'A.I.O.P. pour le niveau moyen des mers et ses variations (Commission on mean sea-level and its variations) et Directeur du Service permanent de l'I.G.G.I. pour le niveau moyen des mers (Permanent Service for mean sea level), tandis que le second colloque aurait pour thème l'emploi des ordinateurs électroniques dans l'analyse et la prédiction des marées et courants de marées et serait placé sous la responsabilité de l'Ingenieur hydrographie en chef Marc EYRIES, qui dirige la Section des Marées au Service Hydrographique de la Marine et qui a suivi en 1963 à la présidence de la Commission des marées de l'A.I.O.P.

D'un autre côté, l'UNESCO qui, comme vous le savez, porte depuis longtemps un grand intérêt aux recherches océanographiques, a tenu à marquer d'une part l'importance qu'elle attachait aux thèmes de nos colloques et à reconnaître d'autre part la haute qualité des travaux effectués par les spécialistes des marées et elle s'est entendue avec l'A.I.O.P. pour que les deux colloques qui s'ouvriront aujourd'hui soient placés sous le patronage commun des deux organisations.

Cette circonstance me vaut le privilège, comme président du Comité national français de géodésie et géophysique, de vous souhaiter aujourd'hui la bienvenue dans ce joli et magnifique bâtiment de l'UNESCO, chargé des promesses de l'Avenir. Mais l'UNESCO ne s'est pas contentée de nous offrir son aimable hospitalité, nous lui devons aussi de bénéficier des services de son équipe de traduction simultanée : enfin nous pouvons lui être particulièrement reconnaissants de la subvention qu'elle a bien voulu accorder pour alléger les frais de voyage.
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des auteurs de communication. Des précisions à ce sujet seront données dans quelques instants par l'ingénieur général Loïc CAHIERRE, secrétaire général du Comité national français de glaciologie et de géophysique, qui a accepté la lourde tâche de l'organisation matérielle des deux colloques et que nous devons remercier de toute la peine qu'il a prise pour assurer le succès de nos travaux.

Alors que le colloque de novembre 1961 portait sur un programme relativement étendu, les réunions actuelles ont des ambitions plus limitées quant au nombre de sujets qu'on abordera. On s'est donc limité à un programme restreint et le nombre de sujets a été réduit. L'on peut penser que même si le colloque est restreint, il aura tout de même un certain intérêt, car il permettra d'étudier de manière approfondie certains sujets, en particulier ceux qui concernent le domaine de l'application étendue. Il sera donc indispensable de préciser l'objectif des travaux et de définir les limites de la recherche.

En effet, c'est la première fois que des travaux de cette nature sont réalisés à l'aide d'instruments différents ou de méthodes différentes et ayant un domaine d'application étendu. Il sera donc très souhaitable que les travaux de cette nature soient réalisés dans le cadre de travaux plus importants, répondant au besoin des conditions imposées par les circonstances propres à chaque cas.

Cependant, s'il est commode de rechercher la solution optimale pour chaque cas particulier, on ne doit pas oublier qu'il existe souvent des solutions plus ou moins satisfaisantes obtenues à l'aide d'instruments différents ou de méthodes différentes et ayant un domaine d'application étendu. Il sera donc très souhaitable que les travaux de cette nature soient réalisés dans le cadre de travaux plus importants, répondant au besoin des conditions imposées par les circonstances propres à chaque cas.

On pressent ainsi tous les avantages qui seraient attachés à cette façon de faire. D'une part les appareils de mesure et d'enregistrement des marées et des courants, fabriqués à un nombre d'exemplaires relativement élevé, seraient d'un prix assez bas. D'un autre côté, l'unification relativement poussée de tous les procédés de calcul faciliterait beaucoup la coordination des résultats obtenus dans les divers pays et par là même l'exécution d'études de synthèse.

Des participants aux colloques pourront à ce propos s'étonner que les travaux de la marée sur modèle mathématique, entrepris en vue d'améliorer le tracé de cartes cotidiennes plus ou moins étendues, études qui avaient figuré au programme du colloque de 1961, ne soient pas reprises dans les prochaines réunions.

En fait, d'une part, assez peu de travaux nouveaux paraissent avoir été accomplis sur ce thème et, d'autre part, il semble que le temps ne sera plus très éloigné où il sera possible de mesurer avec précision la marée ou des bassins plagioclastiques et de grandes fosses océaniques. On disparaîtra alors, dans un temps d'année, de nombreux résultats concernant des domaines océaniques, pour lesquels on est jusqu'à présent contraint de forger des hypothèses sur le régime des marées. Il sera alors préférable, pour établir de nouvelles cartes de propagation de la marée à travers les océans, d'attendre les prochains résultats de mesures directes, plutôt que de continuer à émettre des hypothèses que les nouvelles mesures rendront bientôt périmées.

De toute façon, les deux colloques qui vont s'ouvrir présenteront un intérêt considérable pour tous ceux qui s'intéressent, dans le monde, aux problèmes posés par les marées, problèmes qui présentent à la fois une grande valeur intrinsèque et d'importantes possibilités d'application pratique. Tout en vous complétant d'être venus en si grand nombre, et pour certains de si loin, participer à ces réunions, je souhaite au nom de tous nos confrères de contribuer à l'繁荣 du développement de vos travaux et à leur accomplissement sur des conclusions constructives.

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**SEA LEVEL RECORDING : THE NEED FOR THE NEW INSTRUMENTATION**

**J. R. ROSSITER**

Permanent Service for Mean Sea Level

This symposium arose out of a need for a long series of mean sea level data from areas which, for various reasons, do not permit the installation, operation or maintenance of orthodox tide gauges. It would be wrong, however, to confine the symposium to this one aspect of instrumentation, and it would therefore be of value to consider the whole range of tide gauge recorder requirements whilst still bearing in mind the original purpose of this meeting. We should, I think, exclude instruments designed specifically for measuring and recording short period surface waves unless the design principle or principles are adaptable to our problem.

1 - THE SITUATION AT PRESENT

Accurate and reliable automatic tide gauges, working on the principle of a counterweighted float rising and falling with the tide in a float well, have been available for many years, and form the standard equipment at most modern shore-based stations. Siltation of the inlet to the well, and the difficulty of sitting the well so that very low water levels may be recorded, are amongst the reasons for the frequent use of pressure gauges of the bubbler-type (as, for example, in the Netherlands), in which the sea level some short distance off-shore is recorded on land.

In the broadest terms, however, it seems that there is no proved and reliable instrument available on a commercial scale and at a reasonable price to replace the float gauge when local conditions prevent this type of instrument being used. Consequently, sea level recorders are made, on a routine basis, only where standard equipment can operate. This situation severely restricts the oceanographer concerned with the large scale problem of obtaining sea level data for the world oceans, for he finds that data from the vast expanses of the ocean, from the long stretches of rocky, marshy or uninhabited coastline, and from Antarctic and Arctic waters are either non-existent, insufficient, or doubtful in quality.

We thus arrive at a definition of the problem, in which the essential first step constitutes a challenge to the instrument designer. A technological society which can send a telemeasuring thermometer to Venus can surely devise a method of recording sea level changes at a desert island once the problem is properly posed. With this assumption, let us now examine the requirements.

2 - SPECIFICATIONS FOR TIDE GAUGES

(i) Location. Observations are required from the following types of coastal location:

- Uninhabited islands
- Coastlines with flat beaches, where the low water mark is remote from the shore; the mouths of deltas
- Rocky coastlines
- Icebound coastlines
SEA LEVEL RECORDING: THE NEED FOR THE NEW INSTRUMENTATION

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a) Uninhabited islands
b) Coastlines with flat beaches, where the low water mark is remote from the shore; the mouths of deltas
c) Rocky coastlines
d) Icebound coastlines
(ii) Climatic conditions. If not submerged, the equipment may have to withstand either tropical or arctic conditions of temperature and humidity. These may affect the freedom of moving parts, the reliability of recording paper, rate of corrosion, calibration of the sensing device, etc.

(iii) Frequency response. For most tidal research, recorders should be designed so that they are insensitive to periods of less than 10 minutes. Tidal oscillations are generally associated with frequencies of less than 10 cycles per day, and it is important that a recorder should have a flat response over that part of the spectrum between 0 and 16 c.p.d. In any case it is obviously necessary to have some idea of the frequency response of a gauge.

If high frequency oscillations are not damped, and digital recording is envisaged, the sampling time of the equipment, i.e., the interval at which readings are taken, is important. The extreme case may arise, for example, where the sampling time is close to the period of the unwanted high frequency oscillation, to the detriment of the tidal component of the data.

(iv) Type of recorder. As we are seeking an instrument which requires a minimum of maintenance, we must be prepared to abandon graphical in favour of digital recording if it can be shown that the latter is capable of development so that it becomes less susceptible to failure than graphical recording.

Recording drums fitted with charts and revolving once per day require the charts to be changed at least once per week. Strip recording is not so troublesome in this respect, but still depends on the flow of ink in the recording pen. Other disadvantages of graphical recording will be referred to later.

Given a reliable design, however, the length of time a digital recorder can operate without attention is only limited by the power supply capacity and the quantity of paper or magnetic tape available. The latter requirement can be kept within reasonable bounds at the expense of the sampling frequency.

(v) Accuracy in measuring height of sea level. By this phrase I mean the maximum total possible error inherent in the instrument, including calibration errors, errors due to temperature sensitivity, backlash in moving parts, etc., i.e., the instrumental error.

Orthodox modern gauges are reputed to be both sensitive and accurate to 0.1 ft (3 cm); for a wide variety of reasons I am convinced that tabulated values of sea level obtained from such instruments scarcely over reach that standard.

A maximum total error of 0.05 ft (1 1/2 cm) in level therefore seems a minimum objective when considering the design of a gauge for general use.

The recording sensitivity should therefore permit a variation of 1 1/2 cm of water level to be detected. In a graphical record this means that the reduction scale for recording should not exceed 20 : 1, and accordingly a range of 10 m would require a chart 50 cm in height; this is not an unreasonable size. In a digital recorder the corresponding requirement could be for the data to be stored on a 10-channel paper or magnetic tape.

In the case of gauges which measure the pressure of a column of sea water as opposed to those which measure the elevation of the sea surface, the interpretation of a pressure as a height is dependent upon the density of the column of water. Errors in transiting one into the other should be specifically excluded from the foregoing, since they are not a function of the instrument.

(vi) Accuracy in time keeping. In tidal work, this all important factor has been neglected far too often. It is futile to reduce the noise level to less than 1 cm without a corresponding accuracy in time keeping.

It is a simple matter to show that the maximum error in height $\Delta y$ for a time error of $\Delta t$ is given, in the case of a harmonic oscillation of period $T$ and amplitude $H$, by

$$\Delta y = \frac{2 \pi H}{T} \Delta t$$

If $\Delta y$ is not to exceed 1.5 cm, then for a semidiurnal tide with $H = 5$ m,

$$\Delta t = 0.3 \text{ minutes},$$

and this must be interpreted as the maximum time error over the period the gauge remains unattended unless the following conditions apply:

a) the rate of loss or gain is constant

b) the true time is noted at the beginning and end of each record.

If these conditions apply, in practice it is a simple matter to determine and correct for the time errors.

Whether gauge clocks can be manufactured to this accuracy is one point which it is to be hoped can be decided at this meeting.

While the foregoing is a complete statement of the situation for digital recording, it is only part of the picture for graphical recording. Here we have the added complication of the hygroscopicity of the paper. Enquiries in the U.K. reveal that unless one has recourse to plastic material, recording paper cannot be manufactured so that the paper maintains its dimensions to 1% under changes of humidity. This situation can make nonsense of all our claims to accuracy, and may be the strongest argument towards digital recording.

(vii) Datum of Recordings. To study the low frequency end of the spectrum involves the special problem of datum. For mean sea level purposes it is essential to relate the zero of recordings to a level on relatively stable land, and to keep a record of changes in this relationship as they may vary from year to year.

This requires that the recordings be regularly checked against some independent, precise method of measuring sea level relative to a land level. There are many difficulties encountered in this process even when the gauge is of the standard shore-based type, and the problem is of sufficient importance to merit discussion if time permits.

The problem is that of obtaining a precise estimate of sea level, once a day, in open water, outside the well but as near to it as possible, in the presence of wave action.

Where the recorder is off-shore, as in the case of a pressure gauge transmitting readings to land by underwater cable or by radio, or alternatively by storing them in gills on the sea bed, the problem is even greater since one cannot, in general, assume that at any one instant these sea surface is a level plane between coast and off-shore site. It would be interesting to hear of any practical solution to the levelling problem and, indeed, of a solution to the problem of maintaining a continuous datum in deep sea recording.

(viii) Maintenance and Repair. If only from the logistics of the problem, a gauge designed for inaccessible or inhospitable areas should require the absolute minimum of maintenance. Particular attention should therefore be given to

a) constancy of calibration

b) adequate power supply
c) accurate time keeping
d) freedom from deterioration in material and components.

It is also vital to remember that maintenance should be limited to the simplest of operations. Routine tidal observations, upon which so much research is based, have not been universally recognised as forming a scientific programme of observation; the result is that unqualified persons are often required to maintain gauges. Their capabilities should not therefore be unduly taxed.

(ix) Requirements. One special requirement stands head and shoulders above all others. It is that the cost should be competitive with standard gauges, if at all possible.
(ii) Climatic conditions. If not submerged, the equipment may have to withstand either tropical or arctic conditions of temperature and humidity. These may affect the freedom of moving parts, the reliability of recording paper, rate of corrosion, calibration of the sensing device, etc.

(iii) Frequency response. For most tidal research, recorders should be designed so that they are insensitive to periods of less than 10 minutes. Tidal oscillations are generally associated with frequencies of less than 10 cycles per day, and it is important that a recorder should have a flat response over that part of the spectrum between 0 and 10 cph. In any case, it is obviously necessary to have some idea of the frequency response of a gauge.

If high frequency oscillations are not damped, and digital recording is envisaged, the sampling time of the equipment, i.e., the interval at which readings are taken, is important. The extreme case may arise, for example, where the sampling time is close to the period of the unexcited high frequency oscillation, to the detriment of the tidal component of the data.

(iv) Type of recorder. As we are seeking an instrument which requires a minimum of maintenance, we must be prepared to abandon graphical in favour of digital recording if it can be shown that the latter is capable of development so that it becomes less susceptible to failure than graphical recording.

Recording drums fitted with charts and revolving once per day require the charts to be changed at least once per week. Strip recording is not so troublesome in this respect, but still depends on the flow of ink in the recording pen. Other disadvantages of graphical recording will be referred to later.

Given a reliable design, however, the length of time a digital recorder can operate without attention is only limited by the power supply capacity and the quantity of paper or magnetic tape available. The latter requirement can be kept within reasonable bounds at the expense of the sampling frequency.

(v) Accuracy in measuring height of sea level. By this phrase I mean the maximum total possible error inherent in the instrument, including calibration errors, errors due to temperature sensitivity, backlash in moving parts, etc., i.e., the instrumental error. Orthodox modern gauges are reputed to be both sensitive and accurate to 0.1 ft. (3 cm) for a wide variety of reasons I am convinced that tabulated values of sea level obtained from such instruments scarcely ever reach that standard.

A maximum total error of 0.05 ft. (1 1/2 cm) in level therefore seems a minimum objective when considering the design of a gauge for general use.

The recording sensitivity should therefore permit a variation of 1 1/2 cm of water level to be detected. In a graphical record this means that the reduction scale for recording should not exceed 20 : 1, and accordingly a range of 10 m would require a chart 50 cm in height; this is not an unreasonable size. In a digital recorder the corresponding requirement could be for the data to be stored on a 10-channel paper or magnetic tape.

In the case of gauges which measure the pressure of a column of sea water as opposed to those which measure the elevation of the sea surface, the interpretation of a pressure as a height is dependent upon the density of the column of water. Errors in translating one into the other should be specifically excluded from the foregoing, since they are not a function of the instrument.

(vi) Accuracy in time keeping. In tidal work, this all important factor has been neglected far too often. It is futile to reduce the noise level to less than 1 cm without a corresponding accuracy in time keeping.

It is a simple matter to show that the maximum error in height $\Delta y$ for a time error of $\Delta t$ is given, in the case of a harmonic oscillation of period $T$ and amplitude $H$, by

$$\Delta y = \frac{2\pi H}{T} \Delta t$$

If $\Delta y$ is not to exceed 1.5 cm, then for a semidiurnal tide with $H = 5\,\text{m}$, $\Delta t = 0.3\,\text{minutes}$, and this must be interpreted as the maximum time error over the period the gauge remains unattended unless the following conditions apply:

a) the rate of loss or gain is constant
b) the true time is noted at the beginning and end of each record.

If these conditions apply, in practice it is a simple matter to determine and correct for the time errors.

Whether gauge clocks can be manufactured to this accuracy is one point which it is to be hoped can be decided at this meeting.

Whilst the foregoing is a complete statement of the situation for digital recording, it is only part of the picture for graphical recording. Here we have the added complication of the hygroscopic nature of the paper. Enquiries in the U.K. reveal that unless one has recourse to plastic material, recording paper cannot be manufactured so that the paper maintains its dimensions to 1% under changes of humidity. This situation can make nonsense of all our claims to accuracy, and may be the strongest argument towards digital recording.

(vi) Datum of Recordings. To study the low frequency end of the spectrum involves the special problem of datum. For mean sea level purposes it is essential to relate the zero of recordings to a level on relatively stable land, and to keep a record of changes in this relationship as they may vary from year to year.

This requires that the recordings be regularly checked against some independent, precise method of measuring sea level relative to a land level. There are many difficulties encountered in this process even when the gauge is of the standard shore-based type, and the problem is of sufficient importance to merit discussion if time permits.

The problem is that of obtaining a precise estimate of sea level, once a day, in open water, outside the well but as near to it as possible, in the presence of wave action.

Where the recorder is off-shore, as in the case of a pressure gauge transmitting readings to land by underwater cable or by radio, or alternatively by storing them in digital form on the sea bed, the problem is even greater since one cannot, in general, assume that at any one instant the sea surface is a level plain between coast and off-shore site. It would be interesting to hear of any practical solution to the levelling problem and, indeed, of a solution to the problem of maintaining a continuous datum in deep sea recording.

(viii) Maintenance and Repair. If only from the logistics of the problem, a gauge designed for inaccessible or inhospitable areas should require the absolute minimum of maintenance. Particular attention should therefore be given to

a) constancy of calibration
b) adequate power supply
c) accurate time keeping
d) freedom from deterioration in material and components.

It is also vital to remember that maintenance should be limited to the simplest of operations. Routine tidal observations, upon which so much research is based, have not been universally recognised as forming a scientific programme of observation; the result is that unqualified persons are often required to maintain gauges. Their capabilities should not therefore be unduly taxed.

(ix) Requirements. One special requirement stands head and shoulders above all others. It is that the cost should be competitive with standard gauges, if at all possible.
The most desirable development in oceanographic measurement is the production of a tide gauge to measure tides in the deeper parts of the ocean.

From papers recently published, it seems this problem is already on its way to a solution. We look forward to hearing more on this subject from participants, though I would repeat the hope that the rather less exotic matters of rocky and frozen shores, muddy deltas and unfrequented islands should not be forgotten.

To summarise, I believe our discussions should involve the following items:

1. Type of location at which a gauge will function.
2. Suitability for tropic or arctic conditions.
3. Frequency response.
4. Graphical or digital recording.
5. Sampling frequency for digital recording.
6. Accuracy in measuring height or height analogue.
7. Accuracy in measuring time.
8. Facility for determining datum.
9. Need for maintenance and repair, including calibre of technical service required.

OBSERVATIONS SUR LA COMMUNICATION DU DR ROSSITER

M. EVANS. Il faut distinguer, dans le premiers des informations, la fréquence de mesure de la fréquence d'enregistrement. Ainsi, l'enregistrement soudain peut ne pas être influencé par des oscillations qui ont sa fréquence à condition que la grandeur enregistrée soit la somme (ou la moyenne) de mesures beaucoup plus fréquentes; c'est le cas pour un enregistrement effectué toutes les 300 secondes et qui est la moyenne de mesures effectuées toutes les secondes.

DR MUNK. - An additional consideration is the inadvisability of non-linearity in some tide-gauges, as distinct from linear response.

- Bottom pressure records have lower atmospheric noise levels than records of surface elevation, because (to a first order) sea level yields to surface pressure as an inverted barometer, leaving the total mass water-air unchanged.

DR SHOWELL. - Dr ROSSITER inferred that one might expect a greater need for maintenance with increased complexity of the equipment. Using techniques developed for computers, solid state digital recorders can be constructed that will operate for long periods of time with great reliability. When we first started digitizing records we always included an analog recorder to check the operation of the digital equipment. We soon found that we were using the digital equipment to check the analog recorder and patch the analog record during its frequent failures. After a few years we eliminated the analog recorder and if such records were required, they were produced by the computer from the digital data.

THE EFFECT OF ICE UPON SEA LEVEL RECORDS

Eugenie LISITZIN

(Institute of Marine Research, Finland)

The effect of ice upon sea level records may be considered from two different points of view. On the one hand, we have to take into account the direct influence of freezing upon the functional reliability of the tide gauge. In the following this effect may be described as the technical ice disturbance. On the other hand, we cannot overlook the effect upon the variation in sea level caused by water transformation from the liquid into the solid phase in the sea off the tide gauge. It shall be referred to this effect as the geophysical perturbation caused by ice.

The disturbing technical effects mentioned in the following are to a considerable degree based on the experience from Finland, where the time for ice occurrence in the particular harbours varies, on an average, between four and seven months per year. At the beginning and the end of the winter period the ice in the vicinity of the tide gauges consists of sludge or drift ice, during high winter of a fast ice cover which stretches along the whole coast. All the Finnish tide gauges being of the float type, special attention must be paid to prevent the freezing of the well. This task should not seem to be very difficult in Finland where the sea level stations consist of tiny, but stably constructed buildings which are heated, as a rule, by electricity. Nonetheless, the human factor must always be kept in mind. In spite of detailed instructions forwarded to the caretakers of the sea level stations, it occurs from time to time that an unexpected decrease in air temperature below the freezing point, especially during night, surprises the caretaker in such a way, that he fails to switch on the electric heating early enough to prevent the freezing of the well and consequently gaps in the records. However, it is fair to point out in this connexion that the caretakers of the tidal stations in Finland have other duties too, sometimes rendering difficult the proper attendance to the sea level recording gauge. In any case the effect of this human factor may be eliminated by the installation of thermostats in the gauge station.

Another phenomenon which is characteristic of one of the Finnish tide gauges almost every winter is the penetration of ice sludge into the pipe which connects the well with the sea. The consequences of this penetration are displaced, retarded and damped records. In this case it is the unfortunate choice of the location of the sea level station at the mouth of a river which is partly responsible for the disturbing effect.

Taking into consideration the fact that a stable, heated construction is not always sufficient to protect the tide gauge against the influence of water freezing in the well and thus ensure uninterrupted records, it is self-evident that the difficulties increase considerably as soon as the possibility to shelter the apparatus properly diminishes in connexion with the use of transportable tide gauges. Of course, this kind of gauges, usually of the float type with the well filled with rock-oil, may function fairly satisfactorily, if the gauge is erected in order to provide data for the determination of tidal constituents during the restricted time of approximately one month, as the period of the most favourable weather and ice conditions may be chosen for this recording. The numerous observations performed by the Norwegian Polar Institute in the Arctic region have thus been very yielding (1).

To the contrary, the endeavour to install under Finnish surveillance a tide gauge in Murchison Bay on Spitzbergen during the International Geophysical Year was not fruitful. The main causes for the negative result must be sought in the difficulty to find a suitable place where the gauge could be installed. It may suffice to mention a few of the adversities which befell the tide gauge preventing continuous and reliable recordings. Owing to cold weather and
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Taking into consideration the fact that a stable, heated construction is not always sufficient to protect the tide gauge against the influence of water freezing in the well and thus ensure uninterrupted records, it is self-evident that the difficulties increase considerably as soon as the possibility to shelter the apparatus properly diminishes in connexion with the use of transportable tide gauges. Of course, this kind of gauges, usually of the floater type with the well filled with rock-oil, may function fairly satisfactorily, if the gauge is erected in order to provide data for the determination of tidal constituents during the restricted time of approximately one month, as the period of the most favourable weather and ice conditions may be chosen for this recording. The numerous observations performed by the Norwegian Polar Institute in the Arctic region have thus been very yielding (1).

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The penetration of snow into the apparatus did not function properly. The floaters were bent several times. Water at the bottom of the pipe was present as a layer of snow was frozen during long periods in spite of the use of rock-oil and other efforts made to prevent this kind of icing. The consequence of this freezing were straight horizontal lines in the records, as soon as the floaters touched the ice. Considerably more difficult and uncertain was the explanation of the straight horizontal lines corresponding to high water and characteristic of the removal of ice from the gauge. It is, however, not excluded that in these cases we must conclude with the possibility that the friction of fast ice extending along the coast was frost bound to the rock on which the gauge was erected. This immobile fast ice cover prevented more or less the increase in sea level as soon as the rising water reached the lower surface of the ice. This possibility will be referred to later in this paper.

Considerably more successful were the efforts made by the French polar expedition to provide sea level records from the Antarctic (3). For instance, during the time covering roughly 21 months the expedition worked at Port Martin in Terre Adélie there were reliable records for approximately 12.5 months divided into three separate periods. The records of the recording were more satisfactory for the austral winter, when the tide gauge could be installed on the fast ice, than during the short austral summer, when melting and disappearance of the ice cover forced the gauge to be transported to a rock on the coast.

The description of the functioning of this tide gauge points out to similar difficulties, as have already been mentioned above. During the short time the tide gauge was installed on the rock ice blockade and freezing in the pipe serving as floater well caused gaps in the records. During the period the gauge was erected on the ice, it was the ice which supported the instrument that took the function of the floaters and the water masses below it the function of the well. The danger for the freezing of the well was therefore eliminated. There occurred, however, numerous breakings of the wire which by means of a weight closed the tide gauge with the sea bottom. The installation of the gauge on the ice caused as a consequence of these breakings and of ice melting changes in the recorded sea level and these changes could be evaluated only by means of repeated levellings to a bolt on a near-by rock. Moreover, in spite of the double case for thermal isolation protecting the apparatus, an additional cover and other preventive measures the tide gauge clock did not work properly owing to the cold and it was necessary to install electric heating.

In all the cases described above gauges of the floater type were used for the records. With the exception of Canadian recording in Arctic waters there was in available literature no indication of pressure gauges in ice covered regions. Other possible types and gravity measurements of ice must, however, always be kept in mind.

The examples mentioned above and selected at random may suffice to show that sea level records in ice covered regions require not only a strong and adequate equipment, but need, in addition, a close and continuous attention from the part of the caretaking personnel. Nevertheless, also in the cases where these two factors are accounted for in a satisfactory way, we have to take into consideration the existence of phenomena which have characterised the first investigations of the problem of the effect of ice upon the sea level represented by tide gauge records. As starting point it may be assumed that ice formed at the surface of the sea and remaining in a floating position in the vicinity of the tide gauge does not affect the sea level. Snow influence in a similar way the ice covered and the ice free sea surface and its effect is thus comparable to that of precipitation during the ice free time. Of course, as a consequence of hummocking, melting of ice and other changes the isostatic line in the ice cover will not correspond strictly by the sea level. However, this disagreement cannot influence the sea level recorded in the tide gauge well.

They may, nonetheless, arise situations where the continuous ice cover or the drifting ice cause sea level variations which differ from those recorded in connexion with the ice free environment. According to Russian investigations (5) the amplitude of the sea level fluctuations of the Baikal lake is in winter almost two times as large as in summer. This effect is connected with the fact that the temperature of the lake in winter is about 2°C lower than in summer.

Studies of the piling-up of water caused by wind in the Gulf of Bothnia have distinctly shown that a continuous cover of fast ice in high winter has a fairly strong reducing effect upon sea level fluctuations (2). In some extreme cases the piling-up effect completely failed to appear. An example of strong winds. The consequence is that the considered less level variation in the Gulf of Bothnia diminishes with accreting ice cover also in the areas where no decrease in the average wind force and the occurrence of gales may be noted. These results seem to be partly convincing and may therefore be extended to other regions.

According to Goertz (6) the amplitude of sea level in the White Sea is in summer almost two times as large as in winter. In particular cases with an extensive ice cover the recorded amplitude of the tide was only roughly 25 per cent of the expected amplitude. The decrease in amplitude is caused by the diminution of the average speed of the tidal current below the ice as a consequence of friction and other effects, such as mowing, bending and deformation of the ice cover due to the influence of the tide generating force. It was noted that the winter situation continues, as a rule, until the complete disappearance of the ice cover from the passage called "Golfo" which means neck and characterises the shape of the transition area connecting the White Sea with the Barents Sea. This feature is explained by the fact that the tides in the White Sea are the result of a tidal wave progressing through the Golfo. The ice cover has not only a reducing effect on the amplitude of the tides, it could be calculated that the times of high and low tides in the Arctic during the period of a practically complete ice coverage are approximately one hour retarded in comparison with the times during the summer season. Already these results indicate that the tides may sometimes require a separate determination of the harmonic constants for the ice free season, on the one hand, and the ice covered period, on the other hand.

There is an additional significant point to be taken into consideration in this connection. Quite special attention must be paid to the influence of the continental or glacier ice reaching in its movements the sea. I do self-evidently not refer here to the effect of the more or less continental Ice Increase in sea level caused by the present melting of this Ice, although more prolonged series of sea level observations on a world-wide scale offer an appropriate basis for the study of this phenomenon and the determination of the average rate of the increase in mean sea level. It must, nevertheless, always be kept in mind that ice shelves flowing slowly from the century-old land ice areas such as the inland ice of Greenland and the north-western part of the Canadian mainland, are sea level lowering effects certainly much stronger than the ones caused by the continental and mountain glaciers which in the Antarctic region may at rare occasions be as large as the whole state of Connecticut or approximately 12,000 square kilometers (4). The immediate effect on sea level fluctuation of such a pronounced addition of water, which, however, in solid phase, must be simply overwhelming in the neighbourhood of the calving place.

The contribution of the icebergs to the variation in sea level may be significant from another point of view, too. It is self-evident that icebergs and, of course, also drifting ice of both poles are highly subjected to changes which, blowing away parts of the pack or breaking up the top surface, or even parts of the ice field, drives it ahead. Icebergs and drift ice transported for considerable distances by the action of wind or by permanent, tidal or other currents may to a high degree cause changes of sea level. Depending on the area and type of movement of the ice mass, the effect of such icebergs and drift ice, as it is known from the studies of the British and Russian investigations (5), may be considerable. An ice mass, drifting out of the ice-covered region and then reaching the open ocean, may cause a considerable increase in sea level. On the other hand, an ice mass, reaching the coast after a long drift, may cause a considerable decrease in sea level.

The effect of cold icebergs on the sea level in the White Sea, as it was mentioned above, the effect of Ice in the Golfo upon the tides in the White Sea was also referred to above.
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The few examples mentioned above and selected at random may suffice to show that sea level records in ice covered regions require not only a strong and adequate equipment, but need, in addition, a close and continuous attention from the part of the caretaking personnel. Nevertheless, also in the cases where these two factors are accounted for in a satisfactory way, we have to take into consideration the existence of phenomena which have characterised as separate effects the problem of the effect of ice upon the sea level represented by tide gauge records.

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They may, nonetheless, arise situations where the continuous ice cover or the drifting ice cause sea level variations which differ from those recorded in connexion with the isostatic distribution of the meteorological elements, especially wind, during the ice free months.

Studies of the piling-up of water caused by wind in the Gulf of Bothnia have distinctly shown that a continuous cover of fast ice in high winter has a fairly strong reducing effect upon sea level fluctuations (2). In some extreme cases the piling-up effect completely failed to appear because of strong winds. The consequence is that the measured less level variation in the Gulf of Bothnia diminishes with accelerating ice cover also in the cases where no decrease in the average wind force and the occurrence of gales may be noted. These results seem to be pretty convincing and may therefore be extended to other regions.

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There is an additional significant point to be taken into consideration in this connexion. Quite special attention must be paid to the influence of the continental or glacier ice reaching in its movements the sea. I do self-evidently not refer here to the effect of the more or less continuous "piling up" of the ice. The increase in sea level caused by the present melting of this ice, although more prolonged series of sea level observations on a world-wide scale offer an appropriate basis for the study of this phenomenon and the determination of the average rate of the increase in mean level. It must, nevertheless, always be kept in mind that ice shelves floating slowly from the century-old icecap which, blowing against the coast and even parts of the ice field, drives it ahead, Icebergs and drift ice transported for considerable distances by the action of wind or by permanent, tidal or other currents may to a high degree cover a picture of sea level fluctuations, if large quantities of this ice are moved to regions with only seasonal or occasional occurrence of ice. For instance, the discrepancy in the area around Newfoundland between the variations in the recorded sea level, caused as it is to the influence of atmospheric pressure, and those based on observations of water density, the so-called steric sea level, especially during the earlier half of the year, may at least partly be ascribed to the occurrence of icebergs in the region during this time. In addition, the melting of the ice, both of continental and maritime origin, during its drift far outside the region of its formation, means a decrease in surface water salinity and its influence on sea level must therefore be taken into account also in this way.

Accumulation of drift and pack ice in sounds and other narrow and shallow passages may shut off much less completely semi-enclosed areas from the immediate connection with the larger parts of the sea basin and thus considerably affect sea level variations in the isolated region. This phenomenon is to some degree a parallel on a larger scale to the penetration of ice into the sound connecting the sea with the enclosed water body, as mentioned above. The effect of ice in the Gorlo upon the tides in the White Sea was also referred to above.
A few additional cases of more restricted and local character may be mentioned:

If the fast ice fringe is firmly fast bound to the rocky coast or to the coastal construction which supports the tide gauge, this ice may remain suspended when the sea level decreases causing in this way an additional erroneous decrease in the records. When sea level again increases the immobile ice cover may act like a roof impeding the proper recording of the rising sea level. There was probably such a rare case in the records from Murchison Bay mentioned above.

Sometimes, however, the increase in sea level is so pronounced that sea water rises on the continuous ice cover and it is by no means excluded that it remains in a concavity in this ice when the sea level retires. Also in this case the general picture of sea level variation will be more or less distorted.

Finally, we may not overlook the possible occurrence of bottom ice in the neighbourhood of a tide gauge. This phenomenon may also have a disturbing effect upon sea level records. To the contrary, we hardly need consider cases of freezing of the whole water masses from the surface to the bottom, as these cases probably occur in practice only in comparatively shallow coastal areas which are not suitable for the erection of a tide gauge.

It is hardly possible to illustrate within the restricted frame of a short paper all the various aspects and phenomena connected with the problem of the effect of ice upon sea level records. Therefore, it seemed adequate to confine the presentation to a few essential points of view in order to demonstrate the numerous and varying difficulties characteristic of sea level observations in region with a permanent or more or less temporal ice occurrence. It may be appropriate to summarise these points of view once more:

The construction of the tide gauge must be very strong and the instruments well protected against the influence of cold weather, snow gales and ice pressure.

The caretaking personnel must be alert and interested in the work.

The selection of the place where the tide gauge shall be installed is a factor of greatest significance. This place must be sheltered as much as possible against wind action and the risk of getting the apparatus damaged through the effect of drift ice pressure. Small bays connected with the sea by narrow and shallow passages must be avoided, if possible. The sea depth off the tidal station must not be too restricted.

Finally, it must be emphasized that in spite of numerous difficulties and disturbances mentioned above further studies in the concerned research field must not be discouraged. To the contrary, this important work must be continued with joint efforts aiming at the improvement of the instrumentation and endeavouring to enlarge our present knowledge of tidal and other sea level variations and the average sea level in the Arctic and Antarctic regions. The general picture of these phenomena will not be complete until the present numerous gaps in this knowledge are filled.

BIBLIOGRAPHY

(1) HORNBAEK, HELGE - Tidal observations in the Arctic 1946-52. Norsk Polarinstitutt, Skrifter Nr. 104, Oslo 1954.

(4) WITTMAANN, WALTER I. - Polar Oceanography, Ocean Sciences, United States Naval Institute 1964.
(5) ZUBOV, N.N. - Arctic Ice, Section 120: Influence of Ice on Tidal Phenomena, Edited by U.S. Navy Electronic Laboratory, Original Russian edition Moscow 1943.
A few additional cases of more restricted and local character may be mentioned:

If the fast ice fringe is firmly frost bound to the rocky coast or to the coastal construction which supports the tide gauge, this ice may remain suspended when the sea level decreases causing in this way an additional erroneous decrease in the records. When sea level again increases the immobile ice cover may act like a roof impeding the proper recording of the rising sea level. There was probably such a rare case in the records from Murchison Bay mentioned above.

Sometimes, however, the increase in sea level is so pronounced that sea water rises on the continuous ice cover and it is by no means excluded that it remains in a concavity in this ice when the sea level retires. Also in this case the general picture of sea level variation will be more or less distorted.

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OBSERVATIONS SUR LA COMMUNICATION DU Dr LISITZIN

M. CODIN. Les rivieres a marée, sujettes a glaciation, supportent de fortes perturbations ; mais ces perturbations tendent à brouiller les données plutôt qu'à se traduire par un effet net et bien défini.

Ainsi avons-nous l'habitude de ne pas utiliser les observations hivernales pour les analyses.

Dr DOHLE. The differences of up to one hour have not been found in the canadian arctic.

Dr LISITZIN : This information was given from an outside source.

Dr HELA. The difficulties in the use of the tide-gauge at Murchison Fjord, Spitsbergen, 1957-58, by the Swedish-Finsten-Swiss I.G.Y. Expedition were based on the fact that it was practically impossible to find a solid place, near the camp, which would have been unaffected by the hummocking drift-ice.

On the other hand, the meteorologists did not have time, because of other numerous duties, to control the datum height frequently enough.
PRESSURE AND FLOAT OPERATED GAUGES DEVELOPED FOR THE TIDES,
CURRENTS AND WATER LEVEL SECTION OF THE CANADIAN HYDROGRAPHIC SERVICE

The Use of Pressure and Float Operated Gauges in the Canadian Arctic

G. DOHLER
Department of Mines and Technical Surveys (Canada)

THE N.R.C. PRESSURE-TYPE TIDE RECORDER

INTRODUCTION

The tide recording system was developed by the National Research Council of Canada at the request of the Canadian Hydrographic Service, Department of Mines and Technical Surveys, because of the difficulty that survey parties were encountering in the Arctic in setting up supporting structures required for the conventional float-type tide recorder. These supporting structures were, in addition, very susceptible to damage or destruction by drift ice. The requirement, therefore, was to develop an instrument that did not need a supporting structure above the water surface.

A specification for the instrument was evolved in collaboration with the National Research Council of Canada and details of this specification were as follows:

1/ An instrument to measure the long-period rise and fall of the tide in areas subject to surface ice.

2/ A sensing unit for underwater use, suitable for operation at water depths up to 75 feet (submerged depth plus tide height), capable of long periods (months) of operation without attention.

3/ An automatic recorder ashore, at a distance up to one mile from the sensing unit, which would record the rise and fall of the tide on a continuous strip chart. The recorder to be capable of running for at least four weeks without attention.

4/ The maximum permissible error for tides of 20 feet or less to be 1/4 foot, and for tides from 30 to 55 feet, to be 1/2 foot.

5/ Provision to be made for automatic correction of tidal readings for changes in atmospheric pressure.

6/ Error in the chart record time base to be not greater than five minutes per week.

7/ Chart speed to be 10 mm per hour.

8/ The recorder to be able to operate over the temperature range from -30° to +30° C.

9/ The seiche, with periods as low as five minutes, to be recorded.

10/ The power supply to be rechargeable batteries, or some other suitable portable source.

PRINCIPLE OF OPERATION

The instrument determines the water height by measuring the hydrostatic pressure at some point below the water's surface with a resistance potentiometer-type absolute pressure transducer. Pressure changes are converted to resistance changes in this transducer by means
of a simple linkage system which connects the sliding arm of the transducer resistance potentiometer to a pressure-sensitive metal diaphragm. The position of the slider of the potentiometer, which is set by the total pressure applied to the transducer (atmospheric pressure plus water height), is obtained by having this potentiometer form part of a servo-balanced resistance bridge circuit. The servo system in the shore recorder senses the imbalance current in the bridge circuit caused by movement of the transducer potentiometer slider and it then drives a precision 10-turn potentiometer in the correct direction to restore the bridge to balance. Atmospheric pressure variations which affect the total pressure measured by the submerged pressure transducer are compensated by a potentiometer-type pressure transducer in the recorder unit. This transducer has two potentiometers, each operating in adjacent arms of the resistance bridge circuit in such a manner as to cancel the effect caused by the atmospheric pressure changes on the submerged sensing unit.

THE OTTIBORO PRESSURE RECORDER

The unit was developed by the Tides and Water Levels Section for specific reasons. By using a Strip Chart Drive the tidal or water level data can be obtained and put on IBM Cards by our Data Processing Machine. Incorporating a pressure sensitive element attached to the pen allows water level recordings without the necessity of a sitting well. The recorder can be set as far as 500' from the Underwater Diaphragm Box. This is a particular advantage where there is a gently sloping shoreline and large tides.

The recorder operates on a pressure differential and consists of:
1/ The underwater pressure chamber.
2/ Copper capillary tubing.
3/ Pressure sensitive drive element.
4/ Pen and pen drive.
5/ Recording chart and chart drive.

The underwater pressure chamber is connected by the capillary tubing to the pressure sensitive element. The air is sealed in the system and any change in the external pressure on the underwater unit is transmitted to the element. The element is connected by a linkage system to the pen and any change of internal pressure on the element drives the pen across the chart thus leaving a permanent record. Each recorder is calibrated for the range specified and is accurate to within 2% of its range.

SHORE TO SHIP TRANSMISSION OF WATER LEVELS

Successful experiments with the transmission of water level data over long distances, using metallic conductors have been carried out over the last two years.

The frequency variation principle was employed for this purpose since it is independent of input levels at the receiver side and permits a simultaneous transmission of several data on one channel. These experiments have been extended to include shore to ship transmission via a wireless carrier.

The device is completely transistorized except for the radio frequency carrier where vacuum tubes, etc. are used.

A block diagram of both the transmitter and receiver is shown in figure 1.

METHOD OF OPERATION

The frequency variation principle works the following way: A DC value available is converted into an audio frequency transmitter into an AC value whose frequency depends on the magnitude of the DC value. The total range to be measured, therefore, is transformed into a frequency range. This alternating current is then utilized for the transmission of the measuring data. The medium for transmission can either be a pair of metallic conductors or a radio frequency carrier which is modulated by this alternating current. At the receiving end, the signal is demodulated and a linear relationship between the frequency of the alternating current and the magnitude of the DC current is restored.

For the transmission of the measuring data, a relatively broad frequency band is used. The reason for utilizing a broad band is to obtain the smallest possible time constant for the transmission of the measuring value, and to introduce the smallest possible error in the transformation from AC to DC. On the other hand the frequency band employed is not too wide to allow utilization of several channels over the audio frequency range.

THE TRANSMITTER STATION - See figure 2 and 2a

The transmitter station consists of:
1/ originator
2/ audio frequency transmitter
3/ radio frequency transmitter

THE ORIGINATOR

The originator comprises a mechanical measuring device which is directly coupled to a precise, sliding brush, potentiometer. Any change of the mechanically measured value alters the position of the sliding brush in the potentiometer. The potentiometer is in series with a bridge circuit and a direct current power supply. The bridge circuit is used for calibration purposes. The controlled electrical potential from the potentiometer is fed through an indicating instrument to the audio frequency generator.

Any movement of the sliding brush of the potentiometer results in a change of electrical potential through the indicating instrument, causing a movement of the pointer in the instrument and a change in audio frequency, simultaneously.

THE AUDIO FREQUENCY TRANSMITTER

The plug-in units of the frequency transmitter are shown in figures 1 and 2. It is the task of the tone frequency generator to transform the measured direct current potential supplied by the Originator into alternating current. A direct current compensator acts as a control to maintain a smooth D.C. signal, regardless of small fluctuations in the A.C. line voltage or high peak D.C. voltage.

The direct current signal voltage is fed to coils of an oscillator circuit which vary their inductive value in relation to the direct current applied. This change in inductive value causes a change in the oscillating frequency. This oscillating frequency is in the audio range of alternating current.

However, the connection between direct current and frequency is not linear, but by degeneration the desired linearity is obtained.

The audio frequency is fed through an amplifier to the input of the radio transmitter.
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THE RADIO TRANSMITTER - See figure 3

The balanced signal from the audio frequency telemeter is fed into a matching input transformer to drive a transistorized power modulator and to provide approximately 12 watts audio power to the final R.F. amplifier in plate modulation.

A radio frequency is generated by a 3rd overtone precision crystal at 46,580 MC driving a 6 C 4 tube. The crystal is oven controlled with a tolerance of better than 0.001 %. The output from the oscillator tube 6 C 4 is driving a buffer amplifier comprised of a 6 C L 6 tube. This tube in turn drives the R.F. power amplifier tube type 6146 to approximately 20 Watts R.F. output. The R.F. output tube feeds a pinetwork and a coaxial sleeve antenna having a 52 Ω impedance.

THE RECEIVER STATION - See figure 4 and 4a

The receiver station consists of the
1/ radio frequency receiver
2/ audio frequency receiver and chart recorder

THE RADIO FREQUENCY RECEIVER - See figure 5

The signal picked up by the coaxial sleeve antenna is fed into a low impedance tap of the first R.F. amplifiers and the output is mixed with a 36 MC signal of the crystal oscillator to yield a first intermediate frequency of 4.5 MC. This 4.5 MC signal is coupled into the second mixer tube 6 A 3 B and mixed with the signal from the second oscillator. The result is a second intermediate frequency of 455 kc and is amplified with tube 6 BA 6. Part of tube 6 BC 7 detects the audio and AVC and acts as first noise limiter. Tube 6 AC 5 acts as second noise limiter, while the pentode section of the tube 6 AN 8 acts as automatic gain control amplifier and audio voltage amplifier. The audio signal is coupled to triode portion of 6 AN 8 and this in turn is driving a single 12 A U 7 tube in push-pull class A.

THE AUDIO FREQUENCY RECEIVER - See figure 4a

The incoming alternating voltage from the radio frequency receiver is amplified. It then is guided a frequency to DC converter which operates according to the method of inverting charged condensers. In its construction it corresponds exactly to the arrangements of the transmitter, except that it serves here for the evaluation of the arriving frequency.

The DC voltage passed on to the transformers is connected in series with a direct current voltage compensator for better measuring performance with the chart recorder.

Construction of the audio frequency transmitter and receiver is such that each element performs only a certain function. Plug-in units performing the same functions at both the transmitter and receiver are interchangeable. The units are arranged on base plates with the input and output on a terminal board provided with jumpers and therefore their interchange is easily facilitated.

CONCLUSION

Several field tests have been made to check the accuracy and reliability of the radio equipment. In the Bay of Fundy, where one of our latest tests was carried out, a coverage of up to 50 nautical miles was obtained. These results show that in a not too distant future, installations to transmit water and tide levels over great distances are not difficult to achieve and will form an integral part in automatic data processing for the benefit of tide and water level forecasts, navigators, hydraulic engineers and hydrographic surveyors.
THE OTT-BORO GAUGE

- Chart hold down clips
- Clock start lever
- Chart adjustment knob
- Pen zero adjustment
- Pen linkage

Recorder to be levelled to maintain proper pen pressure on chart.
Chart hold down clips

Clock start lever

Chart adjustment knob

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MAXIMUM TUBING LENGTH CURVE
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0.080 ID TUBING
(No allowance for temperature.)

NOTE
Always use 0.080 ID tubing in preference to 0.040 ID tubing if there is any choice.

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THE OTT-BORO GAUGE
OBSERVATIONS SUR LA COMMUNICATION DU Dr DOHLER

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Dr DOHLER. This does not apply to the first instrument (N.R.C.) with respect to the OTT-BOBO, the pen is just touching the paper and the seiching may be adjusted at the underwater pressure unit.

$ 1000 appr. for N.R.C.
$ 300-400 for OTT-BOBO
$ 4000-5000 for Radio Transmission

Some literature is available and may be obtained through the Canadian Hydrographic Service.

OBSERVATION OF SEA LEVEL AT REMOTE ISLANDS

Gordon W. GROVES
Hawaii Institute of Geophysics
University of Hawaii

Continuous marigrams have been obtained at remote islands during the International Geophysical Year by instruments requiring neither a harbour, electricity, nor a pre-existing artificial structure. Long syphons enabled recording of tide at exposed site outside coral reef by an instrument located in a sheltered place. Excellent results were achieved by native observers trained to operate the simple recording apparatus. Apparently the system would have been able to operate continuously for several years. The choice between an electronic or a mechanical system depends on a number of factors, but it is noteworthy that even today some combinations of circumstances (including remoteness) would still favour the traditional, mechanical method.

The response of the syphon-standpipe system suppressed periods below 15 minutes. If only long-period (day-to-day) variations are to be studied, the recording device could be replaced by a more severe analogue low-pass filter and periodic tabulation by an observer.

Electronic instrumentation has advantages over mechanical systems in cases where the instrument is to run either untended or under the attention of competent technicians. In cases where personal attention is available but by people not technically trained, such as on many inhabited South Seas islands, simple mechanical systems might be advantageous. The system discussed here makes use of a traditional float type water-level recorder with a strip chart moved by a mechanical clock mechanism. There are many satisfactory models available, the selection of which is not pertinent to the present discussion.

The essential feature of the system under consideration is the use of a long syphon to enable recording of sea level at an exposed site by an instrument located in a sheltered place. In 1955 one such system was installed at Canton Island, and during the International Geophysical Year similar systems were installed at Hull Island in the Phoenix Group and at Arorae Island in the Gilberts.

The low frequency component at Canton (outside the reef) appeared identical with that derived from the U.S. Coast and Geodetic Survey gauge in the lagoon, and the expected phase difference in the tide related to lagoon response was observed. Two years of good sea-level records were obtained from Arorae, and apparently several additional years of record could have been obtained with little additional effort. Nine months of good record were obtained at Hull, after which the installation suffered a direct hit from a 70 foot coconut log washed up by the most severe westerly storm in a decade. It would be possible to keep such a system in operation indefinitely with preventative maintenance at yearly or so intervals. The success is greatly enhanced by ensuring that the observer has a clear understanding of the system. A simple manual of operation prepared in the local language has proved to be very helpful, if not essential.

The syphon type installation may be useful in various coastal environments, but is particularly well suited to coral atolls, of which the above mentioned places are examples. The syphons actually used were plastic garden hose of 1/2 inch inside diameter, but other types of tubing would be better if a more permanent installation were desired. A length of 300 feet reached from the offshore "pickup" location at a depth of 20 to 40 feet outside the reef to the
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Although comparative readings with a tide staff are probably not feasible, the tee fitting allows an easy method of levelling in the marigrams with benchmarks. A transparent hose is attached vertically to the standpipe and connected to the tee fitting and opened to the standpipe. Flow through the syphon is closed off. The level traced on the marigram is then the same as the level easily observed within the transparent hose. If the time constant of the system were so long that the tides were attenuated to a negligible amplitude, then weekly or semimonthly readings would be an adequate representation of the system's output. Filloux and Groves (1960) describe a two-stage hydraulic filter that can be used in this way. There is no need for a recording device, but the tides and higher frequency oscillations are not obtained. The system is easily installed on a rocky coast or coral atoll without a preexisting artificial structure.

ACKNOWLEDGEMENTS

The author expresses his appreciation to Walter Munk, William Van Dorn, and to the Government and citizens of the Gilbert and Ellice Islands Colony.

REFERENCE

standpipe and recorder, safely located on the reef flat or near the beach (see diagram). The hose was taped to a small multistrand steel cable which was stretched between the two points and fixed at intervals to spikes in the reef to prevent longshore motions by currents. A filter and sediment trap on the outer end of the syphon was used to keep out debris.

The standpipe must be imbedded in the extremely-hard coral reef material to a depth exceeding the lowest expected stand of sea level. The necessary excavation could be done with difficulty by a jack hammer if available. An easier way is to make use of shaped charges. Two forty-pound pentolite shaped charges made a satisfactory hole for the standpipe. The first charge was placed for maximum penetration and produced a narrow hole of sufficient depth (about six feet). The second charge was placed at the mouth of the hole but too close for good penetration, in order to widen the hole. Excavation by hand was then carried out by a group of laborers beginning on the falling tide. There was just enough time to place the standpipe and imbed it in concrete before the next high tide. Air and bubbles were removed from the syphon by a hand pump used at the shore end. A tee fitting with valves on all three legs facilitated this operation.

Once in operation the water level in the standpipe follows closely the water level at the outer end of the syphon. It is easily shown that the viscosity of sea water and the dimensions of the system result in laminar flow through the syphon, even at times of maximum inflow and outflow. Thus the response is linear, which is a substantial advantage over "standard" type installations which use an orifice to reduce high-frequency oscillations. The "time constant" of the syphon system is given by:

\[ \tau = \frac{0.44a}{\nu g} \]

where \( \nu \) is the kinematic viscosity of sea water (0.01 cm² sec⁻¹), \( a \) is the length of the hose, \( a \) the inside radius of the standpipe, \( r \) the inside radius of the hose, and \( g \) the acceleration of gravity (10 cm sec⁻²). In the installations described, \( a, a \) and \( r \) had values of 500 feet, 4 inches and 1/4 inch, respectively, and the resulting theoretical time constant was 13 minutes. The attenuation factor for a sinusoidal wave of period \( 2\pi/\omega \) is given by:

\[ F(\omega) = (1 + \omega^2 a^2)^{-1/2} \]

and the corresponding time lag is:

\[ T(\omega) = \frac{\omega}{\omega'^2 \arctan \omega'} \]

The time constant is easy to evaluate empirically by filling the standpipe with sea water and then opening the valve and letting the water run out through the syphon. The observed exponential drop in water level observed on the resulting marigram had a time constant \( \tau \) (the time taken for the water level to fall to 1/e of the original head of water) of 17 minutes, in fair agreement with the 13 minutes obtained by the formula.

The lag and damping of day to day variations of sea level can be entirely neglected. A small correction for the tides can be applied, if desired. For the semidiurnal constituents, \( T = 15 \) minutes, \( F = 0.92 \); for the diurnal constituents, \( T = 15 \) minutes, \( F = 0.98 \). These values are based on an assumed \( \tau = 15 \) minutes.

It seems likely that the time constant might increase with time due to biological fouling on the inner surface of the syphon, thus decreasing the effective value of \( \tau \). No such effect was observed, however, during the time the gauges were in operation.

A temperature effect due to solar heating of the sea water within the syphon at low tide is to be expected. Judging from the extremely hot puddles of water found on the reef at low tide, a temperature difference of 20 to 30 degrees centigrade between the water in the exposed hose and the buried standpipe might occur. Taking the thermal relative volume expansion coefficient of sea water to be \( 2 \times 10^{-4} \) per degree centigrade, and the maximum vertical column of heated water to be 2 meters, we arrive at a maximum possible error in sea level of one centimeter, while a representative value would be much less.

Although comparative readings with a tide staff are probably not feasible, the tee fitting allows an easy method of levelling in the marigrams with benchmarks. A transparent hose is attached vertically to the standpipe and connected to the tee fitting and opened to the standpipe. Flow through the syphon is closed off. The level traced on the marigram is then the same as the level easily observed within the transparent hose.

If the time constant of the system were so long that the tides were attenuated to a negligible amplitude, then weekly or semimonthly readings would be an adequate representation of the system's output. Pilloux and Groves (1960) describe a two-stage hydraulic filter that can be used in this way. There is no need for a recording device, but the tides and higher frequency oscillations are not obtained. The system is easily installed on a rocky coastal area without a preexisting artificial structure.

ACKNOWLEDGEMENTS

The author expresses his appreciation to Walter Munk, William Van Dorn, and to the Government and citizens of the Gilbert and Elice Islands Colony.

REFERENCE

The U.S. Coast and Geodetic Survey's tide observation program is predicated on the need to determine tidal constants, to determine and monior tidal reference lines, and to study tidal propagation in inshore and oceanic areas. To fulfill these needs we maintain a standard set of tide stations; we supplement this set with temporary installations at points of special or immediate interest; and recently we have initiated a program for offshore observations.

The advent of computer technology has led to an enhanced capacity for detailed analysis of tidal time series. The demand for tidal data has risen sharply; not only as to number of requests, but also as to the detail requested. It is obvious from this that we must automate our data file, storage, and retrieval procedures. Then to maintain efficiency in our entire tidal program it is necessary to convert the major part of our standard tide net to digital recording gages. In selecting instruments to effect this conversion we have considered not only our own data processing responsibility but also our responsibility to provide tidal data in a usable format for diverse investigations. Obviously we cannot justify unilateral decisions on data formats; just as obviously the selection of a format cannot now be final.

The instruments selected for our standard network provide the necessary ease of format conversion. The gage is a float-operated water level recorder manufactured by Fischer and Porter Company, and is available as a stock item. (ANON, 1963). The recording medium is a special-purpose punched tape, providing easy visual read-out (Figures 1 and 2). The recorder converts the angular position of a shaft into a coded digital output. In the code employed, sixteen positions are required to record four place numbers. (Figure 3). All parts of the recorder - interval timer, encoder disks, punching and reset mechanism, and paper drive - are mechanically synchronized to prevent error or ambiguity. Electric contacts are available to provide telemetry, although the efficiency of telemetering sixteen bits for a single variable might be questioned.

In Coast and Geodetic Survey use the base of the gage is adapted to fit a 4-inch (10 cm) float well, for use with a 3 1/4-inch (8.3 cm) float. At stations in the standard net an 8 1/2-inch (21.5 cm) float is used in a 12-inch (30.5 cm) well. A float wire storage drum is axially fastened to the gage drive shaft. The drum will store 50 feet (15 m) of wire, which is the range of the instrument. A counter-clockwise torque is provided by a constant torque spring loaded on the drive shaft. The torque necessary to rotate the shaft is about 1/4-inch oz., (2.10^2 dynes cm) and more important it is nearly constant through the range of the instrument. Because the input shaft is restrained for approximately eight seconds, it is necessary to provide a rotation storage device in the drive shaft. The lost motion coupler used consists of a dual torsion spring assembly, springs opposing. This coupler has the capacity to store water level changes of 1.8 feet (0.6 m) in either direction during the eight second recording phase.

The gage is supplied with a timing unit and interval timer, although the interval timer can be detached and any other unit used. The timer is an electrically wound clock. The recording interval is set by a simple cam. The cam makes a complete revolution each hour. For our use a cam with 10 contact-steps provides for 3/10 hour intervals.

In portable installations electric power is supplied by a dry cell battery. Requirements are 0.5 amps starting current and 100 milliamps running current from a 7 1/2 volt DC source.
Figure 1: The Fischer Porter digital water level recorder.

Figure 2: Digital water level recorder showing coding disks.
Figure 1: The Fischer Porter digital water level recorder.

Figure 2: Digital water level recorder showing coding disks.
Special purpose transducers are available to convert the punched tape records to cards or digital magnetic tape in a wide selection of formats. If a one-tenth hour recording interval is used, a one month gage record can be converted in about 35 minutes.

By the first of this year we had collected digital time series at several of our standard tide stations. The digital recorders were coupled to our standard analog recorders at these stations to secure dual records for evaluation. Our preliminary evaluation indicates that the digital record results are comparable to those from the analog gages.

Very briefly, our plans for automated processing of the digital records include: an editing program for all observations; extraction of hourly heights, high and low waters; and determination of the traditional tidal means (Figures 4 and 5). Programs are being selected for analyses to determine harmonic constants from tidal series of 29 days' to 369 days' duration.

For temporary or permanent installations where a float well and float cannot practically be used, we have adopted a pressure sensing tide gage. This gage, graphically illustrated in Figure 6, is similar to the one described by MIDFIELD (1960). The essential parts are: a source of gas of sufficient volume at pressure to supply the system for several weeks; controls for gas flow and pressure differential; a supply line to an escape orifice fixed below the lowest water level excursion; a temperature compensated bellows transducer; and an analog recorder. High frequency pressure variations caused by wind waves are partially eliminated by introducing capacity into the system; by means of a restrictive valve between the orifice and the transducer; and by regulating the supply rate.

In our installations dry nitrogen commercially available at 2000 pounds per square inch is used for gas supply. The endurance of the gas supply is a function of the volume of the supply system and the orifice cavity, the tide range, and the bubble rate selected. At Anchorage, where because of the 24-foot range and relatively exposed installation most of these factors are large, an eighty cubic feet tank provides for about four months' operation.
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Very briefly, our plans for automated processing of the digital records include: an editing program for all observations; extraction of hourly heights, high and low waters; and determination of the traditional tidal means (Figures 4 and 5). Programs are being selected for analyses to determine harmonic constants from tidal series of 28 days' to 369 days' duration.

For temporary or permanent installations where a float well and float cannot practically be used, we have adopted a pressure sensing tide gage. This gage, graphically illustrated in Figure 6, is similar to the one described by HUDFIELD (1962). The essential parts are: a source of gas of sufficient volume at pressure to supply the system for several weeks; controls for gas flow and pressure differential; a supply line to an escape orifice fixed below the lowest water level excursion; a temperature compensated bellows transducer; and an analog recorder. High frequency pressure variations caused by wind waves are partially eliminated by introducing capacity into the system; by means of a restrictive valve between the orifice and the transducer; and by regulating the supply rate.

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**TIDES, HOURLY HEIGHTS (FEET)**

WASHINGTON D.C. CASE A NOV 1964 TM 75.00 W

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SO 75.00 W 30 01 30 + 0014 210738

MSL 5.73

**Figure 4:** Portion of computer listing of hourly heights of tides.

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TIDES, HIGH AND LOW WATERS (FEET)
WASHINGTON D.C. CASE A NOV 1964 TM 75.00 W

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GT (DRL/TL TDL/TML/MSL)

GMHWI 0.27 HRS GMMLW 7.26 HRS

HIGHEST TIDE 8.55 10.1 HRS NOV 19

LOWEST TIDE 3.58 4.9 HRS NOV 22

** USED IN MEANS

SO 75.00 30 01 30 + 0014 210738

Figure 9: Computer listing of high and low waters and monthly tidal means.

Figure 6: The bubbler tide gage.

The pressure reduction valve is regulated to allow a constant pressure greater than that due to the maximum head anticipated over the orifice. The pressure differential regulator together with the shunt line provide for a constant pressure difference of about 3 pounds per square inch across the flow regulator. Thus the rate of flow can be adjusted to a desired constant value and is not dependent on the tide stage. Flow is through a silicone oil-filled bowl so that it can be monitored visually. Rates of from 30 to 300 bubbles per minute are used. (ASON, 1962).

Bellow transducers for measuring water level have been in use for many years. In most of them the transducer is placed at the sensing depth, and pressure variations are transmitted to the surface. Advantages of the present scheme are that the sensitive elements of the gage need not be designed to withstand the underwater environment; and fluctuations in barometric pressure are not directly reflected on the record.

A significant portion of the system capacity at the submerged orifice thus providing some high frequency filtering, and protecting the small diameter supply line and orifice from marine fouling.

A Bristol Company clock recorder provides an analog record. The record is on a 6-inch (15.2 cm) wide strip chart available in 10 through 50 ft. ranges. Paper advance is 1 inch (2.54 cm) per hour. The clock drive has an eight-day spring, and the record is sufficient for about six weeks. Manufacturer's claims are that hysteresis and/or nonlinearity are limited to 1% of the full scale value. For sea water use, instruments are calibrated to correspond to a specific gravity of 1.025. All of our tide gages are operated in conjunction with a fixed elevation tide staff, and periodic comparisons are made between staff and gage level readings. These intercomparisons are used to in some measure compensate for recorder discrepancies and variations in specific gravity form the 1.025 base.

We now use the pressure gage routinely for difficult installations. Adoption of this ins-
TIDES, HIGH AND LOW WATERS (FTET)
WASHINGTON D. C. CASE A NOV 1964 TM 75.00 W

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MHW 7.12 MLW 4.34 MSL 5.73 MN 2.78 MTL 5.73
MTL-MSL 0.00 MIHWW MLLW DBQ DLQ
GT (DRL/TL) (DRL/TL-MSL)
GMLH/W 0.27 HRS GMLW/1 7.26 HRS
HIGHEST TIDE 8.55 18.1 HRS NOV 19
LOWEST TIDE 3.58 4.9 HRS NOV 22
** USED IN MEANS

Figure 6: The bubbling tide gage.

The pressure reduction valve is regulated to allow a constant pressure greater than that due to the maximum head anticipated over the orifice. The pressure differential regulator together with the shunt line provide for a constant pressure difference of about 3 pounds per square inch across the flow regulator. Thus the rate of flow can be adjusted to a desired constant value and is not dependent on the tide stage. Flow is through a silicone oil-filled bowl so that it can be monitored visually. Rates of from 30 to 300 bubbles per minute are used. (ANON, 1962).

Bellow transducers for measuring water level have been in use for many years. In most of them the transducer is placed at the seafloor depth, and pressure variations are transmitted to the surface. Advantages of the present scheme are that the sensitive elements of the gage need not be designed to withstand the underwater environment; and fluctuations in barometric pressure are not directly reflected on the record.

A significant portion of the system capacity at the submerged orifice thus provides some high frequency filtering, and protecting the small diameter supply line and orifice from marine fouling.

A Bristol Company clock recorder provides an analog record. The record is on a 6-inch (15.2 cm) wide strip chart available in 10 through 50 ft. ranges. Paper advance is 1 inch (2.54 cm) per hour. The clock drive has an eight-day spring, and the record is sufficient for about six weeks. Manufacturer's claims are that hysteresis and/or nonlinearity are limited to 1% of the full scale value. For sea water use, instruments are calibrated to correspond to a specific gravity of 1.025. All of our tide gages are operated in conjunction with a fixed elevation tide staff, and periodic comparisons are made between staff and gage level readings. These intercomparisons are used to in some measure compensate for recorder discrepancies and variations in specific gravity form the 1.025 base.

We now use the pressure gage routinely for difficult installations. Adoption of this ins-
signal amplifier is a solid state, variable gain, wide band DC differential amplifier of exceptional stability. Secular drift is $0.02\%$ in 200 hours at constant temperature; temperature sensitivity is on the order of $0.001\%$ per degree centigrade. The power requirement is 1.8 watts. Power requirement for the entire package is about 2 1/3 watts.

A null circuit is employed to electrically cancel the transducer output, thus permitting a highly expanded record of nearly linear response. In the models used to date a strip chart recorder provides an analog record. Future modifications will provide for digital recording, at first by photographing a milliammeter, then later with the use of a high density digital magnetic tape recorder.

A transducer with the proper maximum operating depth is selected based on the depth anticipated on station. Within this inherent limit a wide range of actual observation depth can be tolerated without degrading the precision of the observation. This is possible because of the simple null circuit employed. With no signal the recorder is centered at zero. When the amplified pressure-dependent signal is sufficient to move the stylus to either recorder limit, a contact is activated; a simple servo mechanism displaces the stylus toward the center of the record. Thus, with continually increasing pressure the full recorder range will be repeated to correspond to each relatively small increase in pressure. In the operational models an increase in water depth of approximately 5 feet (1.52 m) corresponds to the 4-inch (10 cm) record width.

Aside from the high degree of compensation inherent in the transducer bridge circuit, no temperature compensation is employed. Instead components of extremely small temperature response have been employed, and the assembled components have been laboratory calibrated over anticipated temperature and pressure ranges. A temperature recorder will be included in each package in future installations. (GOODHEART, et al, 1965).

Thus far there have been two successful installations, as reported by HICKS, GOODHEART AND EERLEY (1964). As can be seen in Figure 7, a variety of schemes were used to aid in recovering the instrument package, including surface buoys, a radio transmitter for RDF homing, lights, and time release subsurface buoys. In every installation at least one of these redundancies was worked, so that our recovery record is good; but no single recovery system has been consistent enough to allow elimination of all others. The two successful stations to date  

Figure 7: Mooring and recovery scheme for deep sea tide gage.
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were as shown in Figure 8 at A and B. Station A was at a nominal depth of 690 feet (210 m); the mean tidal range during the 10 semi-diurnal cycles of observations was 3.4 feet (1.03 m); the high water interval 11.85 lunar hours. Station B was in 840 feet (256 m). A mean range of 3.1 feet (0.9 m) was recorded for 10 semi-diurnal cycles, high water interval 11.74 lunar hours.

Special considerations have influenced the selection of components in this gage. The transducer selected is highly stable and should allow long term installations. This allows the possibility of using the gage in tsunami investigations as well as in oceanic tidal propagation studies.

REFERENCES


...)

OBSERVATIONS SUR LA COMMUNICATION DU CR BARBEE

Dr LINNON - I was very interested to hear Comor Barbée's description of the Fisher-Porite tide gage, particularly because at the Liverpool Tide Institute we have recently been testing a Fisher-Porite gage of slightly different design. In our case the float suspension is in a stainless steel tape, containing spool holes, which passes over a sprocket float pully and is allowed to fall again into the well with a counterweight attached to its extremity. We find that this arrangement does not maintain a constant tension on the float suspension so that the buoyancy of the float is affected. Sounding tests have shown that a systematic error occurs amounting to 1 cm at low water. In the instrument used by Comor Barbée the error is avoided but then his instrument uses a gear train to achieve a 2:1 reduction in the rotation of the recording shaft. Our experience has shown that any such gear train introduce other errors due to friction or backlash which have a minimum of the order of 1 cm again, depending upon whether the tide is rising or falling.

CR BARBEE. There may well be difficulty in the introduction of a reduction gear in the drive-train. Is it possible to take it out and procure wire storage drums of 1 foot per rotation of the axle. Note that we have not at this time any P-P gauge in operation in our standard net.

M. DOHLER. Translation time for one month recordings at hourly intervals with the Fisher-Porite gage?

CR BARBEE. Our only experience is for record with one tenth-hour sampling interval. For these, one month's records require 33 min. to translate. Presumably time is pro-rated so 3.5 min. required for 1 month's hourly values.

CR BARBEE. How do you correct time and height errors at the Fisher-Porite gage?

CR BARBEE. We have not to date, but you can always use the same procedures used for analog records.

Dr ROSSITER. In comment on M. Dohler's remarks, I would like to say that the fact that the Fischer-
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M. DOHLER. Translation time for one month recordings at hourly intervals with the Fisher-Porter ?

Cr BARBEE. Our only experience is for record with one tenth-hour sampling interval. For these, one month's records require 35 min. to translate. Presumably time is pro-rated so 3-5 min. required for 1 month's hourly values.

M. DOHLER. How do you correct time and height errors at the Fischer-Porter ?

Cr BARBEE. We have not to date, but you can always use the same procedures used for analog records.

Dr ROSSITER. In comment on M. Dohler's remarks, I would like to say that the fact that the Fischer-
THE AGA TIDAL TELEMETERING SYSTEM

N. L. SPOTTISWOODE

This system, which is used in the U.K. by the Storm Tide Warning Service and also by various Port Authorities, is designed to meet the following specifications:

1) The transmitting equipment must be suitable for attachment to existing Tide Recorders.
2) At the receiving station, the information from each out-station must be presented in the form of a continuous recording on a chart with rectangular co-ordinates.
3) The system must be capable of operating either through the G.P.O., telegraph network or through a radio link. In either case, the relevant regulations for safety and interference must be met.
4) The overall accuracy must be of the order of ± 0.05 ft (± 15 mm) over a range of 32 ft (10 M).
5) Provision must be made for the system to continue to operate during periods of mains failure at the out-stations.
6) The system must provide an independent check in the form of a signal which is transmitted whenever the Rise of Tide crosses a given Datum level (usually O.D. Newlyn). This signal should be derived from a source separate from the Tide Recorder, coder, etc., and should be recorded on the appropriate chart at the receiving station.
7) In the event of failure of the system, means should be provided at the receiving station for checking the G.P.O. line right through to the out-station so as to establish whether the fault is in the line or in the equipment.
8) On restoration of the line after a failure, the system must be automatically self-correcting.

Since continuous information is required, a separate G.P.O. Telegraph line is provided from each out-station to the receiving station. The characteristics of these lines are that they can only transmit two states (+80V and -80V) which can change at a maximum rate of 25 times per second. However, owing to the comparatively slow movement of the tide, the required accuracy can be obtained by transmitting the information once per minute. This has enabled a fairly simple coding system to be used and one which is not sensitive to certain forms of interference and changes in characteristics to which long transmission lines are occasionally subject.

The system operates as follows. The equipment at the out-station causes the outgoing line to change to positive exactly once a minute. The line then remains positive for a period depending on the Rise of Tide, at the end of which it goes negative and remains so until the beginning of the next one minute period. At the receiving station, the length of the positive period is measured and the information is transferred to the Chart Recorder. Since the equipment at the out-stations must be independent of mains failure it is designed to operate from a 12V supply consisting of a storage battery float charged from the mains. An accurate 50 c/s signal is generated by a transistorised tuning fork, and this feeds a synchronous motor which drives the coder cam at 1 r.p.m. In the Coder a pair of fixed contacts send the line positive once per revolution. Another pair of contacts is attached to an arm...
which rotates between 12° and 360° according to the Rise of Tide. The positive signal therefore varies between 2 seconds (lowest reading) and 50 secs (highest reading).

The independent datum check is derived from a metal probe fixed at the exact level, either in the float tube or in a separate tube provided for this purpose. When the water level either reaches or leaves the tip of the probe, a signal is initiated. The signal is not transmitted until the coder cam reaches the 55 second position, so that it always occurs when the line is negative. The datum signal consists of positive and negative pulses at the rate of about 15 pulses/sec and lasting for about 1 second. At the receiving end, the signal appears on the chart as a short horizontal line. A pecked line is printed on the Chart at the datum level, thus enabling the accuracy of the system to be checked four times each day.

Should a mains failure occur at an out-station, it is possible that the local authority will be unaware of it, especially if it is in a fuse supplying only the Tide Recorder hut. It is therefore desirable that information of a mains failure be transmitted to the receiving station. This is achieved by sending the datum signal every minute. The trace on the Chart then appears as a thickened line. Should this continue for more than a short time, the local authority can be alerted by telephone. The batteries supplied at the out-stations enable the equipment to run for about 48 hours without the mains.

Should the signal from an out-station fail to arrive at the receiving station, it is necessary to establish where the fault lies. The three parts of the system (receiving equipment, land line, and transmitting equipment) will probably be serviced by different authorities, and it is therefore necessary to establish in which of these three sections the fault lies. The receiving equipment is checked by means of pilot lights which indicate the arrival of a signal. Should this be arriving correctly (but not appearing on the chart) the fault is local, and if required the signal can be timed with a stop watch and a fairly accurate measure of the tide achieved. Should no signal be arriving, the fault is either in the G.P.O. line or in the transmitting equipment. To check the line, a switch is operated in the receiving equipment which sends a signal to the out-station. If the line is in order, this signal is sent back to the receiving station where it operates a pilot lamp. If this works correctly, the fault must lie in the transmitting equipment, and the Local Authority is contacted by telephone.

In the case of Radio Linke, the remote site is usually isolated and without mains power. The equipment must therefore run for at least three months on self-contained batteries.

In order to conserve power, the system is modified in the following manner. Signals are initiated by changes in Rise of Tide (usually 0.1 ft - 30 mm). In between signals the equipment is shut off and consumes no current.

The period for sending a signal is reduced from 60 seconds to 12 seconds.

The radio equipment is fully transistorised and normally works in the 460 Mc/sec band, using pulse tone modulation.

OBSERVATIONS sur la COMMUNICATION de M. SPOTTISWOODE

Dr WEMELSFLIEGER. In behalf of a floodwarning system, one needs instant information of the height at several gauges. At our Netherlands coast, during storm-surge, strong and highly irregular seiches or squall-oscillations occur, having trough to crest amplitudes of several decimeters and irregular periods, ranging from 10 minutes to 50 minutes and more. In such cases, digital informations are of little use. Only analogue telemetry can reveal what is going on at the coast. So I fully agree with M. Spottiswoode as to the priority, in this special field, of analogue telemetering.

Dr ROSSITER. Could M. Spottiswoode say anything about the troubles experimented with his company’s telemetering equipment.
which rotates between 12° and 360° according to the Rise of Tide. The positive signal therefore varies between 2 seconds (lowest reading) and 50 seconds (highest reading).

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The radio equipment is fully transistorised and normally works in the 460 Mc. a band, using pulse tone modulation.

**OBSERVATIONS sur la COMMUNICATION de M. SPOTTISWOODE**

Dr WEMELSFELDER. In behalf of a flood warning system, one needs instant information of the height at several gauges. At our Netherland coast, during storm-surge, strong and highly irregular seiches or squall-oscillations occur, having trough to crest amplitudes of several decimeters and irregular periods, ranging from 10 minutes to 50 minutes and more. In such cases, digital informations are of little use. Only analogue telemetry can reveal what is going on at the coast. So I fully agree with M. Spottiswoode as to the priority, in this special field, of analogue telemetry.

Dr ROSSITER. Could M. Spottiswoode say anything about the troubles experimented with his company’s telemetering equipment.
DIGITAL RECORDING OF TIDES IN THE DEEP SEA

Frank SNODGRASS

Scraps Institution of Oceanography (United States)
Institute of Geophysics and Planetary Physics University of California
La Jolla, California [U.S.A.]

The purpose of the instrument I wish to describe is to measure the deep sea tides. The unit is designed to fall freely from a surface ship in water depths up to 4000 meters, remain on the bottom for several days then return to the surface upon command from the ship.

Before describing the vehicle, I wish to discuss the transducers. The tide measurement will be made by measuring bottom pressure with a vibrating wire gauge called a vibrotrometer. A tungsten filament about 1 cm in length is stretched in a magnetic field. The wire is encased in a vacuum with one end attached to a diaphragm. Under pressure the diaphragm deflects causing the wire tension and natural frequency of vibration to change. With the wire connected in the feedback of an amplifier a variable frequency oscillator is obtained whose frequency is a function of pressure. By conventional digital techniques the oscillar frequency can be measured to 1 part in 10 million. A pressure gauge with a range of 7000 meters can be resolved for water pressure changes of one millimeter. There is of course always a question of the reliability of such high resolution recordings. The past year has been spent studying records obtained in quiet environments to determine the noise spectrum of the system. The ratio of the noise energy spectrum and the tide energy spectrum is a measure of the quality of the data.

From these tests we have concluded that the measurements will be of good quality if we can compensate for temperature effects. Temperature recordings with a resolution of a millidegree are required but vibrating wire transducers and quartz crystals with intentionally high temperature sensitivity can provide records with resolutions of a micro degree using the measuring system just described.

We therefore have built a system that provides a sensitive pressure recording contaminated somewhat by temperature and a sensitive temperature record that might have a small pressure contamination. If both records are of good quality that is the noise energy spectrum is low we can solve in the computer by cross spectral methods for the true temperature and pressure spectra. In fact we have become interested in the temperature spectrum as it relates to some problem of the heat flow into the ocean from the earth's core.

Further tests to establish the quality of the data can be undertaken with the units installed on bottom. Two pressure or two temperature gauges can be installed and the coherence of the records can be studied, the incoherent dart being the noise. A temperature gauge and a capped pressure gauge can be installed to measure the pressure gauge noise under actual test conditions.

SAMPLE RATE - RECORD LENGTH - PRESENT GAUGE

Slide

The general assembly of the vehicle is shown in this slide although it is wrong in some of the detail. The capsule consists of two 40 cm diameter aluminum spheres with 3 1/2 cm thick walls which withstand the 4000 meters of water pressure and provide 105 pounds of buoyancy. Mounted on the upper sphere is a tripod which carries a radio beacon and flashing light for the recovery of the capsule when it returns to the surface. Recall of the capsule from the bottom is accomplished by transmitting coded acoustical signals from the surface.
ship which are received by a hydrophone also located on the tripod. These signals are decoded by electronic equipment in the top sphere and if correct explodes a release mechanism which drops a ballast weight allowing the instrument capsule to return to the surface by its own buoyancy. If the recall system fails the tide gauge recorder clock will release the capsule. If both of these fail a magnetism link will corrode and eventually release the ballast. With the later devise the release is certain although the time of release can not be set with accuracy of more than 10%.

The ballast weight for this vehicle is automobile batteries connected to the spheres by 20 meters of cable, they also provides power for the recording system. The batteries are unprotected from the sea water except for sealing the battery terminals. Sea water is allowed to flow into the battery to compensate the pressure through a simple valve that reseals to prevent electrical leakage. We have found no loss of power or voltage from adding several hundred c.c. of sea water to the battery.

When the battery ballast is release the small frame on top of the batteries lifts free and returns to the surface with the capsule. Two small batteries, which supply power to the radio beacon and flashing light, and the vibratronics are attached to this frame.

Also shown attached to this frame is a second hydrophone (a pinger) which transmits short acoustical pulses. These pulses provide a homing signal for the ship which tow's a direction sensitive hydrophone. Also the pulse rate is programmed by the capsule. From the pulse rate we can determine whether or not the instruments are recording properly, whether release commands have been received, whether release of the batteries was effected, whether leaks have developed, etc...

If everything I have describe, work perfectly, nothing has been accomplished unless the transducers and the tide recorder mounted in the bottom sphere functions properly and records useful data.

The 20 meters of cable between the ballast and the spheres is provided to remise the heat produced in the recorder sphere from the vicinity of the sensitive temperature transducer.

OBSERVATIONS SUR LA COMMUNICATION DU DR. SNOODGRASS

M. PHYDES, J'ajouterai aux considérations du Dr. Snoodgrass que l'usage de la corde vibrante pour mesurer la pression est très sûr. Un strigeograph à corde est convenable pour une observation prolongée dans les conditions inconfortables.

M. ARGOLO. Can the coordinates of the described instrument (position on the earth) be determined? How accurately can this be done? Thinking about use of the capsule in shallow water, how can it be protected from damage, change of position or sinking below bottom level due to material transport?

Dr. SNOODGRASS. The instrument is designated for the special purpose of deep sea tide recording. The position of instrument, when on the bottom, can be determined with an accuracy not greater than determining the surface ship's position, which is done in the usual way. No special arrangements have been considered, related to the second question, apart from the level difference of the buoyances and the battery supply, connected by a line.

M. HYDE. Y-a-t-il un gaz compresseur dans la chambre du vibraton ?

Dr. SNOODGRASS. Non, c'est un vibraton classique qui mesure la pression absolue.

M. GOUGENHEIM. Est-il nécessaire de prévoir des durées d'enregistrement aussi longues que celles prévues ? Il a été parlé d'une année d'observation, quel est le but particulier de telles mesures de longue durée ?

Dr. SNOODGRASS. L'appareil ne fonctionne pas nécessairement un an. Dans l'avenir, des mesures prolongées seront certainement très utiles.
ship which are received by a hydrophone also located on the tripod. These signals are decoded by electronic equipment in the top sphere and if correct explodes a release mechanism which drops a ballast weight allowing the instrument capsule to return to the surface by its own buoyancy, if the recall system fails the tide gauge recorder clock will release the capsule. If both of these fail a magnesium link will corrode and eventually release the ballast. With the latter devise the release is certain although the time of release can not be set with accuracy of more than 10%.

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When the battery ballast is release the small frame on top of the batteries lifts free and returns to the surface with the capsule. Two small batteries, which supply power to the radio beacon and flashing light, and the vibrotone transducers are attached to this frame.

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M. HYDRES, Y-a-t-il un gaz compresseur dans la chambre du vibrotone ?

Dr SNODGRASS, Non; c’est un vibrotone classique qui mesure la pression absolue.

M. GOUGENHEIM, Est-il nécessaire de prévoir des durées d’enregistrement aussi longues que celles prévues ? Il a été parlé d’une année d’observation, quel est le but particulier de telles mesures de longue durée ?

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TELEMETERING TIDE RECORDER AND TELEMETERING, RECORDING, PRINTING AND INDICATING SYSTEM OF TIDE AND OTHER ELEMENTS

SO CALLED “CALL SYSTEM” IN JAPAN

Hidetaka FUTI
Hakodate Marine Observatory
Japan Meteorological Agency, Tokyo, Japan

The system mentioned here is one of the digital telemetering systems of the multiple data including tides. This system can be used either for the purpose of the routine observation or of the storm surge warning. Observed data from several tidal stations are called automatically or manually at the key station by VHF waves, and recorded there as digital figures. Three sets are now installed for three important basins ; Tokyo Bay, Osaka Bay and Seto Bay. These sets are effectively used in order to obtain the synoptic time-to-time records during the passage of the severe typhoon. Details will be shown by figures.

Figure 1 shows the system installed along the coast of Tokyo Bay. We have a key station at Tokyo, a relay station at Yokohama, and four observing stations at Chiba, Kawasaki, Yokohama and Yokosuka, respectively. The central unit of the key station is equipped in the forecasting room of the Japan Meteorological Agency, Tokyo. Communications among stations are made by two VHF waves in the 60 MHz band.

Figure 1
The mechanism of our pressure-type tide gauge is shown in figure 2. The water pressure is transformed into the lateral displacement of the core between specially designed cords in the differential transformer by a spring inside the bellows. The maximum core displacement is about 2 mm. Thus the pressure change (maximum 1 dp) is measured as the change of the electric voltage (maximum AC 63 mV), and the response is nearly linear. The measurement can be checked by the standard float-type gauge in the same well, and the relative response thus obtained is sufficient except for shorter waves of about one-minute period or less. The error is within 4 cm.

Figure 2

Figure 3 shows the outer appearance of the tide gauge, the measuring unit and the drier. Figure 4 is an example of the gauge equipped in a tidal well. The standard gauge of the Fussa type is also shown in this figure.

The gauge can be equipped at an isolated place where tidal wells are not available. The well is replaced by a protecting tube with small holes in such a case. A station is a few miles distant from the shore and is fixed to the sea bottom. The depth is about 11 m.

Figure 5 shows the block diagram of the measuring system. Tides and other elements are measured as the voltage changes and recorded by one self-balancing recorder by different colours.

Figure 6 shows the block diagram of the whole system. When a observing station is called by the key station, instructions are transmitted to the telemeter and then analog data are converted into digital figures from zero to 1,000 by the A/D converter, and sent back to the key station by using pulse codes. At the key station, these digital informations are rearranged as the final output data, recorded by the printer or indicated on a board by using dixie tubes.

Figure 7 shows the outer appearance of the telemeter, recorder and radio of an observing station. Figure 8 shows the outer appearance of the digital recorders of the key station. The printer and the indicating board are seen. The data call (manual or automatic with selected time intervals) is controlled by switches on the desk below the indicating board.

Examples of the analog and digital records are shown in figures 9 and 10. Analog records are obtained at observing stations, and digital records are given at the key station, respectively.

58
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Figure 3
Radio equipment, control equipment printing and indicating equipment (from left) of motor station.
Figure 8
Radio equipment, control equipment printing and indicating equipment (from left) of motor station.
MARÉGRAPHIE PAR GRANDES PROFONDEURS

Marc EYRIès
Ingénieur Hydrographe en Chef, S.H.M. (France)
Service Hydrographique de la Marine Française

Parmi les phénomènes naturels, la marée astronomique est l'un de ceux qui associent le plus étroitement les mathématiques et la physique, puisque sa description et sa prédiction font appel à la fois aux lois de la mécanique céleste et à celles de l'hydrodynamique.

Les variations de la "hauteur d'eau" à la côte ont été depuis longtemps observées parce qu'elles étaient relativement aisées et qu'elles présentaient un intérêt pratique immédiat pour la navigation dans des profondeurs faibles. Cependant, elles ne traitaient qu'un mouvement secondaire, induit par la marée oceanique, dont les caractéristiques sont plus complexes que ceux de cette marée et surtout dépendent à un haut degré de facteurs physiques du milieu, encore mal connus, et quelques-uns de conditions très particulières comme le débit des fleuves. On est donc réduit, pour la prédiction, à une extrapolation des éléments observés, dans laquelle l'intervention des données astronomiques est réduite : vitesses des ondes dans la méthode harmonique ou coordonnées sol-stellaires dans la théorie Laplacienne, On peut presque dire qu'on "prédit sans expliquer".

Par contre, la marée océanique peut être raisonnablement schématisée de sorte que le problème, du moins dans son énoncé, est assez simplement défini pour être abordé dans son ensemble et que sa solution théorique doit apporter l'explication et permettre la prédiction. Depuis le milieu du XVIIIe siècle cette solution a été établie, puis améliorée, et le développement récent des techniques du calcul doit conduire, dans un proche avenir, à l'élaboration de la grandeur chiffrée d'un ou plusieurs paramètres du mouvement.

Cette prédiction n'est pas sans valeur pratique, car la marée océanique intervient dans la genèse de la turbulence et par conséquent dans le processus de mélange des masses d'eau et aussi dans la répartition instantanée des masses liquides qui affecte le champ de gravité et le champ de pression sur les terres immergées.

PRINCIPE ET DESCRIPTION DU MAREOGRAPHIE AFEGPO

La solution théorique du problème de la marée océanique n'a malheureusement pas pu, jusqu'à présent, être confrontée à l'observation faute d'un instrument adéquat.

Depuis longtemps ce problème technologique a précédé les hydrographes, et dès 1887 le marégraphe français FAVE permettait des observations jusqu'à une centaine de mètres de profondeur. Le marégraphe allemand RAUSCHELMACH (1893) a recelé la limite d'observation jusqu'à 300 mètres environ, mais l'échelle de l'enregistrement analogique se réduit avec la profondeur et le dispositif de mesure de la température interdit l'adaptation de l'instrument à des profondeurs plus grandes.

Dans ces deux marégraphes, le paramètre mesuré est la variation de pression au niveau de l'instrument. La mesure directe d'une hauteur d'eau ou de sa variation est en effet impossible au large ; il faut passer par l'intermédiaire d'une mesure de temps (durée de trajet d'une onde acoustique) ou d'une mesure de pression.

La mesure de temps, très dépendante du pouvoir réflecteur de la surface libre si l'émetteur est mouillé sur le fond, des mouvements propres de l'émetteur, s'il flotte sur la surface, et dans les deux cas de la répartition verticale de la calérité du son, n'a été employé jusqu'à présent que comme un procédé de fortune par profondeur assez faible.
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La mesure de pression est viable pour la mer à partir des hypothèses généralement admises, on montre que la pression dans le fluide est la pression hydrostatique. Dans la plupart des cas, on peut espérer que la relation entre la variation de pression sur le fond, corrigée de la pression atmosphérique, et la variation du niveau de la surface libre reste simple ; cela revient à dire que l'évolution des caractéristiques physiques du milieu marin ou les accélérations verticales dues à des mouvements autres que la marée, n'affectent pas les résultats d'une analyse harmonique ; soit parce que leurs effets sont de l'ordre des erreurs de mesure, soit parce qu'ils sont aperiodiques ou ont des périodes très différentes de celles des composantes de la marée, On remarque d'ailleurs que dans le développement des théories évoquées plus haut, c'est la pression qui intervient comme paramètre descriptif du phénomène plutôt que la hauteur d'eau qui s'en déduit.

L'Association Française pour l'Etude des Grandes Profondeurs Océaniques (AFEGPO), suivant les directives du Service Hydrographique de la Marine et avec l'aide financière de la Délégation à la Recherche Scientifique et Technique et du Sous-Comité Océanographique de l'OTAN, a mis au point un instrument capable de déceler des variations de pression de l'ordre de 10⁻⁴ pascal (1 cm d'eau) autour d'une pression moyenne de l'ordre de 5 10³ pascal (5000 mètres d'eau) ; cette pression moyenne peut être accue sans difficulté majeure par une construction encore plus robuste de l'instrument.

M. L. ERDENEY, qui était chargé de l'élaboration du prototype et de la responsabilité de sa construction avait d'abord envisagé l'emploi de détecteurs piezo-électriques, puis il s'est rallié à celui d'une chambre manométrique dont la pression intérieure, utilisée comme pression de référence, est aussi proche que possible de la pression moyenne "in situ" ; il suffit pour cela de maintenir cette chambre en communication avec un réservoir de gaz pendant toute son immersion puis de l'isoler lorsque l'instrument est mouillé ; la manœuvre de la vanne, assurée par un poids placé le fond dans le manchon GRAV, est électrique dans l'instrument actuel.

Un opercule ferme la chambre manométrique et se déforme élastiquement sous l'effet de la différence des pressions extérieure et intérieure ; le déplacement du centre de cet opercule entraîne une variation de la tension d'une corde, dite corde bathymétrique, tendue entre ce point et le fond de la chambre. La mesure des variations de la période fondamentale de la corde fournit une mesure des variations de sa tension et par conséquent de la différence des pressions.

La pression de référence varie avec la température du gaz de façon notable lorsque la pression moyenne est grande et il devient indispensable de mesurer avec soin ces variations de température. Pour ce faire, la chambre manométrique contient une deuxième corde, dite corde thermométrique, tendue sur un cadre fixe ; la dilatation de cette corde modifie sa tension. La mesure des variations de la période fondamentale de cette corde fournit une mesure des variations de sa tension et par conséquent de sa température ; la variation des deux cordes assure par ailleurs une bonne conduction thermique. L'ensemble des deux cordes n'est pas sans analogie avec l'ensemble des deux "thermomètres" "protégé" et "non protégé" de l'hydrologie classique.

Le prototype a été construit par la Société TELEMAC (17, rue Alfred Roll Paris 17e). Il est constitué par un cylindre de 88 cm de longueur totale et de 8 cm de diamètre, comprenant deux parties de dimensions à peu près égales : la chambre manométrique et la chambre des amplificateurs. La première contient les deux cordes dans sa partie médiane et l'électrovanne dans sa partie antérieure ; l'opercule qui la ferme à sa partie postérieure (acier : diamètre 50 mm, épaisseur 2 mm) subit la pression extérieure par l'intermédiaire d'un liquide tampon et d'une pastille porreuse qui filtre les variations de pression à courtes périodes. Chaque des cordes passe dans l'enfer de deux électro-aimants, le premier servit à l'excitation de la corde, l'autre au comptage de ses périodes et au pilotage du premier. La chambre des amplificateurs contient deux amplificateurs transistorisés, l'un pour la corde bathymétrique, l'autre pour la corde thermométrique et un poste de communication. La sortie vers le poste de comptage de périodes se fait par un câble à 7 conducteurs qui se répartissent comme suit:
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L'Association Française pour l'Etude des Grande Profondeurs Océaniques (AFEGPO), suivant les directives du Service Hydrographique de la Marine et avec l'aide financière de la Délégation à la Recherche Scientifique et Technique et du Sous-Comité Océanographique de l'OTAN, a mis au point un instrument capable de déceler des variations de pression d’ordre de 10⁻¹ pascals (1 cm d'eau) autour d’une pression moyenne de l’ordre de 5 10⁻¹ pascals (5 000 mètres d’eau) ; cette pression moyenne peut être accueillie sans difficulté majeure par une construction encore plus robuste de l’instrument.

M. L. ERDELY, qui était chargé de l’élaboration du prototype et de la responsabilité de sa construction avait d’abord envisagé l’emploi de détecteurs piezo-électriques, puis il s’est rallié à celui d’une chambre manométrique dont la pression interne, utilisée comme référence, est aussi proche que possible de la pression moyenne "in situ" ; il suffit pour cela de maintenir cette chambre en communication avec un réservoir de gaz pendant toute son immersion puis de l’isoler lorsque l’instrument est mouillé ; la manœuvre de la vanne, assurée par un poêle étendant le fond dans le marbre et l’ustensile du point d’instrumentation, est électrique dans l’instrument actuel.

Un opercule ferme la chambre manométrique et se déforme élastiquement sous l’effet de la différence des pressions extérieure et interne ; le déplacement du centre de cet opercule entraîne une variation de la tension d’une corde, dite corde bathymétrique, tendue entre ce point et le fond de la chambre. La mesure des variations de la période fondamentale de la corde fournit une mesure des variations de sa tension et par conséquent de la différence des pressions.

La pression de référence varie avec la température du gaz de façon notable lorsque la pression moyenne est grande et il devient indispensable de mesurer avec soin ces variations de température. Pour ce faire, la chambre manométrique contient une deuxième corde, dite corde thermométrique, tendue sur un cadre invar ; la dilatation de cette corde modifie sa tension. La mesure des variations de la période fondamentale de cette corde fournit une mesure des variations de sa tension et par conséquent de sa température ; la vibration des deux cordes assure par ailleurs une bonne conduction thermique. L’ensemble des deux cordes n’est pas sans analogie avec l’ensemble des deux "thermomètres" "protégé" et "non protégé" de l’hydrologie classique.

Le prototype a été construit par la Société TELEMAC (17, rue Alfred Roll Paris 17e). Il est constitué par un cylindre de 88 cm de longueur totale et de 8 cm de diamètre, comprenant deux parties de dimension à peu près égales : la chambre manométrique et la chambre des amplificateurs. La première contient les deux cordes dans sa partie médiane et l’électrovanne dans sa partie antérieure ; l’opercule qui la ferme à sa partie postérieure (acier : diamètre 50 mm, épaisseur 2 mm) subit la pression extérieure par l’intermédiaire d’un liquide tampon et de pastille porreuse qui filtrent les variations de pression à courtes périodes. Chaque des cordes passe dans l’entrefer de deux électro-aimants, le premier servit à l’excitation de la corde, l’autre au comptage de ses périodes et au pilotage du premier. La chambre des amplificateurs contient deux amplificateurs transistorisés, l’un pour la corde bathymétrique, l’autre pour la corde thermométrique et un poste de commutation, la sortie vers le poste de comptage de périodes se fait par un câble à 7 conducteurs qui se répartissent comme suit :
il sert de câble de mouillage pour le navire et il est arrodi sur un maille de 30 mètres de chaîne (poids 650 kg) étroitement mâché au câble par 5 manchons en treillis métallique, enfi- lés et serrés sur le câble et maitris sur la chaîne ; dans l'intervalles des manchons, le câble est assagi sur la chaîne par des torchons de hord. Ce dispositif peut être utilisé jusqu'à 700 à 800 mètres de fond par un petit navire (150 t) sans installation spécial ; il est destiné à la marégraphie sur la pente littorale.

Le second comprend une charpente métallique en tubes de métal léger, à l'intérieur duquel sont suspendus le marégraphe et la caisse qui contient le réservoir plastique. Le câble conducteur a une longueur de 5 000 mètres ; il sert au mouillage d'une bouée de 5 tonnes (bouée de navire câblé) et est lesté par un maillon de chaîne. Entre la bouée et le navire, le câble est supporté par des floettes ; le navire manœuvre pour réduire au minimum la traction. Ce dispositif peut être utilisé jusqu'à 4 000 mètres de fond par un navire moyen (600 à 800 t) qui doit pouvoir enrouler 5 000 mètres de câble ; il est destiné à la marégraphie océanique.

**ESSAI ET ÉTALONNAGE**

La fréquence d'une corde vibrante est donnée par la relation

\[ N = \frac{C V_{FP}}{1 + \frac{C V_{EA}}{1}} \]

où \( C \) est un coefficient dépendant de la masse volumique de la corde ; \( V \) est le module d'Young ; A l'allongement relatif à partir du repos ; \( F \) la tension de la corde et \( T \) sa longueur.

La lecture \( L \) sur le périodomètre est liée à la fréquence \( f \) par

\[ L = \frac{10^8}{f^2} \]

Le centre de l'opérecle subit des déplacements proportionnels à la différence des pressions intérieure et extérieure ; par suite la variation de cette différence de pressions s'écrit :

\[ \Delta P = B \Delta P \]

où \( B \) est un coefficient dépendant de caractéristiques de l'opérecle et de la corde ; la pression est d'une proportionnelle au carré de la fréquence et par suite inversement proportionnelle au carré de la lecture ; on écrit :

\[ H - H_0 = K \times 10^{11} \left( \frac{1}{L^2} - \frac{1}{L_0^2} \right) \]

où \( H \) est la pression à une épibre donnée en centimètre d'eau (2 \( 10^{11} \) pascale), \( L \) la lecture à la même épibre, \( H_0 \) est la pression de référence, \( L_0 \) la lecture correspondant à la pression \( H_0 \), \( K \) est un coefficient qui dépend des unités et aussi de \( H_0 \) à peu près linéairement, l'éta- lonnage de deux exemplaires du marégraphe a montré que :

- pour le premier \( K = 2.540 \pm 0.470 (H_0 - 25,10^3) - 0.001 (H_0 - 25,10^3)^2 \)
- pour le second \( K = 2.160 \pm 0.630 (H_0 - 25,10^3) - 0.001 (H_0 - 25,10^3)^2 \)

Ainsi la sensibilité de la corde bathymétrique décroît légèrement lorsque la profondeur moyenne (à laquelle règne la pression de référence) augmente ; la variation relative est de 10 % entre 0 < \( H_0 < 5,10^3 \) centimètres d'eau. La variation de pression correspondant à 10 unités de lecture est voisine de 1 centimètre d'eau.

Le "thermomètre à corde" constitué par la corde thermométrique tendue sur le cadre en invar a été étalonné avec les instruments Mueller-Leeda-Northrup de l'Université de Lyon.
1 : électro-vanne
2 : commutation
3 : alimentation des amplificateurs
4 : masses des ampli. et des bobines de cordes
5 : signal

Le gaz utilisé pour obtenir la pression de référence est de l'air ou de l'azote ou encore de l'hydrogène, et non attaché à un intérêt particulier aux variations de la température.

Il a fallu, au moins pour le prototype, choisir entre l'enregistrement des mesures "in situ" et l'enregistrement à bord du navire. Les deux procédés ont des avantages et aussi de sérieux inconvénients.

Le premier est particulièrement adapté à une observation de longue durée (15 jours ou plus) à des points très éloignés de l'océan sur lesquels le navire peut se rendre en interrompant une campagne oceano-graphique qui constitue son activité principale. Il est nécessaire de contrôler le fonctionnement de l'instrument, au moins pendant les premières heures de sa mise en service. Il est souhaitable de ne pas utiliser en surface une bouée qu'il faudrait baliser et qui, laissée sans surveillance pourrait courir de grands dangers. Le mécanisme de rappel de marbre immergé pose alors un problème technologique supplémentaire et sa réalisation est onéreuse ; enfin le risque de perdre l'instrument et les enregistrements enregistrées reste grand.

Le deuxième procédé convient à une observation courte (deux à cinq jours) en des points plus rapprochés, en vue de déterminer les éléments des ondes principales de la mer ou pour en tracer les lignes cotidiennes et les lignes d'égal amplitude. Le navire est alors assujetti à rester mouillé sur le pont pour la prise de manœuvres souvent difficiles et ne peut exécuter d'autres travaux oceano-graphiques. Par contre le fonctionnement du marbre est contrôlé en permanence et sa récupération est quasi certaine tandis que celle des enregistrements enregistrés est assurée.

Pour limiter le problème de la mise au point d'un manomètre différentiel sensible, première étape permise par les moyens financiers, c'est le deuxième procédé qui a été retenu pour le prototype ; le point important est donc de bibliographier exactement à bord du navire qui doit rester relié au marbre par un câble auto-porteur à 7 conducteurs type Schumberger (diamètre 14 mm, poids dans l'air 0.6 kg au mètre, dont la résistance à la rupture est de 8 tonnes). Le signal issu du marbre aboutit à un périodomètre ROCHE (à quartz vibrant à 10 MHz) qui mesure la durée de 10 périodes (échelle de 1/60 de seconde) de l'une ou l'autre des cordes suivant la position du poste de commutation. Une horloge et un circuit programmé très simple permettent de régler l'intervalle entre les séquences d'observation, la durée des séquences ainsi que la commutation sur les cordes. Le dispositif est onéreux relativement au marbre lui-même et une très faible part de ses possibilités est utilisée pour la mesure ; un appareillage plus simple parce qu'il compte les périodes sur une durée d'une dizaine d'heures en cours des points éloignés du navire est en cours de mise au point. La sortie du périodomètre se fait sur une machine imprimante. On obtient en définitive la variation de pression et de température sur le fond à des épochs séparées par des intervalles de temps quel qu'on peut choisir (pourvu qu'ils soient supérieurs à 25 secondes), enregistrée sous forme numérique.

**DISPOSITIF DE MOUILLAGE**

La version actuelle du marbre dont l'enregistrement est en surface comporte deux dispositifs de mouillage :

- Le premier est constitué par un graphe de grandes dimensions (1.4 m x 2 m x 2 m) en méta-léger (AG : poids 200 kg) dont la verge contient le marbre et le réservoir plastique rempli du gaz de compensation. Le câble conducteur a une longueur de 1 000 mètres ;

- Le second comprend une charpente tétraédrique en tubes de métal léger, à l'intérieur duquel sont suspendus le marbre et le caisson qui contient le réservoir plastique. Le câble conducteur a une longueur de 5 000 mètres ; il sert au mouillage d'une bouée de 5 tonnes (bouée de marbre utilisée) et est lesté par un maillon de chaîne. Entre la bouée et le navire, le câble est supporté par des flotteurs ; le navire manœuvre pour réduire au minimum la traction. Ce dispositif peut être utilisé jusqu'à 4 000 mètres de fond par un navire moyen (800 à 800 t) qui doit pouvoir enrouler 5 000 mètres de câble ; il est destiné à la marographie sur la pente littorale.

**ESSAIS ET ETALONNAGE**

La fréquence d'une corde vibrante est donnée par la relation

\[ f = \frac{C}{\sqrt{A}} \]

où \( C \) est un coefficient dépendant de la masse volumique de la corde ; \( A \) est le module d'Young ; A l'allongement relatif à partir du repos ; \( V \) la tension de la corde et \( t \) sa longueur.

La lecture L sur le périodomètre est liée à la fréquence f par :

\[ L = \frac{10^8}{f} \]

Le centre de l'opercule subit des déplacements proportionnels à la différence des pressions intérieure et extérieure ; par suite la variation de cette différence de pressions s'écrit :

\[ \Delta P = B \Delta P' \]

où \( B \) est un coefficient dépendant des caractéristiques de l'opercule et de la corde ; la pression est donc proportionnelle au carré de la fréquence et par suite inversement proportionnelle au carré de la lecture ; on écrit :

\[ H - H_0 = K \times 10^{10} \left( \frac{1}{L^2} - \frac{1}{L_0^2} \right) \]

où \( H \) est la pression à une époque donnée en centimètre d'eau (r 10⁻¹ pascal), \( L \) la lecture à la même époque, \( H_0 \) est la pression de référence, \( L_0 \) la lecture correspondant à la pression \( H_0 \), \( K \) est un coefficient qui dépend des unités et aussi de \( H_0 \) à peu près linéairement, l'etalonnage de deux exemplaires du marbre a montré que :

- pour le premier \( K = 2540 + 0.470 \times (H_0 - 25,10^3) - 0.001 \times (H_0 - 25,10^3)^2 \)
- pour le second \( K = 2100 + 0.650 \times (H_0 - 25,10^3) - 0.001 \times (H_0 - 25,10^3)^2 \)

Ainsi la sensibilité de la corde bathymétrique décroît légèrement lorsque la profondeur moyenne (à laquelle règne la pression de référence) augmente ; la variation relative est de 10% entre 0 < \( H_0 < 5,10^3 \) centimètres d'eau. La variation de pression correspondant à 10 unités de lecture est voisine de 1 centimètre d'eau.

Le "thermomètre à corde" constitué par la corde thermométrique tendue sur le cadre en invar, a été étalonné avec les instruments Mueller-Leehn-Nordrup de l'Université de Lyon.
et sa fidélité a permis un étalonnage à 10 °C près entre 2°C et 13°C ; une variation de 10 °C provoque à 5 000 mètres de profondeur (limite de l’instrument) une variation de la pression de référence de 1,5 centimètre d’eau. En réalité, cet étalonnage n'a été utilisé que pour avoir la variation de température. Pour tenir compte de la variation thermique de la pression de référence on a préféré un étalonnage global donnant, pour une pression mesurée avec la balance manométrique, la lecture bathymétrique en fonction de la lecture thermométrique sans que la mesure des variations de température soit nécessaire. Cet étalonnage a été effectué au Laboratoire National d’Essais pour un ensemble de pressions, mesurées avec la balance, voisines de 1,10 3, 1,10 4, 1,10 5, 1,10 6, 1,10 9, 1,10 10, 1,10 11, 1,10 12, 1,10 13, 1,10 14, 1,10 15, 1,10 16 centimètres d’eau ; il permet de corriger les lectures brutes, relatives à la corde bathymétrique, à partir des lectures relatives à la corde thermométrique, pour obtenir les lectures nettes qui sont utilisées dans la relation écrite plus haut.

Outre les étalonnages, l’instrument a subi deux épures dans la nature par petit fond. La première, dans le barrage hydro-électrique de Génissiat, a été de courte durée mais a permis de constater le bon fonctionnement général de l’installation et de comparer les observations du marégraphe à celles du linigraphè du barrage (lesquelles étaient bonnes car la surface libre de la retenue était calme). La seconde en rade de Cherbourg a duré 8 jours pendant lesquels l’instrument, immergé à 12 mètres a supporté sans défaillance une forte tempête de 72 heures ; l’instrument de comparaison était le marégraphe à flotteur du Port de Cherbourg et il n’a avéré moins précis que le marégraphe en essai dont le fonctionnement entièrement autonome a été excellent.

MESURES PAR GRANDES PROFONDEURS

Les premières mesures au large ont été faites sur le dragueur océanique "Garigliano" de la Marine Française, au mois de juillet 1964. Ce navire, assez lourd (600 t) n’a pu être aménagé pour recevoir 5 000 mètres de câble ; on a donc utilisé le grappin de mouillage décrit plus haut associé à une houe d’une tonne. Le tronçon de câble entre la houe et le navire (150 m) était soutenu par deux goélettes de drague, éclairées la nuit, et basé sur l’avant du navire par une longue houe de 20 mètres environ en fil d’acier.

Cette campanie du "Garigliano" avait pour objet essentiel l’épreuve de cet instrument dans les conditions normales de son emploi. Cependant la situation des stations a été choisie pour que les observations contribuent à la connaissance de la mer au large de nos côtes Atlantiques (voir planche II). Les variations de pression sur le fond, celles de la température, et celles de la pression atmosphérique étaient relevées tous les quarts d’heure ce qui a permis un tracé précis des courbes qui les représentent en fonction du temps. Un exemple de ces courbes figuré sur la planche III, montre que la qualité de la mesure est excellente. L’observation la plus longue a été faite à la station n° 4 : 73 heures par fond de 470 mètres.

Bien que la mer n’ait été agitée (3 à 4 mètres de creux au cours de la station 4) le navire n’a pas éprouvé de difficulté majeure à tenir son poste sans traction excessive sur le câble ; il est vrai qu’il est équipé de deux hélices à pas variables. La manoeuvre serait peut-être plus aisée pour navire plus petit qui pourrait utiliser la traction du câble pour s’éviter tout en limitant cette traction par l’action de son hélice. L’opération la plus délicate reste la mise à l’eau et le relevage de l’instrument et surtout du câble conducteur. Dans le cas présent, il a été mouillé seulement 1 000 mètres de câble et cette longueur avait pu être lovée sur le pont.

Le seul incident, observé dans le fonctionnement du marégraphe a été, à deux ou trois reprises, un brusque décalage en pression de quelques décimètres d’eau. Ces décalages ne gênent en rien l’observation puisqu’ils correspondent à un changement instantané de la référence mais n’ont pu être expliqués avec certitude. Il n’est pas vraisemblable qu’ils résultent d’un décalage vertical de l’instrument, travaillé sur la pente ; l’ dissociation de l’électrovanne peut être mise au cause, mais il est plus probable que la corde bathymétrique est sensible à l’orientation du capteur, ce qui était apparu au cours des essais dans laboratoire.
OBSERVATIONS DU 23 JULIET 1963 - 07H00 - 12H00

Détails de la température (en °C) et de la pression (en hPa)

**CONCLUSION**

Cette analyse montre que la variation de la température et de la pression dans l'atmosphère est liée à des facteurs tels que la latitude, la longitude et l'altitude. Les observations montrent que la température et la pression varient de manière régulière et prévisible, ce qui peut être utilisé pour faire des prévisions météorologiques précises. Les chiffres indiquent une tendance à la baisse de la température et de la pression au cours de la journée, ce qui pourrait être dû à l'effet de la辐照.
Deux mesures par profondeur plus grandes, ont été effectuées au mois d'août 1965, l'une au large du Portugal (38°09'01"N ; 09°16'00"O par profondeur de 580 mètres) ; l'autre au large du Maroc (34°46'05"N ; 09°50'58"W par profondeur de 470 mètres). Ces mesures ont été faites, à l'issue d'une campagne océanographique, par le câbleur "AMPERE", que la Marine Nationale avait loué. Le tonnage important du navire (3000 t) laissait prévoir des difficultés de manœuvre ; par contre, toute la longueur de câble (5000 mètres) pouvait être logée dans ses cuves spéciales.

La première de ces mesures a complètement réussi sans incident. La seconde, effectuée par mer force 4 à 5 a été interrompue après 6 heures de très bons enregistrements par la rupture des conducteurs dans le câble. On a constaté que ce câble ne pouvait être emmagasiné en cuve sans se décomposer dès la seconde manœuvre ; au cours de la mise à l'eau, une coque se formait à chaque tour déroulé et les fils d'acier de la gaine se rompaient en passant sur le davier ; il est nécessaire d'encoller les câbles de ce type sous tension et autour d'un tambour.

CONCLUSION

Il apparaît que l'on dispose maintenant d'un instrument précis pour observer la marée par grandes profondeurs. La version actuelle, avec enregistrement à bord, n'est probablement pas définitive et l'on va chercher à rendre le maniement du marégraphie plus commode en étudiant un enregistrement au fond ou tout au moins dans une bouée immergée. En attendant la mise au point du matériel nouveau, on peut entreprendre avec le matériel actuel des campagnes marégraphiques à condition de les confier à un personnel très expérimenté dans la manœuvre, ce qui a été le cas pour le "Garigliano" et pour l"Ampère", et d'utiliser un type de câble approprié.

OBSERVATIONS SUR LA COMMUNICATION DE M. EYRIES

Mr. NOBLE, Are the measurements taken by the instrument affected by the variations in salinity of the water above it? Would these variations need to be measured if the instrument was used in a river estuary?

Mr. EYRIES, L'instrument mesure une variation de pression ; il est évident que cette variation de pression dépend à la fois de la variation de hauteur d'eau au-dessus de l'appareil et aussi de la variation de la répartition verticale de la densité. Mais la hauteur d'eau est l'un des paramètres du phénomène de la marée et la pression en est un autre qui d'ailleurs s'introduit plus directement que la hauteur d'eau dans les équations hydrodynamiques. Dans un estuaire il est opportun de mesurer simultanément les variations de densité.

Dr. CARTWRIGHT, Has M. Eyries envisaged measuring the thermal variations in the whole column of water above the instrument? This is possible because the ship remains "in situ". In the region of stations no 1-4, for example, there is a considerable internal wave activity at tidal periods. This may have a fundamental influence on deep-water tidal measurements.

M. EYRIES, Ces mesures d'ondes internes sont envisagées en utilisant comme gaz compensateur un gaz thermiquement conducteur (hydrogène par exemple) ; il faut en effet réduire l'inertie thermique du gaz compensateur.

Dr. GROVES, Have the results given in the paper been compensated for temperature? If so, what was the approximate relative magnitude of the correction.

M. EYRIES, Oui, les résultats ont été corrigés de la température, A 5000 mètres d'immersion une variation de température de 0,001° Celaifs donne une correction de : centième. Les variations de température observées à 5000 mètres sont d'environ 0,1° Celaifs, à plus grande profondeur elles sont pratiquement négligeables.
M. GODINHEIM. Le problème des ondes internes qui peuvent exister dans toute l’épaisseur de la mer (dans les bassins océaniques) est l’un des plus importants de l’océanographie actuelle. Les mesures manométriques de maree, aussi bien que celles d’hydrologie classique en sont perturbées. Il y a là toute une série d’études à entreprendre et il appartient à l’A.I.O.P. de les susciter ou tout au moins de les développer.

M. GODIN. La dispersion des résultats est forte pour les ondes diurnes.

M. EYRIERES. Cette dispersion est due à la petiteur d’amplitude de ces ondes relativement à celle des ondes semi-diurnes et aussi au fait que la durée d’observation a été courte.

M. GODENHEIM. L’instrument décrit paraît measurer la pression plus près du fond que celui du Dr SNODGRASS.

Dr SNODGRASS. L’organe sensible est à environ 1 mètre du fond dans les deux instruments.

TIDE GAUGE WITH FM OUTPUT

N. L. BROWN
Manager of Research & Development
The Bissett-Berman Corporation San Diego, California [U.S.A.]

ABSTRACT

A tide gauge is described having an FM output. It utilizes a vibrating wire whose tension is varied directly by the applied hydrostatic pressure. The design of this device differs from previous vibrating wire devices in that the total force due to the hydrostatic pressure on the bellows is balanced entirely by the tensile stresses in the vibrating wire. In other words, the system is a "constant tension" system, whereas previous systems are essentially "constant displacement" systems. The result is a device which has extremely low temperature coefficients and hysteresis and an over-all accuracy of ± 0.03 % for ± 20°C temperature change.

The power requirement for the electronics is 750 microwatts. The frequency range is typically from 300 cycles per second to 1000 cycles per second for pressure range of 2 to 22 psi or for tide heights of 5 to 50 feet. The output frequency is proportional to the square root of pressure. However, using simple digital logic circuitry, a direct readout in feet or meters is achieved.

OBSERVATIONS SUR LA COMMUNICATION DE M. BROWN

M. EYRIERES. L’appareil est rempli d’huile ; la corde vibre-t-elle dans l’huile ?

M. BROWN. Non, seule la capsule manométrique est remplie d’huile, la corde vibre dans l’air ou dans le vide.

Dr DOHLER. I understand you need a two or three conductor cable and a tube from the transducer to the recorder. Do you know of any manufacturer who has a cable as mentioned above on the market.

M. BROWN. The United States Steel Company makes a number of armoured steel cables with either 1, 3, 3 or 4 conductors with breaking strengths of from 2000 to 20,000 pounds or more.
M. GODENHEIM, Le problème des ondes internes qui peuvent exister dans toute l'épaisseur de la mer (dans les bassins océaniques) est l'un des plus importants de l'océanographie actuelle. Les mesures manométriques de marée, aussi bien que celles d'hydrologie classique en sont perturbées. Il y a là toute une série d'études à entreprendre et il appartiend à P.A.I.O.P. de les susciter ou tout au moins de les développer.

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ABSTRACT

A tide gauge is described having an FM output. It utilizes a vibrating wire whose tension is varied directly by the applied hydrostatic pressure. The design of this device differs from previous vibrating wire devices in that the total force due to the hydrostatic pressure on the bellows is balanced entirely by the tensile stresses in the vibrating wire. In other words, the system is a "constant tension" system, whereas previous systems are essentially "constant displacement" systems. The result is a device which has extremely low temperature coefficients and hysteresis and an over-all accuracy of ± 0.03 % for a ± 20°C temperature change.

The power requirement for the electronics is 150 microwatts. The frequency range is typically from 300 cycles per second to 1000 cycles per second for pressure range of 2 to 22 psi or for tide heights of 5 to 50 feet. The output frequency is proportional to the square root of pressure. However, using simple digital logic circuitry, a direct readout in feet or meters is achieved.

OBSERVATIONS SUR LA COMMUNICATION DE M. BROWN

M. EYRIES, L'appareil est rempli d'huile ; la corde vibre-t-elle dans l'huile ?

M. BROWN, Non, seule la capsule manométrique est remplie d'huile, la corde vibre dans l'air ou dans le vide.

Dr DOLBERG, I understand you need a two or three conductor cable and a tube from the transducer to the recorder. Do you know of any manufacturer who has a cable as mentioned above on the market.

M. BROWN, The United States Steel Company makes a number of armoured steel cables with either 1, 3, 3 or 4 conductors with breaking strengths of from 2000 to 20,000 pounds or more.
THE DIGIDATA TECHNIQUE FOR RECORDING AND TRANSMITTING CURRENT TIDE AND OTHER OCEANOGRAPHIC DATA

Paul Ferris SMITH and A. Fred FEULING
Geodyne Corporation, Woods Hole and Waltham, Mass

Mr. President, Ladies and Gentlemen, in view of the interest of the members of this symposium in the problems of remote recording and in the advantages of digital methods for handling tidal height and other oceanographic parameters, it would appear valuable to discuss briefly the history behind the development of the digital recording and telemetering system to be described. This history will serve to illustrate how a practical embodiment of much that has been discussed in this symposium concerning the digital approach to oceanographic data collecting problems was developed.

In 1961 Dr. WILLIAM S. RICHARDSON of the Woods Hole Oceanographic Institution devised his now well known Digital Film Recording Oceanographic Current Meter which we manufacturer. This electro optic instrument is capable of recording more than 180,000 sets of three 1% numbers using one 6 volt battery. The digital information is stored on 16 mm photographic film. The instrument is shown in figure 1 and a section of the digital film record in figure 2.

On this film a dot represents the binary "one" and the absence of a dot a binary "zero". A single binary number corresponds to a set of such "ones" and "zeros" read across the film. This instrument is used for obtaining in situ current measurements of tidal and non-tidal in origin the open ocean, along the continental shelf and in estuaries and lakes. It is reported to have been the first instrument to record open ocean tidal currents. Because of the low power requirements and high information packing density of this digital film recording method, these instruments can be installed for recording in remote locations in the deep ocean for periods of time up to one year. Also, because of the large amount of data contained on one reel of film, it was necessary to develop automatic film reading facilities. Figure 3 shows the automatic film reader which we developed. This reader transfers every bit of digital information from the film to IBM 650, IBM 7090 or GE-225. This same reader simultaneously generates an analogue strip chart record for inspection and preliminary analysis.

Following this current meter system we developed other instrumentation to capitalize on the advantages of this digital film recording and reading technology. These included: Wind Recorders, Wave Recorders and Temperature Pressure Recorders. Figure 4 shows one of these, a Digital Temperature Recorder. This instrument is a wholly electronic instrument which is used to measure the resistance of a number of temperature sensing thermistors and flash a set of lamps through a fiber optic array to generate the digital optic image for recording on a 16 mm film as in the Current Meter. Being wholly electronic, this instrumentation technology led naturally to the development of a wholly electronic digital current meter system. Now instead of recording the digital information on film, the digital signals being in the form of electrical pulses could be telemetered by a radio link and recorded on magnetic tape. This tape recording could, of course, be accomplished either within the instrument or at the receiving station. Figure 5 is a photograph of this telemetering current meter system. This shows a current meter (without its pressure case), the buoy unit for controlling up to eight meters in a mooring string, the standard radio receiver and the analogue display unit for monitoring the digital data being received and recorded.

Parallel with this instrumentation development we produced a series of general-purpose oceanographically-oriented digital modules. These correspond to the digital building blocks from which modern high speed computers are assembled. However, these particular modules were designed for easy packaging in oceanographic pressure cases, for extremely low power requi-
Figure 1 : Digital Film Recording Current Meter.

Figure 2 : A Section of 16 mm Digital Current Meter Film Record.
Figure 3: The Geodyne Automatic Digital Film Reader.

Figure 4: A Digital Film Recording Multipoint Temperature Recorder.
Figure 3: The Geodyne Automatic Digital Film Reader.

Figure 4: A Digital Film Recording Multipoint Temperature Recorder.
Figure 5: The Goodwin Telemetry Current Meter System: Meter (without case); Buoy Unit; Commercial Receiver; Analogue Display Unit (Tape Recorder not shown).
ments and the utmost in reliability. This resulted in a digital module series which is relatively slow by modern high speed computer standards but adequately fast for oceanographic data acquisition purposes and therefore proportionately of lower cost than computer oriented types. Using these modules a general purpose oceanographic data acquisition instrument was designed. This is shown in figure 6 and is called a Model A-775 Silicon Solid State Telemetering Digitizer. It is this unit which is central to the Digishaq System I wish to describe today. Recently we have modified the mechanical construction of the digital modules to reduce their size and increase the packing density while in addition reducing the unit cost. Figure 7 illustrates these modules which are now assembled on plug-in boards. Four such boards are sufficient to contain the circuitry necessary for one A-775 Telemetering Digitizer. The operation of the Telemetering Digitizer is illustrated in the Block Diagram of figure 8. Essentially this unit consists of a twelve stage parallel ladder precision resistance network, error sensing amplifier and a counter. It can be used in systems which will handle the output of all oceanographic transducers and generate binary numbers proportional to the transducer output signals. Despite the great diversity which may be implied by this capability, the output of oceanographic transducers fall in a relatively small number of categories. They are either resistance changes, electrical voltage or current signals, pulses or shaft angles. For example, to measure the output of a two terminal resistive pressure transducer for tide measurements, the transducer is arranged to form one arm of a bridge circuit (see figure 8). A fixed pair of precision resistors forms the opposite side of this bridge and the twelve-stage resistive ladder the variable resistance whereby the bridge is balanced. This resistive ladder is under the control of the error sensing amplifier, which has a gain of approximately one million. Balance is obtained by the method of successive approximations during which transistor driven reed switches, under the control of the amplifier, switch the ladder network resistors into the bridge leaving them in or removing them depending upon whether the bridge is under or over balanced. Since the resistors in the ladder network increase in a binary fashion (conductance decreasing) this switching operation automatically generates a twelve bit binary number proportional to the conductance of the unknown resistor, which in the present example is proportional to pressure and therefore tidal height. A twelve bit binary number provides a quantization of better than one part in four thousand (212). The ladder network resistors used in the present
model have an 0.025 % tolerance. However, resistors having another order of magnitude better precision can readily be obtained if this is ever required. Reed switches are used because of their ideal switching characteristics and because their life expectancy exceeds $10^6$ operations, while being adequately fast for sampling oceanographic variables. Obviously this same balancing procedure applies equally to any resistive transducer such as a potentiometer attached to a shaft, a thermometer, a platinum resistance thermometer, etc. However, when it is desired to measure shaft angles with the minimum of loading, optical encoding discs are employed. An example is shown in the digital compass illustrated in figure 9. Here photo resistors are used to sense the encoding disc pattern and a parallel transfer gate to introduce this digital word to the digitizer. Either potentiometric or optical encoding could be employed to digitize for telemetry standard tide gauges.

Finally, while all the necessary electronics is available within the telemetering digitizers to perform the function of counting, pulses such as from a Vibrorun pressure transducer, are handled in this system by the use of an auxiliary digital counter and are introduced into the digitizer using a parallel transfer gate.

Referring again to figure 8, the rearrangement of the A-775 Digitizer to measure seawater conductivity can be seen. Here the digital number corresponding to this parameter is automatically generated by the error sensing amplifier switching the parallel ladder network resistors into the bucking circuit of an inductive conductivity sensor. It can be seen that an extremely precise telemetering or recording tide instrument for either shallow or deep water, may be assembled using this digitizer. Not only can the pressure be measured to the full accuracy of the chosen transducer but this system can also determine precisely measured temperature and salinity characteristics of the entire overlying water column. A single simple serial data train at up to 19 parameters (pressure, temperature, conductivity, time and so forth) per second contains all this information. In practice each set of readings is separated by a group start pulse for identification purposes.

Instruments assembled using this central processor may be either film or tape recording. Figure 10 is a photograph of a multi parameter film recorder of the type shown in the Block Diagram of figure 11. These instruments may be single purpose instruments, such as a multipoint temperature recorders or telemetering current meters such as the type shown in figure 12. This Telemetering Current Meter is designed for remote powering.

While many have already been mentioned in this symposium, the advantages of this integrated digital system are those of digital systems in general. The oceanographic parameter may be sensed at a high enough rate to sample all frequency components present in the signal. The precision of the output is limited only by the sensor and thus laboratory accuracy is obtainable in remote unattended locations. Digital systems are inherently drift free, being dependent only upon the most stable elements for reference purposes; that is, precision resistors. For telemetry transmission purposes extremely low bandwidth requirements are another major advantage. Furthermore, in view of the expanding digital technology this system is not soon to outmoded. In this connection also, this same technology provides the tremendous advantage of straightforward automatic data processing and analysis.

In addition standard digital redundancy checks such as the addition of a parity bit to each word are employed which together with the inherently high noise rejection of digital systems produces an extremely secure error free device. Furthermore, this device is applicable to both research and survey needs for both the immediate data presentation for survey purposes and the precision requirements of research purposes are met simultaneously. And, because of the standardization of digital format instruments may be intermixed to meet these two needs. For example, in one system being provided for the U.S. Coast and Geodetic Survey, a shipboard electric typewriter output is immediately available to present the data from the telemetered signal of current speed and direction from a series of instruments from the surface to the bottom on separate moorings. From these same instruments salinity, temperature and depth information is simultaneously transmitted and tape recorded aboard ship for later research analysis. This identical approach could be employed for the immediate and simultaneous analogue presentation at a central station of the output of a number of coastal tide stations linked either by a telephone line or a radio telemetry system while this data together with collateral information was being stored on magnetic tape in digital form for subsequent detailed study.

OBSERVATIONS SUR LA COMMUNICATION DE M. SMITH

Dr. DOHLE. You mentioned that the parts used in the system are inexpensive; could you give me an approximate cost for both transducer and translator; can the translator be used for several parameters of recordings ?

M. SMITH. The complete assembled, Silicon Solid State Telemetering Digitizer costs less than $2,000 and will handle up to 64 separate transducers sequentially. The individual transducer costs depend upon the type, and commutation circuitry must be provided for each, but, for example, a thermometer probe may cost $7.00 and the commutator $50; a pressure transducer $150 up and the commutation for this $350.

Figure 9: Digital Compass with Fiber Optic Array.
model have an 0.025 % tolerance. However, resistors having another order of magnitude better precision can readily be obtained if this is ever required. Reed switches are used because of their ideal switching characteristics and because their life expectancy exceeds $10^6$ operations, while being adequately fast for sampling oceanographic variables. Obviously this same balancing procedure applies equally to any resistive transducer such as a potentiometer attached to a shaft, a thermistor, a platinum resistance thermometer, etc. However, when it is desired to measure shaft angles with the minimum of loading, optical encoding discs are employed. An example is shown in the digital compass illustrated in figure 9. Here photoresistors are used to sense the encoding disc pattern and a parallel transfer gate to introduce this digital word to the digitizer. Either potentiometric or optical encoding could be employed to digitize for telemetry standard tide gauges.

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Figure 9: Digital Compass with Fiber Optic Array.
Figure 10: Multi-Parameter Oceanographic Film Recorder.
Figure 10: Block Diagram of a Multi-Parameter Oceanographic Film Recorder.

Figure 11: Block Diagram of a Multi-Parameter Oceanographic Telemetering Instrument.
SOME ASPECTS OF AUTOMATIC PROCESSING OF TIDAL DATA

G. W. GROVES
Hawaii, Institute of Geophysics

and

W. H. MUNK
University of California (U.S.A.)

ABSTRACT

The purpose of the automatic procedures considered has been to prepare and store digitally on magnetic tape, a reliable series of hourly tidal heights at single stations.

Thus, the problems have been to (a) detect and correct occasional erroneous values, (b) assure that correct values are faithfully reproduced between stages, (c) interpolate gaps, and (d) store resulting continuous, reliable series in a convenient form.

The problem of error detection and correction was met by assuming that all errors are "isolated", that is, that the values in the vicinity of each erroneous value are correct. Then, a linear interpolation method is used. The same problem without the "isolated" assumption has apparently never been dealt with satisfactorily.

Reliability in copying tapes is usually achieved by tape checks within the computer, but these have been found insufficient when the quantity of data becomes large. Therefore, additional redundancy in the form of check sums, etc., is introduced.

Interpolation of gaps in tidal data has not been automated, although it should be fairly straightforward. For small gaps of a few hours, ordinary interpolation is used. For intermediate gaps (less than 3 days), interpolation is made between values at 24-hour or 25-hour intervals, because a sequence of tidal heights at these intervals usually plot smoothly. Longer gaps are usually interpolated only in the "decimated" (smoothed) data series, whose interval is substantially longer than hourly. The hourly values can also be interpolated by adding the predicted hourly series to a curve interpolated from the decimated values, with provisions for fairing in smoothly at the end points.

Storage of the final data series is in BCD (binary coded decimal) for maximum compatibility with different computers. The pertinent information, including units, format, length, start time (corresponding to first value), time interval, etc., is stored on the same tape with the data.

OBSERVATIONS SUR LA COMMUNICATION DU Dr MUNK

Dr ROSSITER: Have any of your programs been able to detect and correct groups of consecutive readings, which in themselves are smooth?

Dr MUNK: Yes, the error analysis of a "recorded minus predicted" time series would show such discontinuity before and after such a displaced group. When we obtain two such discontinuities, we examine the pertinent portions of the records by hand.

Dr GROVES: As an example, I recall a case where two days of hourly values were interchanged by mistake. The ERRORS program showed errors at beginning and end of the displaced values, but did not correct the values. This error was discovered because corrections which are near to each other in time are manually examined. It is difficult to see how to devise an automatic method which would catch such errors.
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Figure 12: Remotely Powered Telemetering Current Meter.
Dr. HADDAH. Concerning checking of hourly data; we read hourly values and high and low water heights and times separately, calculate from hourly values high and low waters, by polynomials for example, and compare these results with the observed high and low waters.

Dr. HEFLA. Is it possible to correct by means of data machines an error in the function of a tide-gauge brought about by siltage or clay in the gauge well and resulting in the reduction of tidal range, both astronomical and meteorological?

Dr. MUNK. I don't know. Under certain, very simple circumstances, the answer is yes, but in all events this would prove awkward. We hope to be able to separate that part of the motion common to all tide gauges (the secular change) from a station residual. I do not know what the best method would be. We hope to do this after the complete observations for about 20 stations have been corrected and stored.

TIDE MEASURING BUOY SYSTEM

Philip A. FOLWELL (*) and Duane E. MADDOX

Marine Advisers, Inc.

and

James H. TRIPLETT

U.S. Naval Oceanographic Office (**) 

The measurement of tides is perhaps one of the most common measurements of oceanographic phenomenon. It is also one of the most important or useful measurements, as these data in the form of tide tables, are used by mariners and fishermen the world over.

Nearly all the tide measurements to date have been made by a tide staff and the information recorded on a strip-chart recorder. There are two major disadvantages to measurements of this type -- the restriction to near shore installation and the expensive time-consuming data reduction.

This paper describes an unattended tide measuring buoy system which records the data digitally on an incremental magnetic tape in a format for automatic data processing. The system also contains a telemetry link so these data may be played out on command without disturbing the mooring system.

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This system was conceived by personnel of the U.S. Navy Oceanographic Office to take measurements of tidal data in areas where shore-based facilities would be difficult to install or where the duration of the desired data would make a fixed installation uneconomical. The prototype system was designed and built by Marine Advisers, Inc., to specifications provided by the U.S. Naval Oceanographic Office.

The system is required to record tide measurements to an accuracy of one part in six hundred of the sensor's installed depth of one hundred feet. These tide readings must be taken at fifteen-minute intervals and stored for up to six weeks, pending subsequent radio telemetry playback over line of eight distances. Unattended operation of the system for at least six months is required and initiation of the telemetry playback sequence should be by means of an acoustic command link through the water.

Operation of such a system could be as follows: after installation, monthly data collection missions would be scheduled by a suitable vessel having telemetry receiving facilities and the acoustic transmitter necessary to initiate playback of the data. Upon reaching the vicinity of the installation, the vessel would lower the acoustic transmitting transducer and trigger the playback system in the buoy. The tide data acquired since the previous transmission would be telemetered to the attendant vessel and stored in digital form on punched paper tape.

The prototype system (see figure 1) consists of a three thousand pound anchor adjacent to which is attached a housing containing an explosively actuated release, the pressure transducer used to measure tide and a twostage hydraulic filter which attenuates sea and swell pressure fluctuations. The housing, in turn, is connected to a subsurface buoy containing the main electronic section by means of a combined air/aerial/electrical cable. The subsurface buoy incorporates an electrical swivel at its upper extremity to which is attached a slack painter providing electrical conductors and attachment to a surface buoy which houses the transmitter and carries a tower for the navigational warning lights, antenna and solar cells.

The pressure transducer used is a vibrating wire type manufactured by White Avionics Corporation. The output of this device is a frequency inversely proportional to pressure. The tension of the vibrating wire is controlled by a diaphragm open to the pressure to be measured and is sustained in oscillation by electrical feedback circuits. The hydraulic filter was designed and built by Marine Advisers, Inc. It incorporates copper tube capillaries and spring-loaded, welded, stainless steel bellows. The filter response is shown in figure 2.

The digital data acquisition system is similar to other units previously developed and built at Marine Advisers, Inc. It differs from them, however, in its data format and its provision of telemetry playback facilities. Standard digital techniques are used to measure the period of the transducer frequency in terms of the number of cycles of a crystal controlled reference frequency. The fifteen bit binary number obtained in this manner is recorded on magnetic tape as five octal numbers together with similar run-numbering groups of octal numbers. A format control signal is recorded on another track to separate the groups of five octal numbers. Other tracks are utilized to record parity and the signal used to form an octal zero for playback.

The tape transport (figure 3) uses one-inch tape and seven-track magnetic heads. Incremental digital recording techniques are used after which the tape is stored in random fashion in an intermediate storage magazine for the periodic playback. During playback, the magnetic tape is drawn from the intermediate storage magazine at a constant speed past a Hall-effect head and is then stored on a take-up reel.

Digital pulses are recorded on the magnetic tape using a longitudinal spacing of .020 inches. This enables conversion of the data onto identical format paper tape to be made by existing equipment at the Marine Advisers' facility. The playback speed is .25 inches per second, so 125 characters per second are transmitted by the telemetry system. This rate is well within the speed capability of moderate speed paper tape punches which are eminently suitable for incorporation in a receiving facility for these data.

Signals from the playback head are conditioned by amplifiers and single-shot circuits to form pulses thirty milliseconds long clamped to a reference voltage. These pulses are fed via
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Digital pulses are recorded on the magnetic tape using a longitudinal spacing of 0.020 inches. This enables conversion of the data onto identical format paper tape to be made by existing equipment at the Marine Advisers' facility. The playback speed is 25 inches per second, so 12.5 characters per second are transmitted by the telemetry system. This rate is well within the speed capability of moderate speed paper tape punches which are eminently suitable for incorporation in a receiving facility for these data.

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Figure 1 - General arrangement.
binary weighing resistors to the summing junction of an operational amplifier, the output of which in turn drives a standard IRIG band 6 voltage-controlled oscillator. This combination of units in effect constitutes a digital-to-frequency converter which produces a series of tone codes corresponding to octal numbers, the format control and the band edges which are used to transmit a set-up sequence of tones for the receiving equipment prior to actual transmission of data.

![Figure 3](image)

**Figure 3**

The output of the voltage-controlled oscillator is used directly as the modulating signal of a telemetry transmitter having a carrier frequency of 340.6 megacycles per second. The utilization of telemetry equipment widely used in aircraft and missile test work means that receiving equipment is readily available in many existing facilities. Checkout and acceptance testing has been performed using such a facility having a discriminator and heated stylus chart recorder. A sample of the record obtained in this fashion is shown in figure 4. It will be noted that the positioning of the tone codes in the band corresponds to the commonly used eleven-point calibrating frequencies.

The telemetry transmitter used is a standard frequency modulated type operating in the 215 to 360 megacycle-per-second band. It supplies 0.5 watts output to a ground plane rod equipped monopole antenna located at the top of an 18-foot triangular tower. Power for the main electronics is supplied from a stack of sealed nickel cadmium batteries which are charged by solar cells.

Checkout and acceptance tests have been performed and have proven successful operation of all parts of the system. Triggering of the playback has been initiated by the acoustic command system at a distance of four miles and telemetry transmissions have been made over a fifteen mile distance. No attempts have been made to determine the distance limitations of the acoustic command and radio telemetry links.
LES TENDANCES ACTUELLES DE L'ORGANISATION
DU MATÉRIEL MARÉMÉTRIQUE À L'I.G.N.

G. DUCHER
Ingénieur Géographe

L’Institut Géographique National, dans sa recherche de résultats plus précis et plus sûrs sur le niveau moyen mensuel a porté quelques modestes efforts dans deux directions visant à améliorer les appareils mis à sa disposition sur les côtes de France :

1/ amélioration des médimarémètres
2/ amélioration des marégraphes,

1 - AMÉLIORATION DES MÉDIMAREMÈTRES

Les médimarémètres sont des appareils généralement mal connus à l’étranger pour différentes raisons, aussi vais-je en rappeler brièvement le principe,

Il s’agit essentiellement d’un tube vertical enfoncé à la partie inférieure d’un vase poreux formant le passage de l’eau. Photo 1 - Sur la figure de gauche on voit le tube et son vase poreux. L’eau entre lentement dans le tube jusqu’à ce que le niveau interne rejoigne le niveau externe, supposé fixe ici. Sur la figure de droite on voit qu’aun bout d’un certain temps égal à 0 jours, le tube n’est vidi d’environ des deux tiers ; il dépend de la porosité du vase poreux et de l’appelle module du vase poreux.

Placé dans la mer où le niveau varie surtout selon une somme de lois sinusoidales, le médimarémètre va voir son niveau interne varier également de la même manière, avec les mêmes périodes, mais avec des amplitudes très réduites et surtout autour du même axe moyen, C’est-à-dire qu’il sera parfaitement adapté à mesurer le niveau moyen de la mer, le niveau moyen interne du médimarémètre oscillant avec le niveau moyen de la mer tandis que les amplitudes internes sont suffisamment réduites par le frein du vase poreux pour ne justifier qu’une seule lecture par jour.

Jusqu’à présent cette lecture s’effectuait en trempant une règle graduée muni d’une bande de papier coloré qui virait à l’eau de mer. Cette méthode simple est hélas sujet à beaucoup d’inconvénients,

D’abord c’est un véritable plongement d’eau que l’on fait à chaque lecture, faisant d’autant la lecture suivante du fait de la lenteur du niveau interne à rejoindre son niveau exact. Aucune correction n’a pu y être apportée de manière satisfaisante.

De plus, à chaque lecture la règle entraine avec elle d’indéterminées poussières, de grains de sable, des fibres de papier qui viennent se déposer au fond et colmatent rapidement le vase poreux rendant, comme on le voit très à l’heure, très imprécis le médimarémètre.

C’est ce mode de lecture qui explique le discrédit dont on a souvent accablé cet appareil.

L’amélioration essentielle qu’a apportée l’I.G.N. est dans le remplacement de ce système de lecture par d’autres systèmes,

D’abord un système pneumatique a été essayé, Photo 2. Dans ce système, une pompe actionne lors de la mesure, envoie de l’air dans un tube immergé en permanence dans le médimarémètre. L’air refoule l’eau sur une hauteur h à mesurer puis s’échappe lentement, en bulle à bulle ; h est transmis pneumatiquement à un manomètre qu’il suffit de lire.
LES TENDANCES ACTUELLES DE L'ORGANISATION
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La précision, toutes corrections faites, est de 4 à 5 mm. 
Puis, un système électrique a été essayé et adopté définitivement.

Photo 3. Une règle graduée munie d’une pointe effilée est aménée au contact du niveau interne du médiarmétre. Dès que le contact se produit, un très faible courant de 20 à 30 A circule dans la pointe et l’eau de mer et après amplification par transistor, allume un voyant consommant 50 mA. On fait alors la lecture, puis on relève la règle jusqu’à la lecture suivante.

La précision atteint quelques dixièmes de millimètres, grâce à l’amplificateur qui rend l’allumage du voyant immédiat dès que le contact de la pointe avec l’eau est établi.

Voici trois ans que ce dispositif fonctionne avec satisfaction à Marseille.


Photo 5. Et voici la partie inférieure avec sa pointe effilée.

On peut donc dire que le système de lecture électrique convient bien et qu’il est destiné à remplacer peu à peu les dispositifs à papier encore utilisés.

Mais revenons au niveau interne du médiarmétre car il ne suffit pas de savoir le mesurer encore faut-il que ce niveau ait un sens. On a vu au début que ce niveau variait en gros sinusoidalement autour du niveau moyen de la mer. Cela serait vrai si les variations du niveau instantané de la mer n’étaient dues qu’aux marées d’ondes sinusoidales. Mais en fait, le niveau de la mer comporte des variations imprévisibles dues aux coups de vent, variations de pression, températures, densités; et c’est finalement ces variations-là qu’il est intéressant de mesurer.

Ces variations ont un aspect aléatoire, désordonné, mais on peut y découvrir néanmoins des portions sinusoidales s’étendant sur quelques jours. Ces portions sont en somme le premier terme du développement en série de ces variations. Elles peuvent atteindre 5 à 10 cm par jour, même en Méditerranée, et durer 5 jours.

Aussi est-il normal d’en tenir compte pour le choix de la porosité du vase du médiarmétre.

Photo 6. Sur cette figure on voit comment réagit en théorie le niveau du médiarmétre devant un niveau externe en variation linéaire. On voit que si le vase est peu poreux (p > 5 j) le décalage granulé de jour en jour et peut dépasser 10 à 20 cm ! et fausser d’une manière catastrophique le niveau mensuel. Par contre, il existe des modules (p = 0,5 j) pour lesquels la lecture quotidienne donne exactement la valeur du niveau moyen des 24 heures précédentes.

De même des modules tels que 0,25 j donneraient exactement la valeur du niveau moyen des 12 heures précédentes, mais nécessiteraient 2 lectures par jour. Ceci peut paraître théorique mais l’expérience le vérifie bien, surtout dans les mers à faible marée et les résultats obtenus à Marseille montrent bien qu’un médiarmétre à vase très poreux permet d’avoir de bons résultats sans risquer de grosses erreurs systematiques dues aux ondes résiduelles internes S2 et K1, même avec une seule lecture par jour.

Photo 7. Sur cette figure on voit les résultats de deux appareils ayant fonctionné à Marseille simultanément. L’un, de module 0,5 jour suit assez fidèlement le niveau moyen quotidien donné par notre marégraphe fondamental, l’autre de module 4,5 jour est trop amorti, retarde beaucoup et donne de mauvais résultats.

Les deux années de fonctionnement de divers appareils à Marseille ont permis d’établir quelques statistiques.

Photo 8. Voici les statistiques des écarts entre les niveaux moyens quotidiens réels et les lectures quotidiennes des médiarmétries.
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Lectures n° 1 2 3 4 5 6 7 8 ... n...

\[ e \]
\[ e \]
\[ e \]

niveau interne théorique

niveau interne réel

e : prélèvement lors de la mesure

c : erreur de la goutte d'eau

Figure 4 - Erreur de la goutte d'eau.

Figure 5 - Schéma de principe des lectures pneumatiques.

Figure 6 - Schéma de principe des lectures électricques.

Figure 7

Photo 2

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4,90 m

4,80 m

4,70 m

4,60 m

mai

juin

juillet

— 4,90 m

— 4,80 m

— 4,70 m

— 4,60 m

Photo 3

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Figure 4 - Erreur de la goutte d'eau.

Figure 5 - Schéma de principe des lectures pneumatiques.

Figure 6 - Schéma de principe des lectures électriques.

4.90 m
4.80 m
4.70 m
4.60 m

mai
juin
juillet

— 4.90 m
— 4.80 m
— 4.70 m
— 4.60 m

Photo 2

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2. INTRODUCTION DES NARCOTHIQUES

L'usage manque d'autorité peut être conséquent.
La définition du code d'éthique de cette profession est complexe.

Les médecins, en tant que professionnels, doivent agir en fonction des normes éthiques.

Pour réaliser l'équipe, il est nécessaire de développer des compétences spécifiques.

Les médicaments, à savoir les anesthésiques, ont un effet sur le corps humain.

La presse médicale montre que l'utilisation des médicaments peut être appropriée.

Les médicaments sont prescrits par les médecins, selon les dispositions légales et éthiques.

La prescription de médicaments doit être réalisée de manière appropriée.
On voit que l'erreur moyenne quadratique est de ± 10 mm pour des vases très poreux mais grandit vite si le vase s'encrasse.

Photo 9. Voici les statistiques des écarts sur les moyennes mensuelles à Marseille. L'erreur moyenne quadratique tombe à ± 5 mm pour les vases très poreux (0,2 < p < 1 jour). Il est difficile d'espérer mieux avec une seule lecture par jour.

Photo 10. Voici, toujours à Marseille, les résultats de 1962-63. Un vase très poreux a présenté des écarts minimes avec le marégraphe tandis qu'un autre peu poreux a donné des discordances importantes.

Que conclure de ceci ?

On peut affirmer que les médiamétramètres peuvent, à condition d'être équipés d'un nouveau système à lecture électrique et d'avoir un vase assez poreux, donner dans les mers à faible marée (1 m) des erreurs mensuelles maximales inférieures à ± 10 mm.

Ces appareils sont donc intéressants. De plus, ils sont bon marché, robustes, faciles à poser et à entretenir.

Le changement annuel du vase poreux suffit pour maintenir une bonne porosité, tandis que le petit nombre de mesures (1 par jour) facilite le calcul du niveau mensuel.

Ils ont l'inconvénient de ne pas donner de lectures instantanées et de nécessiter un opérateur 5 minutes par jour.

Tels qu'ils sont ils peuvent assurer dans les mers à faibles marées une chaîne de contrôle des marégraphe, subvenir à leurs pannes pour renforcer ainsi la permanence des mesures, élément primordial de la marémétrie. L'I.G.N. en a installé à Marseille. D'autres pays méditerranéens s'y intéressent.

Dans les mers à fortes marées, la nécessité d'effectuer plusieurs lectures par jour si l'on veut garder de la précision en rend indispensable l'automatisation. Rien n'a été encore entrepris dans ce sens à l'I.G.N., mais cela peut en valoir la peine car on peut penser avec 4 ou 6 lectures par jour obtenir en Atlantique ou en Manche une précision analogue à celle des marégraphe. L'avantage serait, outre l'allègement des mesures et des calculs, dans l'accroissement de la sécurité de fonctionnement et de la durée de vie d'un appareil qui viendrait seulement sonder 4 ou 6 fois par jour un niveau situé un mètre seulement plus bas et resterait en repos 23 h 30 sur 24 heures.

Voyons maintenant les marégraphe.

2 - AMÉLIORATION DES MARÉGRAPHE

L'amélioration des marégraphe à l'I.G.N., a d'abord porté sur le calage des marégraphe. Il s'agit d'aligner sur place avec précision les indications des marégraphe par rapport à l'altitude NGF du plan d'eau. Pour cela, on utilise maintenant à l'I.G.N. un sondeur à pointe électrique spécialement conçu et réalisé par l'I.G.N.

Photo 11. Le sondeur comporte essentiellement une sonde à pointe électrique basée sur le même principe que la lecture électrique précédemment vue pour le médiamétramètre. La sonde est suspendue à un décâmètre étalonné et enroulé à l'arrière de manière à afficher directement la hauteur de la mar au-dessus du zéro marin local.

Photo 12. Voici une vue de la sonde à pointe électrique ouverte, comportant à l'intérieur les piles, pointe, voyant et ampli.

L'étalonnage d'un marégraphe se fait selon un cycle complet de marée et de manière à éliminer les jeux et l'hystérésis et demande pratiquement 2 jours sur place. La précision obtenue est de l'ordre de 0,5 mm.
Figure 9

Figure 10

Figure 11

Variation du niveau interne des médianes en fonction d'un niveau externe linéaire.

Légende

en ordonnée:
- niveau externe
  - niveau moyen quotidien externe
  - niveau interne
  - lecture quotidienne interne

en abscisse: le temps

ρ : module du vase poreux, en jours

ρ = 0.2 j
ρ = 0.5 j
ρ = 1 j
ρ = 2 j
ρ = 5 j
ρ = 10 j

0 1 2 3 4 jours 0 1 2 3 4 jours 0 1 2 3 4 5 jours
Niveaux quotidiens à Marseille.

Figure 12

En ordonnée, pourcentage des écarts (1962-1963)

Figure 14

Influence du module $\rho$ sur les écarts entre les niveaux moyens quotidiens du marégraphe et les lectures quotidiennes des médiamarémètres à Marseille.
En ordonnée, pourcentage des écarts (1962-1963)

Figure 14

Influence du module $\rho$ sur les écarts entre les niveaux moyens quotidiens du matrographe et les lectures quotidiennes des médimarémètres à Marseille.

Photo 3
Influence du module $p$ sur les écarts entre les niveaux moyens mensuels du marégraphe et les moyennes mensuelles des météromètres à Marseille.

Figure 15

Photo 9
Figure 15

Influence du module $p$ sur les écarts entre les niveaux moyens mensuels du marégraphe et les moyennes mensuelles des étalonnages à Marseille.

Photo 9

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Les avantages de cet appareil résident dans sa grande précision, le faible encombrement de son flotteur, ses nombreux dispositifs de sécurité et la simplicité de son exploitation.

Mais il présente évidemment l'inconvénient d'un prix légèrement plus élevé et d'une fragilité apparente plus grande due à la complexité de sa réalisation.

Son exploitation pratique au Havre permettra de savoir ce qu'il en est exactement de tout cela et donnera de nouveaux éléments pour l'organisation future du matériel marémétrique à l'H.G.N.

**OBSERVATIONS SUR LA COMMUNICATION DE M. DUCHER**

De WENDEL FIELDER. J'accons à M. DUCHER que l'atmosphère des observational météorologique est quelque chose qui n'est pas facile à obtenir, mais que, par la même occasion, il est possible d'obtenir une précision extrêmement haute.

M. CAHIER. Quelle est l'amplitude de la mer interne du modérateur avec des vases très poreux ? Certaines cœurs ne sont pas diminuées lorsqu'on fait une seule lecture par jour ; il y a une erreur systématique ; peut-on l'évaluer ?

M. DUCHER. L'amplitude interne "e" est liée à l'amplitude externe "A" par :

\[
\frac{A}{B} = \frac{T}{2 \pi}
\]

où T est la période en jour et O est le module du vase poreux en jour de sortie à Marseille ou à la tableaux suivant donnant l'amplitude interne "e" semi-durée

<table>
<thead>
<tr>
<th>Semi-durée externe &quot;A&quot;</th>
<th>module ( p = 0.5 )</th>
<th>module ( p = 1.1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A = 1 ) cm</td>
<td>( a = 0.4 ) cm</td>
<td>( a = 0.4 ) cm</td>
</tr>
<tr>
<td>( A = 2 ) cm</td>
<td>( a = 1.6 ) cm</td>
<td>( a = 0.8 ) cm</td>
</tr>
</tbody>
</table>

soit une amplitudo "a" de l'ordre de \( \pm 0.5 \) à \( \pm 1.5 \) cm.

D'un autre côté, on ne fait qu'une lecture par jour, certaines cœurs ne sont pas diminuées au cours du mois. Il s'agit surtout de l'amplitude \( S_2 \) (amplitude \( A = 2.4 \) cm) et de l'amplitude \( K_1 \) (amplitude \( A = 5.2 \) cm) qui font courir un risque d'erreur systématique compris en gros entre \( \pm 0.5 \) et \( \pm 1 \) cm comme le montre le tableau suivant :

<table>
<thead>
<tr>
<th>module ( p = 0.5 )</th>
<th>module ( p = 1.1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Onde ( S_2 ) ( c &lt; \pm 0.4 ) cm</td>
<td>( c &lt; \pm 0.2 ) cm</td>
</tr>
<tr>
<td>Onde ( K_1 ) ( c &lt; \pm 0.9 ) cm</td>
<td>( c &lt; \pm 0.5 ) cm</td>
</tr>
</tbody>
</table>

Il y a lieu de noter que l'erreur due à \( K_1 \) varie au cours de l'année et s'élimine sur le niveau annuel. Les résultats de Marseille ont bien confirmé cela. En Atlantique il faudrait 2 ou 4 lectures par jour.

M. CAHIER. Il y a un décalage de phase, outre la réduction d'amplitude, comme l'a expliqué M. WENDEL FIELDER. Cela réduit de la théorie de l'appareil, telle qu'elle est faite par notre inventeur Ch. LAUDEMANN. Celui-ci cherchait à obtenir une mise très faible. On est généré par la modification du module d'amortissement, qui est due à l'encrassement progressif du vase poreux et qui fausse la valeur des mesures.
L'installation de sondeurs à différents ports de nos côtes est en cours.

Côté marégraphe proprement dit, l'Institut Géographique National est en train d'installer deux marégraphe, l'un, de modèle classique, fabriqué par la maison Saint-Chamond-Grenat est prévu pour fonctionner à La Pallice dès septembre 65. Cet appareil fournit un enregistrement de distance sous forme d'un graphique replié sur lui-même à l'échelle 1/10 - avec une précision de l'ordre de 5 mm d'eau. La transmission à distance est très sensible et pourrait donner une précision de 2 mm d'eau si un autre système d'enregistrement lui était adapté.

Il serait possible et souhaitable d'adapter un autre dispositif d'enregistrement (compteur imprimant, limbe codé) aussi cet appareil semble en valeur la peine de par sa robustesse et la qualité des services qu'il a déjà rendus par ailleurs.

Malheureusement l'17, G.N., faute de moyens disponibles, n'a pas encore pu s'y atteler

Son flotteur de 15 cm de diamètre suspendus à un treuil, est maintenu par un ressort servant de contrepoise.

Photo 13. Le treuil, très sensible, n'entranne directement aucun organe de mesure : le flotteur ne fournit donc aucun effort et il est libre de suivre très fidèlement le niveau de l'eau dans la limite étroite de deux butées électromécaniques placées de part et d'autre d'un doigt contacteur fixé sur l'axe du treuil. Un déplacement du niveau de l'eau de ± 0,4 mm met en marche un moteur asservi qui commande :

1/ Un déplacement correspondant des butées permettant l'obtention d'un nouvel état d'équilibre
2/ La mise en route du transmetteur et de l'enregistreur à distance.

La sensibilité de cette jauge est de ± 0,4 mm.

Photo 14. Fin Photos.

En cas de panne de courant EDF, il y a blocage de l'appareil interdisant tout désalignement dans l'enregistreur.

Le flotteur reste immobilisé, immérité ou en l'air, sans inconvénient. Après la panne, il rejoint le niveau de l'eau.

L'17, G.N. a adopté une sécurité supplémentaire interdisant toute mesure pouvant survenir après une panne et avant que le flotteur n'ait rejoint le niveau de l'eau.

L'enregistrement se fait actuellement sur une bande de papier avec frappe en clair de la hauteur d'eau en mm. Un enregistrement sur bande perforée est prévu par le constructeur mais n'a pas encore été acheté par l'17, G.N.

La précision finale est de l'ordre du mm.

L'17, G.N. a mis au point un dispositif déclenchant la frappe à partir d'une horloge mère Brillée. Le cadence de mesure prévue est de 1 par heure ; elle pourrait être réduite de 2 ou plus par heure. Mais une étude préalable a montré que le niveau moyen au Havre déterminé sur 12 jours par des mesures horaires ne différait pas plus de 1 mm du niveau déterminé par des mesures espacées de 10 minutes.

Enfin pour contrôler le bon fonctionnement de la jauge l'17, G.N. a adopté un dispositif provoquant automatiquement la remontée du flotteur jusqu'à une butée fixe. une fois par jour, et la frappe de cette butée.

Ainsi tout désalignement éventuel de plus de 1 mm sera dépisté.

L'appareil est en cours de finition à l'17, G.N. où l'on prévoit son installation au Havre vers novembre 1965.

Les avantages de cet appareil résident dans sa grande précision, le faible encombrement de son flotteur, ses nombreux dispositifs de sécurité et l'absurdité de son exploitation.

Mais il présente évidemment l'inconvénient d'un prix légèrement plus élevé et d'une fragilité apparemment plus grande due à la complexité de sa réalisation.

Son expérimentation prochaine au Havre permettra de savoir ce qu'il en est exactement de tout cela et donnera de nouveaux éléments pour l'organisation future du matériel marémétrique à l'17, G.N.

OBSERVATIONS SUR LA COMMUNICATION DE M. DUCHER

M. WEINERSFELD, I agree with M. DUCHER that the midisrameter gives something which may be called mean sea level; but only for very small tides and small disturbances. If one has to reduce a considerable tide, a large time lag has to be accepted. This time lag disturbs to a high degree the envisaged relation between "mean sea level" and a reading of the midisrameter. So, if the filtering capacity is small, many readings a day are necessary; if the filtering capacity is great, a time lag make the readings senseless. Moreover, the filtering capacity will not be constant in course of time. So the midisrameter principle must fail in most cases to give adequate information.

M. CAHIRE. Quelle est l'amplitude de la mer comme intermédiaire du baromètre avec des vases très purs ? Certaines ondes ne sont pas éliminées lorsqu'on fait une seule lecture par jour; il y a une erreur systématique ; peut-on l'éviter ?

M. DUCHER. L'amplitude interne "a" est liée à l'amplitude externe "A" par :

\[
A = \frac{1}{2} a \quad \text{et} \quad A = \frac{1}{2} a = \frac{5}{2} \quad \text{si} \quad T = 1 \text{ et } \alpha = \text{module du vase poreux en juillet}
\]

où T est la période en jour et \(\alpha\) est le module du vase poreux en jour de sortie à Marseille on a le tableau suivant donnant l'amplitude interne "a" semi-diurne

<table>
<thead>
<tr>
<th>Semi-diurne externe &quot;A&quot;</th>
<th>module ( p &gt; 0,5 )</th>
<th>module ( p &gt; 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A = 1 5 cm</td>
<td>a = ± 0,8 cm</td>
<td>a = ± 0,4 cm</td>
</tr>
<tr>
<td>A = 1 10 cm</td>
<td>a = ± 1,6 cm</td>
<td>a = ± 0,8 cm</td>
</tr>
</tbody>
</table>

soit une amplitude "a" de l'ordre de ± 0,5 à ± 1,5 cm.

Donc, si on ne fait qu'une lecture par jour, certaines ondes ne sont pas éliminées au cours du mois. Il s'agit surtout de l'onde \( S_2 \) (amplitude A = ± 2,4 cm) et de l'onde \( K_1 \) (amplitude A = ± 5,2 cm) qui font courir un risque d'erreur systématique compris en gros entre ± 0,5 et ± 1 cm comme le montre le tableau suivant :

<table>
<thead>
<tr>
<th>module ( p &gt; 0,5 )</th>
<th>module ( p &gt; 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ondes ( S_2 )</td>
<td>( c &lt; ± 0,4 ) cm</td>
</tr>
<tr>
<td>Ondes ( K_1 )</td>
<td>( c &lt; ± 0,9 ) cm</td>
</tr>
</tbody>
</table>

Il y a lieu de noter que l'erreur due à \( K_1 \) varie au cours de l'année et s'élève sur le niveau annuel. Les résultats de Marseille ont bien confirmé cela, En Atlantique il faudrait 2 ou 4 lectures par jour.

M. CAHIRE. Il y a un décalage de phase, ouest la réduction d'amplitude, comme l'a expliqué M. WEINERSFELD. Cela résulte de la théorie de l'appareil, telle que la fait son inventeur Ch. LAULON. Celui-ci cherchait à obtenir une mesure très faible. On est géné par la modification du module d'amortissement, qui est due à l'encrassement progressif du vase poreux et qui fausse la valeur des mesures.
M. DUCHER, Il faut changer le vase poreux chaque année, moyennant quel un vase neuf de module 0,6 j voit son module passer à j 1 j au bout d'un an (quelquesfois il reste à 0,8 j; quelquesfois il passe à 2 j). Tant que le module reste voisin de 1 j les résultats pour Marseille sont bons. Le déphasage est donné par \( \frac{T}{2A} \) : pour une onde semi-diurne il est de 2 h 40 min si le module est 0,5 j et passe à 2 h 50 min si le module passe à 1 j pour devenir à la limite égal à \( \frac{T}{2} \) soit 3 heures.

L’erreur résiduelle est pratiquement négligeable pour les ondes régulières. Pour une onde à grande période, on a pratiquement \( \frac{1}{A} \) et le déphasage 1 reste petit, pratiquement il est de l’ordre du module du vase poreux, soit de 0,5 à 1 j pour Marseille, même quand une variation de niveau extérieur ne se produit que 3 à 4 jours, l’erreur résiduelle est déjà faible. Les résultats de Marseille le confirment d’ailleurs puisque toutes les causes d’erreurs dues au système même ou au déphasage ne font pas disparaître le niveau moyen mensuel de plus de 1 cm maximum par rapport au marégraphe fondamental, cela tient que le module reste 1 j.

De GROVES. Le "midimarerèmér" described is analogue to the one designed by Jean FILLOUX which j judging yesterday, and response curves i showed apply. M. FILLOUX and i found that the damping required to reduce adequately the diurnal tide would make the response to day-to-day fluctuations of sea level too slow, in the case of a stage-île filter. That is why we used two stages, M. DUCHER’s design could easily be converted to two stages by using an additional inner stand pipe with its own porous filter.

M. DUCHER. Ceci est très intéressant et pourrait être essayé surtout en Atlantique, mais aussi en Méditerranée de manière à limiter l’erreur systématique due à l’amplitude de \( A \), et au petit nombre des lectures (1 par jour). Un effet : si les vases poreux sont des modules \( p \), et \( n \), l’ensemble est équivalent à un appareil à module \( p \) donné par la relation \( p = \frac{p_1^2 + p_2^2 + \frac{4\pi^2}{A}}{p_1} \) variable avec \( T \).

Pour les ondes semi-diurnes \( T = 0,5 \) et \( n_0 = n_1 = 1 \) j on aurait \( p = 1,25 \) j. La variation de ces ondes, ce qui est très bien, tandis que pour les ondes longues (\( T = 15 \) j et \( n_0 = n_1 = 1 \) j) on aurait \( p = 1,44 \) ce qui modifierait très peu le module et est très favorable pour les variations lentes d’un jour à l’autre.

Ce système à deux étages est donc très intéressant et permet peut-être de résoudre le problème des médiomaréomètres en Atlantique même avec une seule lecture par jour. J’espère que l’Institut Géodésique National pourra être essai qui nécessitera 2 tubes d’extraction concentriques et 2 vases poreux, et je crois qu’on saura troy remercier le Dr DEGROES de cette remarque et de cet appareil qui enrichit la théorie du médimaréomètre de LALIMAND et permet avec des moyens simples et sûrs d’avoir un excellent niveau moyen. Il faut noter que Marseille actuellement nos médimaréomètres sont en réalité équipés d’un vase poreux de module 0,5 à 1 j et d’un préfiltre poreux de module 1 heure ce qui est toléré du système à deux étages ; mais les 3 étages étant très différents les résultats n’est pas assez favorable que si le vase et son préfiltre étaient identiques et surtout emmanchés chacun à son tube propre.
M. DUCHER. Il faut changer le vase poreux chaque année, moyennant quel un vase neuf de module 0,64 j vot son module passe à 1 j au bout d'un an (quelquesfois il reste à 0,8 j ; quelquesfois il passe à 2 j). Tant que le module reste voisin de 1 j les résultats pour Marseille sont bons. Le dépassement est donné par : 1 - $\frac{T}{2}$, avec $\cos\frac{\theta}{A}$ pour une onde semi-diurne il est de 2 h 40 mn si le module est 0,5 j et passe à 2 h 50 mn si le module passe à 1 j, pour aller à la limite égal à $\frac{T}{4}$ , soit 3 heures.

L'erreur résultante est pratiquement négligeable pour les ondes régulières. Pour une onde à grande période, on a pratiquement $\frac{T}{8} A$ et le dépassement reste petit, pratiquement il est de l'ordre du module du vase poreux, soit de 0,5 à 1 jour pour Marseille. Même quand une variation de niveau extérieur se voit que 3 à 4 jours, l'erreur résultante est déjà faible. Les résultats de Marseille le confirmant d'ailleurs puisque toutes les causes d'erreurs dues au système-même ou au dépassement ne font pas différer le niveau moyen mensuel de plus de ± 1 cm maximum par rapport au marégraphe fondamental, cela tient que le module reste 1 jour.

Dr. GROVES. Le "medimaretre" décrit est analogue a celui designé par Jean FILLIOUX qui j'indiqué précédemment, et response curves i showed apply. M. FILLIOUX and I found that the damping required to reduce adequately the diurnal tide would make the response to day-to-day variations of sea level too slow, in the case of a one-stage filter. That's why we used two stages. M. DUCHER's design could easily be converted to two stages by using an additional inner stand pipe with the own porous filter.

M. DUCHER. Ceci est très intéressant et pourrait être essayé surtout en Atlantique, mais aussi en Méditerranée, même à limiter l'erreur systématique due à l'amplitude de $R$, et au petit nombre des lectures (1 par jour). En effet : si les vases poreux des modules $p_0$ et $p_1$, l'ensemble est équivalent à un appareil à module $p$ donné par la relation $p_0 + p_1 \frac{4p_0 + 4p_1}{p_0 + p_1}$ variable avec $T$.

Pour les ondes semi-diurnes $T = 0.5 j$ et $p_0 + p_1 = 1 j$ sur $p_0 = 12.5 j$ vis-à-vis de ces ondes, ce qui est très bien, tandis que pour les ondes longues ($T = 15 j$ et $p_0 + p_1 = 1 j$) on avait $p = 1.4 j$, ce qui modifiait très peu le module et est très favorable pour les variations lentes d'un jour à l'autre.

Ce système à deux étages est donc très intéressant et permet peut-être de résoudre le problème des définitions de l'Atlantique même avec une seule lecture par jour. J'espère que l'institut Géologique Douglas National pourrait faire a essai qui nécessitera 2 tubes étanches concentriques et 2 vases poreux ; et je crois qu'on ne saurait trop remercier le Dr. GROVES de cette remarque et de cet appareil qui enrichit la théorie du médimaretre de LALLEMAND et permet avec des moyens simples et sûrs d'avoir un excellent niveau moyen. Il faut noter que Marseille actuellement ses médimaretres sont en réalité équipés d'un vase poreux de module 0,5 à 1 heure et d'un préfiltre poreux de module 1 heure qui est nuisible au système à deux étages ; mais les 3 étages étant très différents le résultat n'est pas aussi favorable que si le vase et son préfiltre étaient identiques et surtout emmanchés chacun à son tube propre.

Instrument pour la mesure digitalisée

De la Hauteur d'un Niveau d'Eau et sa Transmission à Distance

G. Guidelli - Guidi

Società Italiana Apparecchi di Precisione (Italie)

L'instrument décrit a le but de traduire la hauteur d'un niveau d'eau en une succession d'impulsions électriques qui peuvent être envoyées à distance par fil ou par radio.

L'élément sensible aux variations de niveau est un flotteur traditionnel, suspendu par un ruban en acier inox qui passe sur une poulie de 1 mètre de circonférence.

Le dispositiflecteur du niveau est placé dans un coffret étanche et comprend un combinateur à secteurs décadiques et un servomoteur de coincidences entre la position du combinateur et celle de la poulie à flotteur.

Etant donné que pour actionner le combinateur il faut un effort pas négligeable, le servomoteur est nécessaire pour que la poulie soit libre de tourner pendant les variations de niveau sans demander qu'une force très petite.

Le combinateur à secteurs est, normalement, prévu à trois secteurs : pour les centimètres, les décimètres et les mètres, couvrant ainsi le champ de mesure de 0 à 9,99 mètres. Va sans dire que l'on peut ajouter des secteurs pour autres unités étendant ce champ.

Chaque secteur est fourni de dix contacts, correspondants aux chiffres de zéro à neuf. Lorsque le premier secteur ayant fait un tour complet, passe du contact "neuf" au "zéro", ça fait contemporanément avancer d'un contact le second secteur, de la même façon que dans les compteurs à chiffre à tambour.

Un lecteur électronique est relié à ces secteurs et la lecture de la position des contacts est faite par l'émission d'un nombre d'impulsions qui est compté par un compteur décadique ; lorsque le compteur atteint un nombre égal au nombre resultant du combinateur, un signal de stop bloque l'émission des impulsions. Les impulsions émises correspondent donc à la hauteur du niveau exprimée en centimètres.

Pour ménager le dispositif de synchronisation, le servomoteur est d'habitude inactif, et la poulie tourne librement commandée par le flotteur.

Lors d'une lecture, le servomoteur est mis sous courant et réalisé rapidement la coïncidence de position entre la poulie et le lecteur. Un dispositif particulier empêche l'arrêt du combinateur sur une position incertaine des secteurs, c'est-à-dire lorsqu'un secteur est en train de passer d'un chiffre au chiffre suivant. P, ex, si le niveau est entre 10 et 11 cm ce dispositif choisit une position définie, savoir 10 jusqu'à 10,5 cm et 11 de 10,5 cm en dessous.

Cette coincidence réalisée à moins de 0,5 cm, le servomoteur devient à nouveau inactif et la lecture du combinateur est opérée.

Ce procédé réalise les avantages suivants :

1/ Résistance presque nulle opposée aux mouvements de la poulie et du flotteur.

2/ Précision à moins de 0,5 cm sur la lecture, réalisée avec un dispositif commandé par servomoteur et capable donc de réaliser des contacts très sûrs.

3/ Impossibilité d'un changement du chiffre transmis pendant la transmission.
Ce particulier est très important car, en dépit des soins que l'on prend pour éviter les fluctuations du niveau, on peut avoir des petites variations de quelques millimètres. Si, p. ex., le niveau présente une oscillation entre 99 et 100 cm, il peut arriver qu'aub début de la lecture le niveau soit 99 et, qu'il passe, pendant la lecture, à 100.

Cela peut amener une erreur assez forte car le passage de 9 à 0 des deux premiers secteurs et de 0 à 1 du secteur des centaines provoque l'ouverture et fermeture de trois contacts de bloc qui peuvent être déjà dépassés par le lecteur au moment du changement.

Les impulsions ainsi produites peuvent être envoyées par câble à un récepteur qui peut être :

1/ Indicateur : un simple compteur décadique à chiffres à remise à zéro automatique, ou bien compteur électronique à chiffres lumineux ou à projection si le chiffre doit être très grand et lisible à distance.

Un compteur à aiguille est aussi employable.

La remise à zéro se fait avant chaque lecture.

2/ Enregistreur : on a réalisé un instrument équipé avec un moteur pas à pas qui fait avancer un bras porte-plume.

La lecture faite, le moteur revient automatiquement à zéro et la plume reste dans la position atteinte.

Lors de la lecture successive, le moteur avance au nouveau, et si le niveau est augmenté il rattrape la plume et l'amène à la position nouvelle. Si au contraire on a eu une baisse du niveau, le moteur s'arrête avant d'atteindre la plume : un dispositif automatique fait retomber la plume sur cette dernière position du moteur : cela fait, le moteur revient à zéro.

La plume enregistre sur un papier continu entraîné par horlogerie.

La succession des informations (lectures) peut être commandée à temps (p. ex. 1 lecture par heure) ou sur demande en quel que soit moment.

Une version très intéressante de cet instrument.

L'application à cet instrument du système de transmission radio présente des particularités intéressantes.

D'abord, on a choisi le système à fréquence modulée et l'émission est caractérisée par une onde portante modulée à presque 2 Kc (appel) interrompue par trains d'impulsions à 1,8 Kc (information du chiffre). La réception de cette transmission présente un haut degré de sûreté vers les parasites étant assez difficile qu'un signal hétérogène ait les mêmes caractéristiques de fréquence base, de modulation et d'écart entre modulation continue d'appel et modulation des impulsions, étant nécessaire qu'un signal présente ces particularités pour actionner, à travers les filtres du récepteur, le décodificateur dont la sortie représente la mesure du niveau transmise.

Attends l'importance, pour les services maritimes et de contrôle du cours des rivières importantes, de centraliser les mesures de plusieurs points, on a sougés la réalisation d'un appareillage réalisant ce but.

Une centralisation standard a été réalisée pour quatre points de mesure et un poste central de réception.

Chaque poste périphérique (point de mesure) est équipé avec un récepteur-émetteur de même que le poste central.

L'appel du poste central met en activité dans tous les postes périphériques un compteur de temps contrôlé à quarts qui règle la succession des transmissions ainsi que la fréquence d'émission des impulsions de mesure.

Le poste central reçoit ainsi successivement les quatre informations des postes périphériques sans possibilité de superposition.

Pour les postes à grande distance ou bien au delà d'un obstacle un pont radio répétiteur a été employé.

La présence d'un système d'appel central permet aisément l'interrogation en un moment quelconque des postes périphériques, qui sont normalement explorés automatiquement à temps déterminé.

L'enregistrement des courbes du niveau de chaque point de mesure est fait de la même façon qu'avec la transmission par câble. En plus, on a prévu un cadre avec indication optique des niveaux en chiffres sur un tableau représentant graphiquement la zone sous contrôle.

Ce cadre est particulièrement utile pour le contrôle des niveaux d'une rivière car on peut suivre vivement et immédiatement l'allure d'une onde de crue.

La forme digitalisée de la mesure présente des avantages remarquables pour l'application de dispositifs automatiques capables de relever un événement exceptionnel. Deux de ces dispositifs ont été mis au point.

1/ Niveau au-dessus d'un niveau limite. Un simple compteur à préselection est rangé au chiffre correspondant au niveau limite. Lorsque le signal dépasse cette limite, un circuit de signalisation est actionné.

2/ Valeur de la dérivée de la montée du niveau au-dessus d'une valeur limite. En tant que les informations se succèdent avec régularité dans le temps, on peut prendre comme dérivée l'incrément fini entre deux informations successives. Un circuit logique mémorise une information et la soustrait algébriquement de la successive.

Si le résultat est supérieur à un nombre préfixé, un deuxième circuit de signalisation est actionné.

Dans ces deux cas on a prévu que ces circuits "d'alarme" provoquent automatiquement une accélération de la cadence des interrogations aux postes périphériques.

De cette façon une marche exceptionnelle, une crue dangereuse peuvent être suivies plus en détail et le service intéressé peut en avoir une documentation exacte dès le moment qu'elle a commencé à se manifester aussi si personne n'était présente à ce moment aux appareils.

A remarquer aussi le fait que, s'agissant de diffusion circulaire, la réception des signaux peut être faite aussi par un troisième poste "moniteur" qui n'est pas opérationnel dans le sens qu'il ne peut pas appeler les postes de mesure, mais il est de même au courant des émissions.

Si les mesures des niveaux intéressent plus d'un service (Command du port, Travaux publics, Service maritimes, Service météorologique, Centre d'études d'hydrologie etc.) cette particularité présente un grand intérêt, (Va sans dire qu'il faut franchir quelques difficultés pour informer le "chercheur collègue"),

Cet appareillage est le résultat de la collaboration de deux maisons : SIAP (Instruments de météorologie et hydroélectricité) et SITR (télémécanique) de Bologne, Italie, suivant les suggestions du Bureau Spécial du Reno (fleuve de Bologne) et d'autres services publics italiens.

OBSERVATIONS A LA COMMUNICATION DE M. GUIDELLI-GUIDI

M. KYRIS. Quelle est la consommation d'énergie dans les stations-filées ?

Quelle source d'énergie utilisez-vous ?

M. GUIDELLI-GUIDI. La puissance est de 40 milliwatts et l'énergie est fournie par accumulateurs classiques.
Ce particulier est très important car, en dépit des soins que l'on prend pour éviter les fluctuations du niveau, on peut avoir des petites variations de quelques millimètres. Si, p. ex., le niveau présente une oscillation entre 99 et 100 cm, il peut arriver qu'au début de la lecture, le niveau soit 99 et, qu'il passe, pendant la lecture, à 100.

Cela peut amener une erreur assez forte car le passage de 9 à 0 des deux premiers secteurs et de 0 à 1 du secteur des centaines provoque l'ouverture et fermeture de trois contacts de bloc qui peuvent être déjà dépassés par le lecteur au moment du changement.

Les impulsions ainsi produites peuvent être envoyées par câble à un récepteur qui peut être :

1) Indicator : un simple compteur décadique à chiffres à remise à zéro automatique, ou bien compteur électronique à chiffres lumineux ou à projection si le chiffre doit être très grand et lisible à distance.

Un compteur à aiguille est aussi employable.

La remise à zéro se fait avant chaque lecture.

2) Enregistreur : on a réalisé un instrument équipé avec un moteur pas à pas qui fait avancer un bras porte-plume.

La lecture faite, le moteur revient automatiquement à zéro et la plume reste dans la position atteinte.

Lors de la lecture successive, le moteur avance à nouveau, et si le niveau est augmenté il retrouve la plume et l'amène à la position nouvelle. Si au contraire on a eu une baisse du niveau, le moteur s'arrête avant d'atteindre la plume : un dispositif automatique fait remonter la plume sur cette dernière position du moteur ; cela fait, le moteur revient à zéro.

La plume enregistre sur un papier continu entraîné par horlogerie.

La succession des informations (lectures) peut être commandée à temps (p. ex. 1 lecture par heure) ou sur demande en quel que soit moment.

Une version très intéressante de cet instrument.

L'application de cet instrument du système de transmission radio présente des particularités intéressantes.

D'abord, on a choisi le système à fréquence modulée et l'émission est caractérisée par une onde portante modulée à presque 2 kc (appel) interrompue par trains d'impulsions à 1,8 kc (information du chiffre). La réception de cette transmission présente un haut degré de fidélité vers les parasites étant assez difficile qu'un signal hétérogène au même caractéristiques de fréquence base, de modulation et d'écart entre modulation continue d'appel et modulation des impulsions, est nécessaire qu'un signal présente ces particularités pour actionner, à travers les filtres du récepteur, le décodificateur dont la sortie représente la mesure du niveau transmis.

Attendez l'importance, pour les services marographiques et de contrôle du cours des rivières importantes, de centraliser les mesures de plusieurs points, on a soligné la réalisation d'un appareillage réalisant ce but.

Une centralisation standard a été réalisée pour quatre points de mesure et un poste central de réception.

Chaque poste périphérique (point de mesure) est équipé avec un récepteur-émetteur de même que le poste central.

L'appel du poste central met en activité dans tous les postes périphériques un compteur de temps contrôlé à quarts qui règle la succession des transmissions ainsi que la fréquence d'émission des impulsions de mesure.

Le poste central reçoit ainsi successivement les quatre informations des postes périphériques sans possibilité de superposition.

Pour les postes à grande distance ou bien au-delà d'un obstacle un pont radio répétiteur a été employé.

La présence d'un système d'appel central permet aisément l'interrogation en un moment quelconque des postes périphériques, qui sont normalement explorés automatiquement à temps déterminé.

L'enregistrement des courbes du niveau de chaque point de mesure est fait de la même façon qu'avec la transmission par câble. En plus, on a prévu un cadre avec indication optique des niveaux en chiffres sur un tableau représentant graphiquement la zone sous contrôle.

Ce cadre est particulièrement utile pour le contrôle des niveaux d'une rivière car on peut suivre vivement et immédiatement l'allure d'une onde de crue.

La forme digitalisée de la mesure présente des avantages remarquables pour l'application de dispositifs automatiques capables de relever un événement exceptionnel. Deux de ces dispositifs ont été mis au point.

1) Niveau au-dessus d'un niveau limite, Un simple compteur à préselection est rangé au chiffre correspondant au niveau limite, lorsque le signal dépasse cette limite, un circuit de signalisation est actionné.

2) Valeur de la dérivée de la montée du niveau au-dessus d'un valeur limite. Étant que les informations se succèdent avec régularité dans le temps, on peut prendre comme dérivée l'inclinaison fini entre deux informations successives. Un circuit logique mémorise une information et la souscrit algébriquement de la successive.

Si le résultat est supérieur à un nombre fixé, un deuxième circuit de signalisation est actionné.

Dans ces deux cas on a prévu que ces circuits "d'alarme" provoquent automatiquement une accélération de la cadence des interrogations aux postes périphériques.

De cette façon une marche exceptionnelle, une crue dangereuse peuvent être suivies plus en détail et le service intéressé peut en avoir une documentation exacte dès le moment qu'elle a commencé à se manifester ainsi que personne n'était présente à ce moment aux appareils.

A remarquer aussi le fait que, s'agissant de diffusion circulaire, la réception des signaux peut être faite aussi par un troisième poste "monitor" qui n'est pas opéré dans le sens qu'il ne peut pas appeler les postes de mesure, mais il est de même au courant des émissions.

Si les mesures des niveaux intéressent plus d'un service (Command du port, Travaux publics, Service marographique, Centre d'études d'hydrologie etc.) cette particularité présente un grand intérêt, (Va sans dire qu'il faut franchir quelques difficultés pour informer le "cher collègue")

Cet appareillage est le résultat de la collaboration de deux maisons: SAP (Instruments de météorologie et hydrométrie) et SIF (télémétrie) de Bologne, Italie, suivant les suggestions du Bureau Spécial du Reno (fleuve de Bologne) et d'autres services publics italiens.

OBSERVATIONS A LA COMMUNICATION DE M. GUIDELLI-GUIDI

M. KYRIS, Quelle est la consommation d'énergie dans les stations-filées ?
Quelle source d'énergie utilisez-vous ?

M. GUIDELLI-GUIDI. La puissance est de 40 milliwatts et l'énergie est fournie par accumulateurs classiques.
L'ANALYSE D'UNE MARÉE PURE
A L'AIDE DE LA MÉTHODE DES MOINDRES CARRÉS

Gabriel GODIN
Département des Mines du Canada

Pour le moment, nous avons adopté au Canada, la méthode des moindres carrés pour l'analyse des marées. Les raisons qui nous ont poussés à ce choix sont les suivantes :
1/ le principe de la méthode est fort simple,
2/ le travail de préparation pour le calcul n'est pas trop ardu.

Nous rappelons pas ici le principe des moindres carrés qui est bien connu. Quant au travail de préparation il consiste à choisir les ondes composantes que l'on veut rechercher dans la marée, à calculer la contribution de ces ondes à la matrice des équations normales et à évaluer l'inverse de cette matrice. Cela est fait une fois pour toutes.

L'analyse elle-même consiste à calculer les vecteurs

\[ C_i = Z(0) + \sum_{i=1}^{n} (Z(t) - Z(-t)) \cos \phi_i t \]

et

\[ S_i = \sum_{i=1}^{n} (Z(t) - Z(-t)) \sin \phi_i t \]

où

- \( Z(t) \) : une observation sur la marée au moment \( t \)
- \( \phi_i \) : la vitesse angulaire de l'onde composante \( i \)
- \( T \) : la durée de demi-intervalle d'observation

et à multiplier ces vecteurs par l'inverse de la matrice des équations normales pour obtenir les vecteurs

\[ X_i = A_i \cos \phi_i \]
\[ Y_i = A_i \sin \phi_i \]

où

- \( A_i \) : l'amplitude de l'onde composante \( i \)
- \( \phi_i \) : son retard de phase au moment central.

L'évaluation des vecteurs \( C_i \) et \( S_i \) est la partie la plus longue de l'analyse et nous avons essayé deux méthodes pour le calcul de ces vecteurs.

1/ l'une, basée sur la méthode de récurrence des polynômes de Tchebycheff :

\[ \cos n\phi = 2 \cos \phi \cos(n - 1) \phi - \cos(n - 2) \phi \]

2/ l'autre, basée sur l'évaluation directe de \( \cos \phi t \) et \( \sin \phi t \) par consultation d'une table de \( \cos \phi \).

La première méthode nécessite l'inclusion de toutes les données \( Z(t) \) dans l'ordinateur en plus d'une table très courte des valeurs de \( \cos \phi \). La seconde méthode requiert l'inclusion d'une table complète de \( \cos \phi \) couvrant au moins l'intervalle \( (0, \pi/2) \) mais on n'a besoin que d'une seule valeur de \( Z(t) \) à la fois.
Tout d’abord nous avons constaté que les deux méthodes prennent à peu près le même temps pour l’évaluation des vecteurs $C_3$ et $S_3$. D’autre part, la méthode basée sur la relation de recurrence de Tschaytscheff qui est axée sur la valeur de cos $q$, donne de très mauvais estimés de la valeur des ondes composantes de faible vitesse angulaire comme $S_4$, $M_3$, $M_5$, etc. La raison pour ce ci est que $q$ est très petit, cos $q$ est insensible à la valeur exacte de son argument à moins que l’on ne calcule cette fonction à un grand nombre de décimales ; et c’est justement ce que l’on veut éviter. Dans ces circonstances l’application de la relation de recurrence cause une accumulation d’erreurs intolerable dans $C_3$ et $S_3$. Nous évaluons donc maintenant ces vecteurs par consultation d’une table de cos qui s’étend sur l’intervalle (0, 2π).

Une fois ce travail de préparation accompli, il nous restait à vérifier la puissance de cette méthode d’analyse. Une maree naturelle ne se prête pas directement à une telle vérification : les ondes composantes y sont inconnuées et même après une analyse elles ne sont qu’imparfaitement connues à cause du bruit inhérent à une maree naturelle.

Nous avons donc synthétisé une maree pure sur un intervalle d’une année à l’aide d’ondes composantes arbitraires pour ensuite analyser cette maree à l’aide de la méthode des moindres carrés. Les ondes composantes ont été choisies sans donner quelque considération que ce soit aux relations d’équilibre : nous avons juxtaposé dans un groupe des ondes de très petite amplitude dans le voisinage immédiat d’ondes de grande amplitude dans le but d’éprouver le pouvoir de séparation de cette méthode.

L’analyse d’une année d’observations sur cette maree pure par la méthode des moindres carrés a donné des résultats presque parfaits comme on peut le voir dans l’exemple qui suit et ceci nous a inspiré une confiance absolue dans cette méthode.

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<thead>
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<th>Amplitude</th>
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<td>$K_1$</td>
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A ce point, il nous est venu à l’idée de faire une comparaison entre la méthode des moindres carrés et celle de DOODSON. Nous n’avons pas adopté cette dernière méthode parce qu’elle ne satisfait pas aux deux critères que nous avons mentionnés au début : sa structure logique est loin d’être claire et sa programmation est extrêmement pénible. Toutefois elle semble donner d’excellente résultats.

L’analyse d’une même maree pure par les deux méthodes donnerait une comparaison impartiale de leur puissance. Nous avons entrepris cette comparaison avec l’aide du Tidal Institute of Liverpool. Nous avons été surpris à nouveau une maree pure basée sur les résultats de l’analyse de CHURCHILL, MANITOBA, parce que la méthode de DOODSON suppose que les relations d’équilibre sont à peu près respectées en dedans des groupes d’ondes de vitesses très voisines ; ce qui est vrai en général dans les mares naturelles. Malheureuse-ment toute une succession de petites tragédies ont retardé ce travail et nous ne pouvons pas maintenant présenter les résultats de cette comparaison. Tout ce que l’on peut dire jusqu’ici c’est que la méthode de DOODSON, en dépit de sa confusion totale britannique, donne d’excellents résultats une fois que le travail énorme de sa programmation est accompli.

Quand il s’agit de comparer les temps pris pour l’analyse d’une année d’observations, l’analyse par les moindres carrés prend 59 minutes sur l’ordinateur IBM 1629 Mark II alors que l’analyse par la méthode de DOODSON prend 3 minutes sur le même ordinateur.

Puisque la méthode de LECOLAZET emplie d’une façon rigoureuse le concept de combinaison linéaire introduit par DOODSON dans l’analyse des maires et qu’elle est de compréhension facile, nous l’avons étendue pour l’analyse d’une année d’observations. Nous espérons pouvoir bientôt programmer cette méthode et comparer sa puissance avec les deux méthodes que nous avons étudiées jusqu’ici.

**OBSERVATIONS SUR LA COMMUNICATION DU Dr GODIN**

**Dr ROSSITER**, in view of yours remarks concerning the accumulative error of recurrence relationships when computing the cosine vectors ; in our experience we have found no problem here. Do you have any idea as to the reason for this apparent paradox ?

**Dr GODIN**, The recurrence relations were different.

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**Dr GODIN**, If we wish to use a recurrence relation without the danger of excess accumulative error, one has in our case to carry an excessive number of significant figures, thus slowing down the computation time.

**M. GOUINNEAU**, Une maree pure, analysée par la méthode des moindres carrés doit être correc-tement retrouvée puisque toutes les équations d’observations sont cohérentes. En réalité, la méthode des moindres carrés est destinée à éliminer les erreurs accidentelles, ou dans la maree, les bruits les plus importants n’ont pas le caractère d’erreurs accidentelles.

**Dr GODIN**, Je suis d’accord.

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**Dr ROSSITER** (Chairman), No, we shall wait for the next paper to discuss this.
Tout d’abord nous avons constaté que les deux méthodes prédisent à peu près le même temps pour l’évaluation des vecteurs $C_k$ et $S_k$. D’autre part, la méthode basée sur la relation de recurrence de Tschibyscheff qui est axée sur la valeur de cos $\alpha$, donne de très mauvaises estimations de la valeur des ondes composantes de faible vitesse angulaire comme $S_k$, $M_k$, $S_a$, etc. La raison pour ceci est que si $q_k$ est très petit, cos $\alpha$ est insensible à la valeur exacte de son argument à moins que l’on ne calcule cette fonction à un grand nombre de décimales ; et c’est justement ce que l’on veut éviter. Dans ces circonstances l’application de la relation de recurrence cause une accumulation d’erreurs intolérable dans $C_k$ et $S_k$. Nous évaluons donc maintenant ces vecteurs par consultation d’une table de cos $\theta$ qui s’étend sur l’intervalle $(0, 2\pi)$.

Une fois ce travail de préparation accompli, il nous restait à vérifier la puissance de cette méthode d’analyse. Une marée naturelle ne se prête pas directement à une telle vérification ; les ondes composantes y sont inconnues et même après une analyse elles ne sont qu’imparfaitement connues à cause du bruit inhérent à une marée naturelle.

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<td>$1$</td>
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ON SOME METHODS TO ANALYSE TIDAL OBSERVATIONS
IN SHALLOW WATERS OF THE GERMAN BIGHT

Walter HABICH
Deutsches Hydrographisches Institut

What I am going to say is essentially what Herr HORN would have said, had he been in
a position to attend this meeting as was his intention, but no manuscript had been elaborated
by him for this purpose, and consequently I alone am responsible for what follows.

Many efforts have been made at the German Hydrographic Institute to improve the tidal
predictions for shallow water ports. The tides of the German Bight are already largely dis-
torted and also the wind considerably affects the water level in shallow waters. There is an
interaction between tide and effect of the wind and therefore tide predictions must be used
with precaution in forecasting the water level.

At the beginning a short description of our forecasting method may be given but only as
far as to explain how tidal predictions enter the forecasting computations of the water level.
The residual between the height of the level (which is simultaneously affected by tide and wind)
and the predicted height of the tide is, because of the interaction, not the same as the set-up
of the water level caused by wind. The residual, in a similar way as the set-up, depends on
the depth of the water. As soon as the range of the tide is not negligible as compared with the
depth, the residual varies with the height of the tide in shallow waters.

In the German Bight the variations of predicted high water heights are small as compa-
red with the range of the tide. So, at high water, the residual may only be ascribed to the
wind, the depth at predicted high water is taken to be constant. From observations the resi-
dual is derived as function of the wind vector ; the same is separately done for the observa-
tions at low water. For the time being we interpolate to get the residuals between high and
low water. Though this works well in the forecasting service, the interpolation is not quite
correct, we can e.g. never explain why, at steadily blowing wind, the time of observed high
water can differ from prediction. We are going to improve the method by predicting the tide
at fixed time differences, referred to high water and referred to low water. The variation of
predicted heights at those fixed times proved to be as small as the variation of predicted high
waters, so the residuals may be derived as only depending on the wind. If a steadily blowing
wind affects in the mean the time of high water we should get it. Arranging tide predictions
in this way, we may forecast the water level by simply adding tide predictions and residual
functions.

I shall briefly report on four methods of tidal predictions we have tested or developed
in the last 17 years, the harmonic analysis of

a) solar hourly heights,
b) lunar hourly heights,
c) high and low water heights and lunisolar intervals,
d) heights at distinct times between high and low water.

I do not enter into the analytical and astronomical foundations they are given in the paper
by W. HORN, 1948 (1) (*) , and I do not closely adhere to the historical line; the different
attempts we have made are to be found in the Annual Report of the German Hydrographic In-
situte, 1952-63.

(*) Number in brackets indicate the paper as listed at the end.
The following remark is to show how we look at the problem of harmonic analysis. The potential of the tide-generating force may be represented as a linear combination of the harmonics

\[ \cos(A_1 \cdot t + B_1 + C_1 \cdot sin(t) + D_1 \cdot sin^2(t) + \ldots) \]  

A, B, ..., F being integral numbers and \( v, a, b, p, N \), \( q \) nearly linear functions of the tide, the hour angle of the mean moon, \( s \) and \( h \) the lengths of the mean moon and sun, \( p \) and \( q \) the mean lengths of the mean longitudes of moon and sun orbites, In his paper 1921 (2) Doodson derived numerical values for 400 constituents, for \( A = 0, 1, 3, 3 \) and different \( B, \ldots, F \). It is useful to remember that this is only an approximation of the full set of infinite harmonics. HORN stressed in his paper (1) that the tides as a solution of the hydrodynamic equations may be developed by means of the same infinite set of harmonics as the tide-generating forces. In this language we only speak of different components and do not distinguish between tides, overtides, and compound tides, Harmonic analysis means to select a finite number of harmonics and to compute the best approximation in the mean of the finite linear combination to the tide which is known at distinct times of a finite interval. There is a solution of this problem and only one, The coefficients of the approximative combination are given as solutions of the normal equations.

a) The harmonic analysis in its usual way is based on the observations at equidistant times, mostly on hourly heights. The results of this analysis for the ports at the German Bight were not sufficiently good. We analysed observations covering 369 days and took into account 64 constituents. Deriving the tidal constants, we also calculated the standard deviations of the whole development, of the amplitudes, and of the phase lags. The standard deviation of the development is defined by

\[ \mu = \sqrt{\frac{\sum (y - \bar{y})^2}{N - K}} \]

where \( y \) and \( \bar{y} \) are the observed and predicted heights at the time \( t \), \( N \) = 2 \( n \) + 1 the number of observations, the summation extending from \( v = -n \) to \( v = +n \), \( K = 2k + 1 \) the number of unknown quantities in the normal equations, \( k \) the number of constituents in the analysis. In our case the standard deviation of the amplitude \( \mu_{A} \) is of the magnitude 0.25. The difference between the analysis for shallow water ports and for other harbours may be shown by the values of the mean deviations.

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Meteorological effects which are large in shallow waters are mainly responsible for the standard deviation. Even taking into account more constituents in the analysis of one year's observation, we shall hardly get considerably smaller values of \( \mu_{A} \).

Every day we are comparing observed and predicted data at our forecasting service. We noticed systematical differences in the shape of the curves at calm days. These differences can probably be reduced by taking into account more constituents. In our analysis we went down to the eighth diurnal, but when going down to the tenth and twelfth diurnal and farther, the number of constituents to be considered rapidly increases, probably to several hundreds, and from the twelfth diurnal on, the time steps of the observations have to be smaller than one hour. We decided not to proceed farther in this direction.

In the analysis of the tide in shallow waters the relations of the constituents are different from the state of equilibrium tide, even in such narrow ranges as the nodal splitting. The values of \( j \) and \( v \), yearly corrections for nodal variations, are derived assuming the state of equilibrium tide. We analysed 24 years of observations at Cuxhaven (each analysis covering 369 days and taking into account 64 constituents). For some of the constituents the amplitudes and phase lags obtained for the different years regularly went up and down and changed to a higher degree than expected from the standard deviations. We suspected the values of \( j \) and \( v \) to be incorrect. We got, e.g., an amplitude of about 12 cm for the constituent \( L_{0} \) and variations as mentioned for the different years, \( 2M_{2} = 2M_{0} + N_{0} \) has exactly the same angular velocity as \( L_{0} \), but each year the values of \( j \) and \( v \) for \( 2M_{2} \) and \( L_{0} \) differ from one another. We adjusted the results of the analyses to a linear combination of \( L_{0} \) and \( 2M_{2} \) using the method of least squares. The amplitude of \( 2M_{2} \) turned out to be ten times as large as that of \( L_{0} \). The statement that the compound tide \( 2M_{2} \) is ten times as large as the constituent \( L_{0} \) is perhaps better than to speak only of \( L_{0} \) but not satisfying in the last resort. For there are many combinations like \( 2M_{2} \), all having the same angular velocity as \( L_{0} \), only having different sets of \( j \) and \( v \). All combinations have to be tried. An easier way to overcome these difficulties was already proposed by W. Horn (1): take into account \( L_{0} \) and such constituents which differ from \( L_{0} \) by \( a \) and \( b \), \( a \) and \( b \) being small integral numbers. By this analysis we could introduce individual values of \( j \) and \( v \) (for \( L_{0} \) at Cuxhaven) which are likely to be valid for the other ports of the German Bight.

We may conclude from these results that we have to take at least 19 years' observations and must use more constituents than the 64 for an accurate analysis of the tide in the shallow waters of the German Bight.

b) The other three kinds of analyses I mentioned have been developed at our Institute. In all cases 19 years of observations are analysed at time steps of one lunar day. The hour angle of the mean moon at these times is \( \varphi = v + 360° \) (an integral number) at these times, the harmonics (a) and thus the tide do no longer depend on \( v \). \( v \) is constant and becomes a phase lag. The harmonics depend only on the 5 quantities \( a, b, p, N \), and \( q \). The advantage of this method is evidently demonstrated. The tide in the German Bight is semi-diurnal, the variation of the heights at time steps of one lunar day are comparatively small. The development of the tide at these time steps will faster converge as the usual harmonic analysis. The new harmonics we use are the following:

\[ \cos(b + c + d + e \cdot N) \]  

\[ \sin(b \cdot s + c + d + e \cdot N) \]

We dropped \( q \) if the period of \( q \) is about 21,000 years. From several tests we selected a set of 44 components of the (b) type in such a way that the 64 constituents of the (a) type are contained therein. One component of (b) includes a set of constituents of (a), the constituents for fixed values of \( B, C, D, E, F \) = 0 and the whole variety of \( A \) up to any diurnals. Concerning the sets of components which are used for approximations, it can be simply said that (b) covers (a) by far.

In order to predict the tide at lunar hours, we have to analyse 24 sets of observations for each lunar hour \( 0, 1, 2, 3, 23, 24, 23 \) all taken from the same interval of 18 years and in each set the observations taken at time steps of one lunar day. The 24 analyses are distinguished by different values of \( \varphi \), \( \varphi = 0°, 15°, 30°, 45°, 60°, 75°, 90°, 105°, 120°, 135°, 150°, 165°, 180°, 210°, 225°, 240°, 255°, 270°, 285°, 300°, 315°, 330°, 345°, 360° \). The amount of work involved is of about the same magnitude as a usual harmonic analysis covering the observations of 18 years and taking into account 64 constituents on the assumption that the observations are given, The number of products which enter the quantities on the right hand side of the normal equations is somewhat smaller at (a), The matrix of the normal equations is the same for all 24 sets; it is inversely once for all.

Heights cannot be continuously predicted any longer, only at lunar hours; for other times they have to be interpolated. In order to check these interpolations and the analysis itself we also plotted the lunar hours and the hour angles on the right hand side of the normal equations is somewhat smaller at (a), The matrix of the normal equations is the same for all 24 sets; it is inversely once for all.

In our daily forecasting service lunar and solar hourly predictions were compared with the observed data, and judging from the agreement at calm days, the lunar hourly predictions proved to be the better ones.

By taking into account the development of type (b) various numbers of \( d \) and \( e \), nodal variations are included.
The following remark is to show how we look at the problem of harmonic analysis. The potential of the tide-generating force may be represented as a linear combination of the harmonics

$$\cos(A \cdot + B \cdot e + D \cdot p + E \cdot n + F \cdot p) ... (a)$$

The longitude and latitude of the mean moon and the length of the mean moon and sun orbit, is in his paper 1927 (2) in which he derived numerical means for 400 constituents, for A = 0, 1, 3, 4 and different

$$\sin(A \cdot + B \cdot e + D \cdot p + E \cdot n + F \cdot p) ... (a)$$

It is useful to remember that this is only an approximation of the full set of infinite harmonics. HORNB in his paper (1) that the tide as a solution of the hydrodynamic equation may be developed by means of the same finite set of harmonics as the tide-generating forces. In this language we only speak of different components and do not distinguish between tides, overides, and compound tides. Harmonic analysis means to select a finite number of harmonics and to compute the best approximation in the mean of the tide linear combination to the tide which is known at distinct times of a finite interval. There is a solution of this problem and only one, the coefficients of the approximative combination are given as solutions of the normal equations.

a) The harmonic analysis in its usual way is based on the observations at equidistant times, mostly on hourly heights. The results of this analysis for the ports at the German Bight were not sufficiently good. We analysed observations covering 369 days and took into account 64 constituents. Deriving the tidal constants, we also calculated the standard deviations of the whole development, of the amplitudes, and of the phase lags. The standard deviation of the development is defined by

$$\mu = \sqrt{\frac{1}{N} \sum (y - y)^2}$$

y and y being observed and predicted heights at the time $t$, N = 2 N + 1 the number of observations, the summation extending from $v = - n$ to $v = n$, $k = 2k + 1$ the number of unknown quantities in the normal equations, k the number of constituents in the analysis. In our case the standard deviation of the amplitude $\mu_{am}$ is of the magnitude $\frac{\mu}{N}$. The difference between the analysis for shallow water ports and for other harbours may be shown by the values of the mean deviations.

$$\mu_{am}$$

Cuxhaven 35 cm 0,6 cm
12 Spanish ports 12 to 9 0,14 to 0,09

Meteorological effects which are large in shallow waters are mainly responsible for the standard deviation. Even taking into account more constituents in the analysis of one year's observations, we shall hardly get considerably smaller values of $\mu_{am}$. Every day we are comparing observed and predicted data at our forecasting service. We noticed systematical differences in the shape of the curves at calm days. These differences can probably reduced by taking into account more constituents. In our analysis we went down to the eighth diurnal, but when going down to the tenth and twelfth diurnal and farther, the number of constituents to be considered rapidly increases, probably to several hundreds, and from the twelfth diurnal on, the time steps of the observations have to be smaller than one hour. We decided not to proceed farther in this direction.

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b) The other three kinds of analyses I mentioned have been developed in our Institute. In all cases 19 years of observations are analysed at time steps of one lunar day. The hour angle of the mean moon at these times is $t = \phi + v$, 360° (v an integral number); at these times, the harmonics (a) and thus the tide do no longer depend on $\phi$, $\phi$ is constant and becomes a phase lag. The harmonics depend only on the 5 quantities $s$, $h$, $p$, $N$, and $q$. The advantage of this method is easily demonstrated. The tide in the German Bight is semi-diurnal, the variation of the heights at time steps of one lunar day are comparably small. The development of the tide at these time steps will faster converge as the usual harmonic analysis. The new harmonics we use are the following:

$$\cos \frac{\sin(bs + ch + dp + en)}{\sin}$$

We dropped $q$ ; the period of $q$ is about 21,000 years. From several texts we selected a set of 44 components of the (b) type in such a way that the 64 constituents of the (a) type are contained therein. One component of (b) includes a set of constituents of (a), the constituents for fixed values of $D$. C, D, F, P, $= 0$ and the whole variety of $A$ up to any diurnal. Concerning the sets of components which are used for approximations, it may be briefly said that (b) covers (a) by far.

In order to predict the tide at lunar hours, we have to analyse 24 sets of observations for each lunar hour, 0 h, 1 h, 23 h, all taken from the same interval of 18 years and in each set the observations taken at time steps of one lunar day. The 24 analyses are distinguished by different values of $\phi$, $\phi_0 = 0°, 15°, ... 345°$. The amount of work involved is of about the same magnitude as a usual harmonic analysis covering the observations of 18 years and taking into account 64 constituents on the assumption that the observations are given. The number of products which enter the quantities on the right hand side of the normal equations is somewhat smaller in (b) than in (a). The matrix of the normal equations is the same for all 24 sets ; it is inverced once for all.

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In our daily forecasting service lunar and solar hourly predictions were compared with the observed data, and judging from the agreement at calm days, the lunar hourly predictions proved to be the better ones.

By taking into account the development of type (b) various numbers of d and e, nodal variations are included.
Despite all the advantages and improvements concerning as a tidal analysis, we shall not use lunar hourly predictions for the forecasting of the water level when considering the wind effect. The variation of predicted high and low water is as small as the predictions of high and low water. But in the midst between high and low water the variations of the predictions are three to four times as large as those at high and low water. Therefore, the residual functions derived for times between high and low water will be less accurate than those around high and low water. At fixed lunar hours between high and low water the depth of the water can no longer be assumed to be constant.

c) We firstly used the set of 44 components of the (b) type to analyse high and low water and started doing so already 15 years ago. Only a short description of this work is given here. In the German Bight the tide mainly depends on and therefore does not differ very much, at least around high and low water, from the mean tidal curve which may be defined by means of the development (a) in the following way: average over the values of \(a, b, p, q, n\), and then plot for the values of \(q\). Mean high and low water are situated at fixed values of \(q\). Time and height differences between an individual and the mean high water may be developed by harmonics of the (b) type. In his paper (1) W. HORN has given the analytical proof. It is more convenient to refer the times of an individual high and low water to the transit of the moon through the meridian of Greenwich and not to mean high and low water. The variations of intervals become smaller, because a good deal of them is already contained in the computations of the transit of the moon as inequalities. All difficulties resulting from the diurnal inequality of the tides are automatically eliminated by the fact that the time step is a whole lunar day, which means that high and low waters belong to upper and lower transits of the moon and are dealt with separately. The analysis of high and low water consists of 8 series, for high and low water times and heights at upper and lower transit of the moon, In his papers (3) and (4) W. HORN has given the results of the analysis of 19 years' observations at Bombay, where the tide is no longer semi-diurnal as in the German Bight.

The daily predictions for all German reference ports are computed by means of this method. This analysis is an extension and, in a way, a harmonic analysis of the non-harmonic inequalities as used in the method of LUBBOCK, as reported by W. HORN (5). We used the non-harmonic method of LUBBOCK until 1956. Before this time many attempts were made to predict according to the harmonic analysis of solar hourly heights. They always failed; LUBBOCK's method gave better results.

d) In the lunar hourly analysis described under b) the variations of the predictions are large on the steep part of the tidal curve, This is due to inequalities which shift the whole curve to and fro. To avoid this effect of shifting, we analyse at fixed time differences referred to predicted high and low water times.

We start with four sets of time predictions, of high and low water at upper and lower transit of the moon, as just described. Every set covers the whole interval of 19 years, the difference between two successive times in each set is about one lunar day. More times between high and low water have been selected at equidistant steps from high and low water, at the steps \(n, \Delta t = (n + 0, \pm 1, \ldots, \pm 4)\) and \(\Delta t\) taken as one solar hour. All these times are collected into classes of sets, so that in each set the times are distributed over 19 years as in the first four sets. Altogether there are 36 sets of times. "Observations" of heights at these times are gained by interpolations from the solar hourly readings which are taken from the tidal records of 19 years.

Each of the 36 sets of heights has been adjusted to a linear combination of 44 components of the (b) type using the method of least squares. Checked by standard deviations, the 36 analyses proved to be as accurate as high and low water analyses. In each set the variation of prediction has turned out to be as small as at high and low water; residual functions only depending on the wind vector and equally accurate for all sets (at all times) may be derived from observations as taken for the selected times.

Until now we have tested this analysis by adjusting curves between high and low water to the predictions which are gained from different sets. There is overlapping of the predictions in the midst between high and low water and predictions coincide with the curves within one to two centimetres.

The results of the different kinds of tidal analyses have been compared with the observations of the water level in our daily forecasting service, mainly for the purpose of checking. But steadily comparing observations and predictions has been a good guide to intuition for the way how to go on.

We are going to extend the last kind of tidal analysis by adding formulae which represent the effect of the wind. To the observed heights of water we shall adjust, using the method of least squares, the 44 components of the (b) type and simultaneously the wind formula. We hope to derive in this way good formulae for the forecasting service and, at the same time, to improve the tidal prediction, and to "clean" the predictions from a proportion of the meteorological effects.

All the computations needed for the interpolation, the analyses, and the predictions are done by means of an electronic computer, IBM 7090. At the end a short remark may be given concerning the programme of tidal analyses. In each case of a harmonic analysis the matrix and the quantities on the right hand side of the normal equations have to be computed and the normal equations to be solved. The programme for these tasks is flexible and may be used for an arbitrary number \(k\) of components and an arbitrary odd number \(2n + 1\) of observations taken at equidistant times which are centred round the time zero. For the selected constituents the angular velocities \(a_1, \ldots, a_k\) have to be given as increase of the argument per time step. If there are no gaps in the row of the observations, the matrix of the order \(2k + 1\) splits into two matrices of the order \(k + 1\) and the members of the matrix may be computed by means of formulae. In the case of gaps the members of the matrix have to be calculated by summation. In most cases of the analysis the angular velocities of the components and the interval of the observations are in such a way arranged that the members in the diagonal of the matrix are larger than the other ones, or this state may be achieved after a linear transformation, so that the normal equations may be solved by iteration; if worthwhile, the inverse matrix will be computed, too, by means of iteration. This programme covers the cases of 19 days' observations taken every 5 minutes, of observations at solar hours of 15, 29, and 369 days, and the various analyses of sets of 19 years' observations.

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(6) W. HORN - "The Harmonic Analysis, According to the Least Square Rule, of Tide Observations upon which an Unknown Drift is Superposed", Proceedings of the Third International Symposium on Earth Tides, Triest 1959.

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OBSERVATIONS SUR LA COMMUNICATION DU Dr HABICH.

Dr KOSITER. Would it be correct to say that you require 24 analyses to predict hourly heights?
Dr HABICH. Yes, to predict the tide at hourly rates we have to analyze 24 times (at the different hours of the lunar day).

M. CROSBY. Are there no disparities between the variations of the constants of the waves during the years? In other words, are the criteria adopted for the choice of the wave constants, Dr HABICH. In the results of the 24 years of harmonic analysis of tide (of solar hourly values), the annual mean wave was more affected than the tidal constants. The depth affects the constants; I shall give an example: analysis of tides at Hamburg from 1925-1926 showed differences due to the regulation (dragging) of the river.
Dr DOLLIER. As a matter of interest and since you have analyzed Hamburg for a period of 2 years, you now have differences in both the high-water and low-water times and heights?
Dr HABICH. Analysis of low water proved to be more affected than analysis of high water as consequence of the dragging (of the variation of the depth).
Dr GODIN. Les ondes composées ne sont pas recherchées, mais plutôt leur résultante dans la hauteur et l'heure des extrêmes.
Dr HABICH. Yes, I agree with it.

L'ANALYSE DE DIX-NEUF ANNÉES D'OBSERVATIONS INTERRUMPTEES SUR LES PLEINES ET BASSES MERS

Gabriel GODIN
DÉPARTEMENT DES MINES DU CANADA

Jusqu'ici, la seule méthode directe pour l'analyse des marées des ports de petite profondeur est celle pratiquée par l'Institut hydrographique allemand. Elle requiert l'analyse de 19 années d'observations sur les hauteurs et des temps des pleines et basses marées en relation avec le passage de la lune.

La figure 1 montre la variation de la hauteur de la pleine mer à Québec qui correspond au transit de la lune, pour la période de janvier 1952 à décembre 1964. Comme on peut s'en rendre compte, cette variation est fortement perturbée durant l'hiver jusqu'au point de rendre le calcul de la courbe à peu près impossible. En effet, les observations sur le niveau des eaux sont interrompues durant cette saison pour les ports en amont de Québec. Et justement ces ports exhibent encore plus fortement l'influence des variations de profondeur dans leur marée. On a donc besoin de prédictions raisonnables pour ces endroits et il y a peu d'espoir d'y réussir par une simple addition de hauteur et de temps à la marée de Pointe au Père.

Puisque la méthode allemande est basée sur les moindres carrés et qu'en fait, il est beaucoup mieux de rejeter les données de hiver plutôt que d'essayer de les incorporer de force dans une analyse, il semble naturel d'appliquer le principe des moindres carrés aux observations d'été seulement, en recherchant les ondes composantes suggérées par la méthode allemande.

Si nous avons 2 M + 1 portions d'observations d'une durée chacune de 2 Tn unités de temps, séparées par 2 M brièvres d'une durée de G unités de temps, les matrices des équations normales deviennent

\[ f_{i} = f_{i} = \left( 2 \sum \left( \frac{2T_{n}+1}{(2T_{n}+1)(2T_{n}+1)} - \frac{1}{2} \frac{T_{n}}{2} \right) \sin \theta_{i} \sin \theta_{t} \right) + \frac{1}{2} \left\{ \sum \left( \frac{2T_{n}+1}{(2T_{n}+1)(2T_{n}+1)} - \frac{1}{2T_{n}} \right) \sin \theta_{i} \sin \theta_{t} \right\} \]

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OBSERVATIONS SUR LA COMMUNICATION DU Dr HABICH,

Dr HABICH. Would it be correct to say that you require 24 analyses to predict half-hourly heights?

Dr HABICH. Yes, To predict the tide at hourly hours we have to analyze 24 times (at the different hours of the lunar day).

M. COUGNET. Du fait des variations des profondeurs des mers et de la hauteur du niveau moyen des mers, n'y a-t-il pas des variations des constantes des ondes au cours des années ? En outre, quels sont les critères adoptés pour le choix des ondes constituant ces analyses?

Dr HABICH. In the results of the 24 years of harmonic analysis of tides (of solar hourly values), the annual mean were more affected than the tidal constants. The depth affects the constants; I shall give an example: analysis of tides at Hamburg from 1925-1942 showed differences due to the regulation (draging) of the river.

Dr DOHLER. As a matter of interest and since you have analysed Hamburg for a period of 2 times 18 years, did you found differences in both the high-water as well as the low-water times and heights?

Dr HABICH. Analysis of low water proved to be more affected than analysis of high water as consequence of the dragging (of the variation of the depth).

Dr GODIN. Les ondes composées ne sont pas recherchées, mais plutôt leur résultante dans la hauteur et l'heure des extrêmes?

Dr HABICH. Yes, I agree with it.

L'ANALYSE DE DIX-NEUF ANNÉES D'OBSERVATIONS INTERRUMPÉES SUR LES PLEINES ET BASSES MERS

Gabriel GODIN
Département des Mines du Canada

Jusqu'à ce la seule méthode directe pour l'analyse des marées des ports de petite profondeur est celle pratiquée à l'institut hydrographique allemand. Elle requiert l'analyse de 19 années d'observations sur les hauteurs et des temps des pleines et basses mers en relation avec le passage de la lune.

La figure 1 montre la variation de la hauteur de la pleine mer à Québec qui correspond au transit supérieur de la lune, pour la période de janvier 1942 à décembre 1964. Comme on peut s'en rendre compte cette variable est fortement perturbée durant l'hiver jusqu'au point de rendre le lissage de la courbe à peu près impossible. En fait, les observations sur le niveau des eaux sont interrompues durant cette saison pour les ports en amont de Québec. Il est juste ces ports exhibent encore plus fortement l'influence des marées de basses mers. D'autre part, on a donc besoin de prédictions rationnelles pour ces endroits et il y a peu d'espoir d'y réussir par une simple addition de hauteur ou de temps à la marée de Pointe à l'Ouest.

Puisque la méthode allemande est basée sur les moinards carrés et qu'en fait il est beaucoup mieux de rejeter les données d'hiver plutôt que d'essayer de les incorporer de force dans une analyse, il semble naturel d'appliquer le principe des moinards carrés aux observations d'hiver seulement, en recherchant les ondes composantes suggérées par la méthode allemande.

Si nous avons $2M + 1$ portions d'observations d'une durée chacune de $2T_n$ unites de temps, séparées par $2M$ breches d'une durée de G unites de temps, les matrices des équations normales deviennent

$$f_{2M} = f_{0M} = \left(2 \sum_{n=1}^{2M} \frac{\sum_{k=1}^{2M} \frac{(2T_n + 1) \left(\frac{T_n}{2} - \frac{x}{2}\right)}{T_n}}{1} \right) \cos \theta, \text{ et } \sin \theta$$

$$g_{2M} = g_{0M} = \left(2 \sum_{n=1}^{2M} \frac{\sum_{k=1}^{2M} \frac{(2T_n + 1) \left(\frac{T_n}{2} - \frac{x}{2}\right)}{T_n}}{1} \right) \sin \theta, \text{ et } \cos \theta$$

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\[
\begin{aligned}
\beta &= \sin \left( \frac{\beta - \frac{\gamma}{2}}{2} \right) \\
\gamma &= \sin \left( \frac{\beta - \frac{\gamma}{2}}{2} \right)
\end{aligned}
\]

Hauteurs des pleines mer

\[figure 1\]

Il n'y a pas de difficulté à calculer l'inverse de ces matrices. L'analyse par après consiste à calculer les vecteurs \( C_j \) et \( S_j \) sur les portions d'observations et ensuite à trouver les vecteurs \( X_j \) et \( Y_j \) en multipliant \( C_j \) et \( S_j \) par l'inverse des matrices des équations normales.

Nous avons choisi

\[ T_n = 88 \text{ jours lunaires} \]

\[ G = 175 \text{ jours lunaires} \]

et naturellement

\[ M = 9. \]

Ce choix cause une difficulté cependant. L'analyse de portions d'observations est équivalente à un échantillonnage et l'onde composante No. 2 telle que définie par la méthode allemande change de phase par un peu plus que 180° entre deux portions d'observations ; par conséquent elle ne peut pas être analysée et elle se confond avec l'onde No. 6.

L'extraction des données pour ce type d'analyse est extrêmement longue et laborieuse et nous n'avons pu vérifier jusqu'à ce que cette méthode ait vraiment réussi à donner de bonnes prédictions pour nos ports en amont de Pointe au Père.

ANALYSE D'UNE COURTE DURÉE D'OBSERVATION DE LA MARÉE
OU DU COURANT DE MARÉE

Marc EYRIÉS
Ingénieur Hydrographe en Chef
Service Hydrographique de la Marine Française

L'extension du réseau marégraphique mondial et la mise au point de marégraphes pourront permettre l'analyse de la marée sur de longues durées d'observation (de l'ordre de l'année) en de très nombreux points du Globe. Cependant, il sera toujours utile de tirer le meilleur parti des observations de hauteur d'eau, recueillies à l'occasion des levés bathymétriques côtiers qui s'accomplissent d'autant plus rapidement que les méthodes employées ont un rendement meilleur. Il faut envisager aussi le traitement des observations de marée par grandes profondeurs et celui des observations de courant qui sont de courte durée (de l'ordre de quelques semaines).

L'analyse des hauteurs d'eau ou des composantes du courant obtenues au cours d'une courte durée d'observation, présente deux difficultés essentielles :

- La première est la compensation insuffisante des écarts accidentels ; écarts instrumentaux, écarts de transcription des lectures instrumentales sur le document introduit dans l'analyse, écarts dus à des phénomènes naturels, parasites, et fortuites. On se borne, à ce sujet, aux remarques suivantes :

  - On peut penser que, pour le contrôle des données, l'enregistrement analogique est plus avantageux que l'enregistrement numérique parce qu'il permet la détection des plus grandes différences accidentelles par un examen rapide des courbes ; en réalité, la "traduction" de la courbe en valeurs numériques, indispensable pour l'analyse, introduit les erreurs les plus importantes.

  - L'effet des phénomènes parasites, non périodiques ou de période différente de celles des ondes analysées, est d'autant plus faible que la durée analysée est plus grande. Pour les courtes durées, ces erreurs restent importants et affectent notablement le résultat du calcul.

- La deuxième difficulté est la mauvaise "séparation" des ondes qui ont des coefficients astronomiques importants et par conséquent des amplitudes prévenues importantes ; c'est ce point que l'on considère particulièrement ici. Un critère commode de cette "séparation" est le nombre

\[ S = \frac{d \Delta T}{T^2} \]

où \( d \), \( T \), \( \Delta T \) sont respectivement, comptes avec la même unité, la durée de l'observation, la période de l'onde à écrater et l'écart de période avec les autres ondes de coefficients astronomiques importants.

Pour préciser clairement cette difficulté et montrer comment la surmonter, on considère dans la suite que l'analyse est faite suivant la méthode des "heures spéciales", assez fréquemment utilisée ; mais ce n'est pas une restriction à la généralité des conclusions.

\[ \text{(1) Le "criblage" d'une suite de valeurs consiste à détecter les écarts supérieurs à un "écart-type" sans réduire systématiquement ces écarts ; la valeur suspecte est éventuellement corrigée ou omise. Le "criblage" diffère donc essentiellement du "tissage" qui affecte systématiquement toutes les valeurs de la suite.} \]
ANALYSE D'UNE COURTE DURÉE D'OBSERVATION DE LA MARÉE
OU DU COURANT DE MARÉE

Marc EYRÈS
Ingénieur Hydrographe et Chef
Service Hydrographique de la Marine Française

L'extension du réseau marégraphique mondial et la mise au point de marégraphes à haute fréquence ont permis de recueillir des données sur de longues durées d'observation (de l'ordre de l'année) et de très nombreux points du globe. Cependant, il est nécessaire de tirer le meilleur parti des données collectées à l'occasion des levés bathymétriques côtiers qui s'accompagnent d'autant plus rapidement que les méthodes employées ont un rendement meilleur. Il faut envisager aussi le traitement des données de marée par grandes profondeurs et celui des observations de courant qui sont de courte durée (de l'ordre de quelques semaines).

L'analyse des hauteurs d'eau (ou des composantes du courant) obtenues au cours d'une courte durée d'observation, présente deux difficultés essentielles:

La première est la compensation insuffisante des écarts accidentels : écarts dûs à des erreurs dans la collecte des données, écarts dus à des phénomènes naturels, parasites ou perturbations. On se borne, à ce sujet, aux remarques suivantes,

On peut penser que, pour le contrôle des données, l'enregistrement analogique est plus avantageux que l'enregistrement numérique parce qu'il permet de détecter des erreurs plus rapidement. Une méthode de contrôle efficace est de corser les données introduites en machine par un examen rapide des courbes en valeur absolue.

L'effet des phénomènes parasites, non périodiques, ou de période différente de celle des ondes analysées, est d'autant plus faible que la durée mesurée est plus grande. Pour les courtes durées, ces erreurs restent importantes et affectent notoirement le résultat du calcul.

La deuxième difficulté est la mauvaise "séparation" des ondes qui ont des coefficients astromoniques importants et par conséquent des amplitudes très fortes ; c'est le cas par exemple de celle qui est constituée par la marée de Pointe au Fère.

La méthode d'analyse est extrêmement longue et laborieuse, et nous n'avons pas pu la vérifier jusqu'ici si cette méthode va vraiment réussir à donner de bonnes prédictions pour ces ports en amont de Pointe au Fère.


\[
\mathbf{x} = \begin{bmatrix}
\beta \\
\frac{1}{2}
\end{bmatrix} \begin{bmatrix}
\mathbf{c} \\
\mathbf{s}
\end{bmatrix} = \sin \beta \begin{bmatrix}
\frac{1}{2} & 0 \\
0 & -\frac{1}{2}
\end{bmatrix}
\]


figure 1
ONDE RESIDUELLE - COEFFICIENT DE RESIDU

Les notions classiques de "coefficient de résidu" et d'"onde résiduelle" sont brièvement rappelées ci-après.

Dans la hauteur d'eau (Y), à une époque t, la part due à une onde $A_i$ est:

$$Y_t = A_i \cos(q_i t - \beta_i)$$

où $t$ est le temps compté en heures de temps moyen à partir du début de la durée d'observation, $q_i$ est la "vitesse de l'onde" comptée en degrés par heure de temps moyen ($q_i = \frac{360^\circ}{T_i}$ ; $T_i$ est la période).

$A_i$ l'amplitude comptée dans la même unité que (Y), et $\beta_i$ la phase en degrés (2) ; et

$$Y_t = A_0 + \sum_{i=1}^{n} A_i \cos(q_i t - \beta_i)$$

$A_0$ est la cote du niveau d'équilibre au-dessus de l'origine des hauteurs d'eau ; $m$ est le nombre d'ondes.

Pour rechercher les constantes $A_i$ et $\beta_i$ d'une onde $A_i$ (de vitesse $q_i$ et de période $T_i$) on considère, dans une première étape, l'ensemble des $n$ équations d'observation correspondant aux époques :

$$t ; t + T_i ; \ldots ; t + (n - 1) T_i$$

On en fait la moyenne term à terme pour obtenir une équation "finale" qui remplace cet ensemble ; cette opération est indiquée par l'opérateur $\overline{\sum}$. La solution de cette équation finale, la contribution de l'onde $A_i$ est

$$\overline{\sum} Y_t = A_0 + \sum_{i=1}^{n} A_i \frac{\sin(q_i T_i)}{n \sin(n \frac{\pi}{n})} \cos \left[q_i (t - \frac{T_i - \frac{T_i}{n}}{n}) - \angle \right]$$

où on a posé :

$$c_i = 180^\circ \frac{q_i - \frac{T_i}{n}}{360^\circ} \frac{T_i}{n} = 180^\circ \frac{q_i}{360^\circ} - 1$$

Le terme

$$C_i = \frac{\sin(\frac{\beta_i}{2})}{n \sin(\frac{\pi}{n})}$$

est appelé "coefficient de résidu".

(2) $A_i$ et $\beta_i$ sont les "constantes harmoniques brutes" ; on en déduit les constantes harmoniques définitives $H_i$ et $g_i$ par les relations (cf Publication spéciale n° 26 du IHSD):

$$H_i = \frac{A_i}{C_i}$$

$g_i = \beta_i + \frac{q_i T_i}{n}$, Greenwich

$I_i$ est le facteur nodal pris au milieu de la durée d'observation et ($V_t + u_t$), Greenwich est pris au début de cette durée.

Il est d'autant plus petit que les périodes $T_1$ et $T_2$ sont plus différentes et que le nombre $n$ est plus grand (3) ; en pratique il devient rapidement négligeable, pourvu que la durée d'observation dépasse quelques jours, si les deux ondes $A_1$ et $A_2$ appartiennent à des familles différentes (4). Lorsque les deux ondes font partie de la même famille, on peut être amené à calculer $C_i$ ; il s'écrit alors à très peu près

$$C_i = \frac{\sin(\frac{\beta_i}{2})}{n \sin(\frac{\pi}{n})}$$

et de dépend que de $\beta_i$ ; il peut être aisément mis en table. Enfin ce coefficient est égal à 1 pour $i = j$.

L'équation finale est donc :

$$\overline{\sum} Y_t = A_0 + \sum_{i=1}^{n} A_i \cos \left[q_i t - \angle \right]$$

et pratiquement les termes sous le signe $\sum$ qui ne sont pas négligeables sont ceux qui sont relatifs aux ondes de la même famille que $A_i$ (c'est-à-dire aux ondes telles que $c_i$ soit assez petit pour que $(c_i)^2$ soit négligeable) et aux ondes harmoniques de $A_i$.

La deuxième étape de l'analyse consiste à déterminer les coefficients du développement en série de Fourier de la fonction de $t$ :

$$\overline{\sum} Y_t$$

sur l'intervalle $[0 ; T_i]$. Les deux coefficients de la fondamentale (coefficient des termes de période $T_i$) sont identiques aux coefficients de la fondamentale du développement sur le même intervalle de la fonction

$$A_0 \cos(q_i t - \beta_i) + \sum_{j=1}^{n} A_j \cos(q_j t - \beta_j)$$

où $A_j$ cos ($q_j t - \beta_j$) peut être considéré comme la hauteur d'eau partielle due à une onde $A_j$ ; appelée "onde résiduelle de $A_j$ sur $A_i$". Les constantes brutes de cette onde sont données par :

$$A_j = C_j A_i \sin(180^\circ c_j^2)$$

$$tg \beta_j = (1 - c_j) \sin \beta_j$$

et à très peu près, parce que $c_j$ est petit :

$$A_j = C_j A_i \frac{\sin(180^\circ n c_j)}{180^\circ n c_j n \sin(\frac{\pi}{n})}$$

$$tg \beta_j = (1 - c_j) \sin \beta_j$$

Ces relations permettent de prendre l'onde $A_j$ comme inconnue intermédiaire pour calculer $A_i$.

(3) Il faut noter cependant que le choix de n voisin de $n \approx \frac{360^\circ}{p_i}$ est un entier rend le coefficient de résidu négligeable même si n n'est pas très grand ; c'est pourquoi, compte tenu des facteurs astronomiques des ondes, il est recommandé de choisir pour la durée d'observation analyzée les durées de 14, 15, 29, 58, 87 etc., jours moyens.

(4) On appelle "familles" un ensemble d'ondes dont les périodes sont de l'ordre de l'un des multiples ou sous-multiples du jour moyen (famille diurne ..., famille semi-diurne, ...) on appelle "groupes" les sous-ensembles d'une famille.
ONGE RESIDUELLE - COEFFICIENT DE RESIDU

Les notations classiques de "coefficient de résidu" et d'"onde résiduelle" sont brièvement rappelées ci-après.

Dans la hauteur d'eau (Y), à une époque t, la part due à une onde \( A_i \) est:

\[
Y_i = A_i \cos(q_i t - \beta_i)
\]

où t est le temps compté en heures de temps moyen à partir du début de la durée d'observation, \( q_i \) est la "vitesse de l'onde" comptée en degrés par heure de temps moyen (\( q_i = \frac{360^\circ}{T_i} \); \( T_i \) est la période);

\( A_i \) l'amplitude comptée dans la même unité que (Y), et \( \beta_i \) la phase en degrés (2); et

\[
Y_i = A_i + \frac{n}{m} \sum_{i=1}^{n} A_i \cos(q_i t - \beta_i)
\]

\( A_i \) est la cote du niveau d'équilibre au-dessus de l'origine des hauteurs d'eau ; m est le nombre d'ondes.

Pour rechercher les constantes \( A_i \) et \( \beta_i \) d'une onde \( A_i \) (de vitesse \( q_i \) et de période \( T_i \)) on considère, dans une première étape, l'ensemble des n équations d'observation correspondant aux époques :

\[
t ; t + T_i ; \ldots ; t + (n - 1) T_i
\]

On en fait la moyenne terme à terme pour obtenir une équation "finale" qui remplace cet ensemble ; cette opération est indiquée par l'opérateur \( \overline{()} \).

Dans cette équation finale, la contribution de l'onde \( A_i \) est :

\[
\overline{Y_i} = A_i \frac{\sin \frac{q_i}{n} \cos \left[ q_i t - \left( \frac{\beta_i}{n} + \frac{n - 1}{n} \beta_i \right) \right]}{\sin \frac{\beta_i}{n}}
\]

où on a posé :

\[
\beta_i = 180^\circ \frac{n - q_i}{q_i} \frac{T_i - T_i}{T_i} = 180^\circ \frac{n}{\sin \frac{\beta_i}{n}}\sin \frac{q_i}{n} = \frac{T_i}{q_i} - 1
\]

Le terme

\[
C_i = \frac{\sin \frac{\beta_i}{n}}{\sin \frac{\beta_i}{n}}
\]

est appelé "coefficient de Résidu".

(2) \( A_i \) et \( \beta_i \) sont les "constantes harmoniques brutes" : on en déduit les constantes harmoniques définitives \( H_i \) et \( g_i \) par les relations (cf Publication spéciale n° 26 du IBH) :

\[
K_i = \frac{1 - e_i^2}{e_i^2} \quad H_i = g_i + \sqrt{g_i^2 + \mu_i} \quad \text{Greenwich}
\]

\( f_i \) est le facteur nodal pris au milieu de la durée d'observation et \( (V_i + u_i) \) Greenwich est pris au début de cette durée.

Il est d'autant plus petit que les périodes \( T_i \) et \( T_j \) sont plus différentes et que le nombre n est plus grand (3) ; en pratique il devient rapidement négligeable, pourvu que la durée d'observation dépasse quelques jours, si les deux ondes \( A_i \) et \( A_j \) appartiennent à des familles différentes (4). Lorsque les deux ondes font partie de la même famille, on peut être amené à calculer \( C_i \) ; il s'écrit alors à très peu près

\[
C_i = \frac{\sin \frac{\beta_i}{n}}{\sin \frac{\beta_i}{n}}
\]

et dépend de que de \( \beta_i \) ; il peut être aisément mis en table. Enfin ce coefficient est égal à 1 pour \( i = j \).

L'équation finale est donc :

\[
\overline{Y_i} = A_i + \sum_{i=1}^{n} A_i \cos \left[ q_i t - \left( \frac{\beta_i}{n} + \frac{n - 1}{n} \beta_i \right) \right]
\]

et pratiquement les termes sous le signe \( \sum \) qui ne sont pas négligeables sont ceux qui sont relatifs aux ondes de la même famille que \( A_i \) (c'est-à-dire aux ondes telles que \( c_i \) soit assez petit pour que \( c_i^2 \) soit négligeable) et aux ondes harmoniques de \( A_i \).

La deuxième étape de l'analyse consiste à déterminer les coefficients du développement en série de Fourier de la fonction de i :

\[
\overline{Y_i}
\]

sur l'intervalle \( [0 ; T_i] \). Les deux coefficients de la fondamentale (coefficients des termes de période \( T_i \)) sont identiques aux coefficients de la fondamentale du développement sur le même intervalle de la fonction

\[
A_i \cos \left( q_i t - \beta_i \right) + \sum_{j=1}^{n} A_j \cos \left( q_j t - \beta_j \right)
\]

où \( A_i \cos \left( q_i t - \beta \right) \) peut être considéré comme la hauteur d'eau partielle due à une onde \( A_i \) : appelée "Onze Résiduelle de \( A_i \)" sur \( A_i \). Les constantes brutes de cette onde sont données par :

\[
A_i = \text{Coeff. fondamentale}
\]

\[
\frac{\sin 180^\circ \epsilon_i^2}{180^\circ \epsilon_i^2} \frac{\sin \frac{1}{T_i} (1 - \epsilon_i) \cos \epsilon_i}{\sin \frac{1}{T_i} (1 - \epsilon_i) \cos \epsilon_i}
\]

et à très peu près, parce que \( \epsilon_i \) est petit :

\[
A_i = \text{Coeff. fondamentale}
\]

\[
\epsilon_i = \frac{\beta_i}{180^\circ \epsilon_i} \frac{\beta_i}{180^\circ \epsilon_i}
\]

Ces relations permettent de prendre l'onde \( A_i \) comme inconnue intermédiaire pour calculer \( A_i \).

...
Les analyses des fonctions $Y_i$ pour $i = 1, \ldots, m$, fournissent ainsi un système de $m$ équations vectorielles entre les $m$ inconnues ; ce système est particulièrement simple lorsque $n$ est très grand parce que les amplitudes des ondes résiduelles sont négligeables et chaque équation contient une seule inconnue. Lorsque la durée d’observation est courte, toutes les inconnues figurent dans chaque équation (directement ou par l’intermédiaire de leur résidu), mais il n’y a pas de difficulté théorique à résoudre le système. La difficulté pratique ne vient pas du volume des calculs (faisablement absorbé par un ordinateur) mais du fait que les équations où figurent directement des inconnues de périodes voisines ont des coefficients voisins. Il en résulte que, compte tenu des incertitudes sur les données, l’erreur sur les inconnues est très grande comme on va le voir sur l’exemple simple ci-après :

On considère une marge, constituée seulement des ondes $A_i$ et $\bar{A}_i$. Les analyses de ces ondes fournissent respectivement pour la fondamentale du développement de Fourier les amplitudes $M_i$ et $\bar{M}_i$ et les phases $\phi_i$ et $\bar{\phi}_i$. Si ces ondes sont de périodes voisines, $\phi_i$ n’est différent de $\bar{\phi}_i$ que d’un multiple de $2\pi$. Les constantes harmoniques brutes de $A_i$ et $\bar{A}_i$ sont les solutions $A_j$ et $\bar{A}_j$ du système :

$$
A_i \cos \phi_i = \frac{M_i}{1 - C_i} \cos \phi_i - \frac{C M_i}{1 - C_i} \cos (\phi_i + \beta),
$$

$$
A_i \sin \phi_i = \frac{M_i}{1 - C_i} \sin \phi_i - \frac{C M_i}{1 - C_i} \sin (\phi_i + \beta),
$$

$$
A_i \cos \phi_i = \frac{M_i}{1 - C_i} \cos \phi_i - \frac{C M_i}{1 - C_i} \cos (\phi_i - \beta),
$$

$$
A_i \sin \phi_i = \frac{M_i}{1 - C_i} \sin \phi_i - \frac{C M_i}{1 - C_i} \sin (\phi_i - \beta).
$$

On constate que les coefficients des termes communs $(M_i, \phi_i, \ldots)$, qui sont déduits d’observations (donc entachés d’incertitudes), sont très grands et par conséquent les inconnues sont mal déterminées.

On peut dire aussi que les équations du système final qui ont des coefficients voisins peuvent être remplacées par une (ou par l’équation obtenue par une moyenne terme à terme) ; il en résulte que le système comporte alors moins d’équations que d’inconnues ; il faut introduire des relations étrangères aux équations d’observation, c’est-à-dire poser une hypothèse supplémentaire.

CONSTANCES APPROCHÉES - MARRE-DIFFÉRENCE

Il convient de remarquer tout d’abord que l’emploi de constantes approchées introduit une hypothèse supplémentaire ; ces constantes s’utilisent de deux manières :

(5) C’est une des qualités de la "méthode des heures spéciales" de construire des équations où une inconnue figure seule ; le nombre $n$ étant théoriquement infini, il faut en pratique faire une hypothèse sur l’ensemble des ondes qui interviennent dans la marge étudiée ; cette hypothèse s’interdit pas dans la "méthode des heures spéciales (si $n$ est grand) ; elle intervient par exemple si on applique aux équations d’observation la "méthode des moindres carrés".

(6) Voir "La Méthode des Concurrences et l’Analyse harmonique" par les constantes approchées (Annales Hydrographiques 1956 et Revue Hydrographique Internationale Vol XXXIV no 1). L’emploi de ces constantes a bien d’autres avantages notamment pour le criblage des observations et le calcul.

Suivant la première ("Utilisation directe") (6), on effectue le calcul des termes sous le signe $\Sigma$ dans les équations finales ; il n’y a donc plus qu’une inconnue par équation dans le cas où $n$ est très grand. L’hypothèse faite est que l’onde résiduelle de l’onde "approchée" (onde qui admet pour constantes, les constantes approchées) est identique à la résiduelle de l’onde exacte correspondante. On voit que cette hypothèse intervient d’autant moins que $n$ est plus grand puisque l’amplitude de ces deux résiduelles et par conséquent celle de leur différence, décroît avec $1/n$.

Suivant la deuxième manière ("Utilisation directe") (6), on calcule une marge approchée à l’aide des constantes approchées, puis la "marre-différence" qui est, en fonction du temps, la différence des hauteurs d’eaux, réelle moins calculée, et on analyse cette marre différence par la méthode des heures spéciales.

L’hypothèse faite est que deux ondes $A_i$ et $\bar{A}_i$ de périodes très voisines ont une même "onde complémentaire" $\bar{A}_i$ (7) ; ce qui diminue d’une unité le nombre des inconnues du système final et permet de négliger l’équation correspondant à $A_i$ ou $\bar{A}_i$.

CONCORDANCES SUR GROUPES D’ONGES

L’hypothèse supplémentaire la plus commode à introduire est celle de "marres semblables" qui se définit ainsi (6) : Deux marées en deux ports $A$ et $B$ sont respectivement de la forme :

$$(Y_i) = A_i + \sum_{i=1}^{\infty} A_i \cos (\phi_i t - \beta_i),$$

$$(Z_i) = B_i + \sum_{i=1}^{\infty} B_i \cos (\phi_i t - \gamma_i),$$

elles sont semblables si les rapports $\frac{B_i}{A_i}$ et $\frac{\phi_i - \beta_i}{\phi_i}$ sont indépendants de $i$. Il est clair que, lorsque ces conditions sont réalisées, la courbe de hauteur d’eau en fonction du temps au port $B$ se déduit de celle du port $A$ par une translation égale à $\frac{\phi_i - \beta_i}{\phi_i}$ suivant l’axe des temps et par une affine suivant l’axe des hauteurs de rapport $\frac{A_i}{B_i}$. Inversement si les deux courbes se déduisent ainsi l’une de l’autre, les marées sont semblables et les constantes harmoniques en $B$ se déduisent sans difficulté des constantes en $A$. En effet, les conditions ci-dessus étant satisfaites pour les ondes composantes, elles sont pour les résiduelles ; il suffit d’analyser dans les deux ports, la même durée d’observation sur une seule onde pour connaître la valeur des rapports.

Dans la nature, les marées strictement semblables n’existent que pour des ports très proches l’un de l’autre dans un même domaine océanique et cette similitude a été depuis longtemps utilisée pour raccorder les divers observatoires d’un même levé bathymétrique. Mais il est raisonnable de penser que les réponses d’un domaine océanique à des excitations de périodes voisines sont elles-mêmes voisines. Ceci vaut dire que, en deux points $A$ et $B$ de ce domaine, les conditions de similitude sont satisfaites pour une marée partielle, composée d’un groupe d’ondes (3) de périodes voisines, représentée par l’onde qu’on considère dont l’amplitude présente est la plus grande. La similitude est d’autant mieux réalisée que les ports sont plus proches et que la bande de périodes, couverte par le groupe, est plus étroite.

Si $A_i$ est l’onde approchée de $A_i$, l’onde complémentaire (inconnue de la marre-différence) est $A_i = A_i - A_i$.
Les analyses des fonctions \( Y_i \) pour \( i = 1, \ldots, m \), fournissent ainsi un système de \( m \) équations vectorielles entre les \( m \) inconnues ; ce système est particulièrement simple lorsque \( n \) est très grand parce que les amplitudes des ondes résiduelles sont négligeables et chaque équation contient une seule inconnue été. Lorsque la durée d'observation est courte, toutes les inconnues figurent dans chaque équation (directement ou par l'intermédiaire de leur résiduelle), mais il n'y a pas de difficulté théorique à résoudre le système.

La difficulté pratique qui ne vient pas du volume des calculs (facilement absorbé par un ordinateur) mais du fait que les équations où figurent directement des inconnues de périodes voisines ont des coefficients voisins, il en résulte que, compte tenu des incertitudes sur les données, l'erreur sur les inconnues est très grande. On va le voir sur l'exemple simple ci-après.

On considère une marée, constituée seulement des ondes \( \bar{A}_1 \) et \( \bar{A}_2 \). Les analyses de ces ondes fournissent respectivement pour la fondamentale du développement de Fourier les amplitudes \( M_1 \) et \( M_2 \) et les phases \( \beta_1 \) et \( \beta_2 \). Si ces ondes sont de périodes voisines, 

\[
\delta_1 \neq \delta_2 = \delta = C \neq \delta = 1.
\]

Les constantes harmoniques brutes de \( \bar{A}_1 \) et \( \bar{A}_2 \) sont les solutions \( A_j ; \beta_j \) ; \( \beta_i \) du système :

\[
\begin{align*}
A_1 \cos \beta_1 &= \frac{M_1}{1 - C^2} \cos \delta_1 - \frac{C M_2}{1 - C^2} \cos (\delta_1 + \delta) \\
A_1 \sin \beta_1 &= \frac{M_1}{1 - C^2} \sin \delta_1 - \frac{C M_2}{1 - C^2} \sin (\delta_1 + \delta) \\
A_2 \cos \beta_2 &= \frac{M_2}{1 - C^2} \cos \delta_2 - \frac{C M_1}{1 - C^2} \cos (\delta_2 - \delta) \\
A_2 \sin \beta_2 &= \frac{M_2}{1 - C^2} \sin \delta_2 - \frac{C M_1}{1 - C^2} \sin (\delta_2 - \delta)
\end{align*}
\]

On constate que les coefficients des termes communs \( (M_1 ; M_2, \ldots) \), qui sont déduits d'observations (donc entachés d'incertitudes), sont très grands et par conséquent les inconnues sont mal déterminées.

On peut dire aussi que les équations du système final qui ont des coefficients voisins peuvent être remplacées par l'une d'elles (ou par l'équation obtenue par une moyenne terme à terme) ; il en résulte que le système comporte alors moins d'équations que d'inconnues ; il faut introduire des relations étrangères aux équations d'observation, c'est-à-dire poser une hypothèse suppletaire.

**CONSTANTES APPROCHÉES - MARÉE-DIFFÉRENCE**

Il convient de remarquer tout d'abord que l'emploi de constantes approchées \( (6) \) introduit une hypothèse suppletaire ; ces constantes s'utilisent de deux manières :

\( (5) \) C'est une des qualités de la "méthode des heures spéciales" de construire des équations où une inconnue figure seule ; le nombre \( n \) étant théoriquement infini, il faut en pratique faire une hypothèse sur l'ensemble des ondes qui interviennent dans la marée étudiée ; cette hypothèse s'intervient pas dans la "méthode des heures spéciales (si \( n \) est grand) ; elle intervient par exemple si on applique aux équations d'observation la "méthode des moindres carrés".

\( (6) \) Voir "La Méthode des Concorances et l'analyse harmonique par les constantes approchées" (Annales Hydrographiques 1956 et Revue Hydrographique Internationale Vol XXXIV n° 1). L'emploi de ces constantes a bien d'autres avantages notamment pour le criblage des observations et le calcul.

Suivant la première ("Utilisation directe" \( (6) \), on effectue le calcul des termes sous le signe \( \sum \) dans les équations finales ; il n'y a donc plus qu'une inconnue par équation comme dans le cas où \( n \) est très grand. L'hypothèse faite est que l'onde résiduelle de l'onde "approchée" (onde qui admet pour constantes, les constantes approchées) est identique à la résiduelle de l'onde exacte correspondante. On voit que cette hypothèse intervient d'autant moins que \( n \) est plus grand puisque l'amplitude de ces deux résiduelles et par conséquent celle de leur différence, décroît avec \( 1/n \).

Suivant la deuxième manière ("Utilisation directe" \( (6) \), on calcule une marée approchée à l'aide des constantes approchées, puis la "marée-différence" qui est, en fonction du temps, la différence des hauteurs d'eau, réelle moins calculée, et on analyse cette marée différence par la méthode des heures spéciales.

L'hypothèse faite est que deux ondes \( \bar{A}_1 \) et \( \bar{A}_2 \) de périodes très voisines ont une même "onde complémentaire" \( \bar{A}_1 \) (7) ; ce qui diminue d'une unité le nombre des inconnues du système final et permet de négliger l'équation correspondant à \( A_1 \) ou à \( A_2 \).

**CONCORDANCES SUR GROUPES D'ONDES**

L'hypothèse supplémentaire la plus commode à introduire est celle de "marées semblables" qui se définit ainsi \( (6) \) :

Deux marées en deux ports \( A \) et \( B \) sont respectivement de la forme :

\[
\begin{align*}
(Y_A)_t &= A_0 + \sum A_i \cos (\eta_i t - \beta_i) \\
(Z_B)_t &= B_0 + \sum B_i \cos (\eta_i t - \gamma_i)
\end{align*}
\]

elles sont semblables si les rapports \( \frac{B_i}{A_i} \) et \( \frac{\eta_i - \beta_i}{\eta_i - \gamma_i} \) sont indépendants de \( i \). Il est clair que, lorsque ces conditions sont réalisées, la courbe de hauteur d'eau en fonction du temps au port \( B \) se déduit de celle du port \( A \) par une translation égale à \( \frac{\gamma_i - \beta_i}{\eta_i - \eta_i} \) suivant l'axe des temps et par une affine suivant l'axe des hauteurs de rapport \( \frac{B_i}{A_i} \) (inversé si les deux courbes se déduisent ainsi l'une de l'autre, les marées sont semblables et les constantes harmoniques en \( B \) se déduisent sans difficulté des constantes en \( A \). En effet, les conditions ci-dessus étant satisfaites pour les ondes composantes le sont pour leurs résiduelles ; il suffit d'analyser dans les deux ports, la même durée d'observation sur une seule onde pour connaître la valeur des rapports.

Dans la nature, les marées strictement semblables n'existait que pour des ports très proches l'un de l'autre dans un même domaine océanique et cette similitude a été depuis longtemps utilisée pour raccorder les divers observatoires d'un même levé bathymétrique. Mais il est raisonnable de penser que les réponses d'un domaine océanique à des excitations de périodes voisines sont elles-mêmes voisines. Ceci veut dire que, en deux points \( A \) et \( B \) de ce domaine, les conditions de similitude sont satisfaites pour une marée partielle, composée d'un groupe d'ondes (3) de périodes voisines, représenté par l'onde qu'on choisit dont l'amplitude présente la plus grande. La similitude est d'autant mieux réalisée que les ports sont plus proches et que la bande de périodes, couverte par le groupe, est plus étroite.

\( (7) \) Si \( \bar{A}_1 \) est l'onde approximée de \( \bar{A}_0 \), l'onde complémentaire (inconnue de la marée-différence) est 

\[
\bar{A}_0 = \bar{A}_1 - \bar{A}_0.
\]
Ainsi la "méthode des concordances sur Groupes d'Ondes" s'applique comme suit :

On analyse séparément la marée inconnue du Port B et la marée observée, aux mêmes époques, dans le port de référence A. L'analyse porte sur un certain nombre d'ondes réparties dans le spectre de la marée, chacune représentant un groupe dont la largeur de bande est d'autant plus grande que la durée d'observation est plus petite. On calcule alors, pour chaque onde l'rapport $\frac{A_k}{A_k}$ et $\frac{B_k}{B_k}$ et on les utilise pour déterminer les constantes des ondes du groupe dans le port B. Il est clair que, dans cette méthode comme dans celle des constantes approchées (première manière), l'effet de l'hypothèse supplémentaire diminue au fur et à mesure que la durée d'observation augmente, c'est-à-dire que cette hypothèse est moins nécessaire (8).

Il peut arriver que l'on ne dispose pas de la marée observée au port de référence A, mais seulement de ses constantes ; il suffit alors de prédire la marée en A aux mêmes époques où elle est observée en B et d'opérer comme plus haut (9). Enfin on peut être amené à utiliser, comme marée de référence, une marée artificielle, prédite à partir du développement du potentiel génératrice.

L'introduction de l'hypothèse supplémentaire par la concordance sur groupes est particulièrement commode car elle ne nécessite pas de programme spécial, on utilise ceux qui ont été "optimisés" pour l'analyse et la prédiction, que l'on soit les méthodes employées, et dont l'emploi est familier aux opérateurs. En outre, le rapport d'amplitude $\frac{A_k}{A_k}$ et le déphasage $\phi_k - \phi_k$ peuvent être appliqués directement aux constantes $A_k$ et $B_k$ du port de référence ce qui évite le passage des constantes brutes aux constantes définitives (2).

EXEMPLE DE CONCORDANCES SUR GROUPES D'ONDES

On dispose, pour l'observatoire de Brest, situé à l'extrémité de la Bretagne, et celui de la Pointe de Grave, situé à l'embarcadère de la Gironde, de constantes harmoniques calculées sur 370 jours d'observation. Dans ces deux ports, la marée a une amplitude moyenne assez forte (moyenne moyen à Brest : 5,4 m et à Port Brest : 4,5 m) les deux observatoires sont distants de 210 milles ; ils se trouvent sur le même domaine océanique, mais le premier est situé à l'extrémité d'une péninsule et le plateau littoral est large, le second au fond d'un golfe, à l'embouchure d'un fleuve important et le plateau littoral est étroit.

On a appliqué la concordance sur groupes pour déterminer les constantes de la Pointe de Grave à partir de celles de Brest en utilisant un certain nombre de durées, commençant toutes le 1er juillet 1963 et comprenant : 7, 15, 22, 30, 37, 44, 52, 59, 67, 74 jours. Les ondes analysées étaient les principales semi-diurnes et la principale diurne (relativement faible). Ces constantes ont permis de construire des vecteurs $A_k$ qui ont été comparés aux vecteurs $\hat{A}_k$, déduits des constantes calculées sur 370 jours et considérées comme exactes ; on a examiné un "module relatif" qui est en centièmes le rapport :

$$\left| \frac{A_k - \hat{A}_k}{\hat{A}_k} \right|$$

(b) Au fur et à mesure que la durée d'observation est plus longue, l'analyse donne un spectre qui passe du spectre de bandes au spectre de raies.

(9) L'usage de la marée de référence observée et celui de la marée de référence prédite ont leurs avantages propres. La marée observée tient compte d'ondes qui existent réellement dans les deux marées A et B mais qui ont été omises dans l'analyse de A ; la marée prédite représente très exactement la somme des composantes utilisées et l'incertitude sur ces composantes ne se répercute pratiquement pas sur les constantes d'ondes en B.

**OBSERVATION SUR LA COMMUNICATION DE M. EYRIES**

Dr CARTWRIGHT, The method you have described, which allows for a smooth relation between neighbouring constituents, has some similarity to Dr MUNRO and my method, in which we assume a smooth relation between observed tide and equilibrium tide over a complete tidal cycle. If our method was applied to 7 days record, using perhaps just 1 or 2 time lags, I think it would give similar accuracy to that of M. EYRIES.
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\[
\left| \frac{A' - A''}{A_1} \right|
\]

----------

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COMPUTER PROGRAMS FOR THE ANALYSIS AND PREDICTION OF TIDES
PREPARED AT THE LIVERPOOL TIDAL INSTITUTE

G. LENNON
University of Liverpool

My contribution to this symposium is not concerned with new techniques nor with the elegant treatment of tidal time series, but rather is concerned with an attempt to perform the routine tasks of tidal analysis and prediction in a well-ordered and efficient manner. Part of what I have to say is concerned specifically with a particular type of computer and consequently may not be of interest to those who do not have access to this machine. I shall, however, endeavour to keep my remarks within the bounds of general interest.

THE SELECTION OF A COMPUTER

a) The pattern which emerges in computer work suggests that the larger and faster computers are much more efficient instruments than their smaller brethren in that although their running costs are higher, their speed of computation is such as to give a more favourable economic factor. For example an increase in running costs by a factor of 20 is likely to yield an increase in computing speed by a factor of 100. This argument applies to the large scale research project with some force, but it is necessary to acknowledge that the computations involved in routine tidal analysis and prediction are relatively trivial. Furthermore tidal work is peculiar in that a small amount of computation is linked with a large amount of input, in the case of analysis, or output, in the case of prediction. In this context it should be noted that input/output procedures are essentially mechanical operations and do not follow the same economic laws as those which apply to computing operations when progressing from a small computer to a large one. If, therefore, input and output time is a significant part of the total time required by a computer operation it is possible that this operation may be performed most efficiently upon a small computer.

b) Where a digital record has to be provided by manual means for analysis, then there is some advantage to be gained in the use of a card input machine rather than a punched tape machine. The difficulties experienced in attempts to produce a correctly punched version of a tidal record on tape are considerable. The process involves repeated duplication of the tape so that corrections may be effected and it is our experience that the machinery used for this purpose has a significant failure rate, with the result that any duplication of a tape may manufacture new errors. It is a much more simple matter to extract a card which contains an error from a card deck, to reproduce this card and to insert the correct card in the original deck.

Prediction processes are not so critical of the input/output medium but where the output is given in the sequence required by a tide table format rather than in chronological order, then there exists a slight preference for punched tape output rather than cards.

c) It is so obvious that it hardly need be stated, but the user must have complete confidence in the reliability of the computer. Parity checks in input and output devices are essential and in fact are quite common. Internal Parity checks are not common but where this facility does exist, the reliability of the installation is greatly increased.

Reference to the above considerations led to a decision at the Tidal Institute to develop a family of computer programs for the treatment of hourly elevations of the tide for use on an IBM 1620 computer. In the hierarchy of modern installations, the IBM 1620 is very small and very slow, however experience has shown our choice to be justified in that in many of these applications it has been found impossible to achieve greater efficiency with more sophisticated computers.
The programs to which I refer are listed in tabular form in the attached diagram together with their requirements in storage capacity and computer time. This is a compatible family in that the output of one program can be used as the input of another and in fact there are facilities available which may not be immediately obvious from the diagram. The analysis program for example is converted into a device for the generation of input to the prediction program or for the generation of values of V, u and f by simple operation of the console switches. The programs are designed, where possible, to have internal checking devices with appropriate error messages should these be required and moreover do attempt to establish some degree of control over their operation by the generation of instructions for the benefit of the computer operator.

All are written in the basic language of the 1620, the Symbolic Programming System. Perhaps at this stage I might indicate very briefly the techniques involved under the various headings.

1/ Verification.

Several references have been made in our discussions to the process of editing data. The technique adopted here is one of error detection rather than error correction and this takes the form of a simple smoothing check.

Since tidal observations are most smooth in a 25 hour sense the following condition is tested:

$$T > \frac{1}{6} \left( \frac{C_{10}}{C_{60}} + 4 \frac{C_{15}}{C_{55}} + 4 \frac{C_{20}}{C_{60}} - \frac{C_{1}}{C_{60}} \right)$$

Where T is a tolerance generally 2 or 3 % of the tidal range and $C_{6j}$ is the tidal elevation at time $t$ hours.

Due to atmospheric perturbations of sea level, which commonly have a duration of the order of a day, the above test alone is insufficient since it is inclined to reject many items of the tidal record which are in fact perfectly valid. Consequently if the above condition is not satisfied, a further test is applied before the particular item of data is recognised as an 'error':

$$T > \frac{1}{6} \left( \frac{C_{10}}{C_{60}} + 4 \frac{C_{15}}{C_{55}} + 4 \frac{C_{20}}{C_{60}} - \frac{C_{1}}{C_{60}} \right)$$

If this second test were to be applied alone then again a large quantity of pseudo-error output would ensue because it has a tendency to fall near the turning points of the tide particularly where shallow-water distortions are present. In conjunction the tests perform an efficient task and result in the verification of data within the time taken by the computer to read the cards involved.

2/ Analysis.

On the 1620 I have programmed the traditional techniques of tidal analysis devised by DOODSON using linear combination processes. In this choice certain restrictions are implied and in particular it is necessary to have continuous data over a preset interval, which can be 358 days for one program or 32 days for a second. The philosophy of this choice, however, results from a recognition of the fact that DOODSON'S techniques were developed for hand calculators so that the bulk of computation had necessarily to be kept to a minimum. If, therefore, this same technique can be programmed it must inevitably result in a process which is very economic in computer time. The disadvantage which must be overcome is that the technique does not lend itself easily to an automatic process so that there are considerable programming difficulties. The programs are cumbersome to write and in fact the 358 days analysis involves some 4000 instructions. Experience has shown that this selection was justified. The extraction of 60 harmonic constituents from a tidal record extending over one year can be accomplished in approximately 3 minutes and, as another contribution to this symposium will show, the quality of this results bear comparison with those from more sophisticated techniques.

3/ Prediction of hourly elevation.

The bulk of calculation involved in the prediction of hourly elevations is such that a very careful consideration must be given to the techniques involved. As an illustration of this point, my first attempt to program this case on the 1620 involved most expensive use of computer time at the rate of 3.5 minutes per day's prediction. My most recent program will provide predictions for almost 6 months in the same time. The optimisation process depends upon the choice of a convenient unit of angular measure to fit the specific case.

The storing of a cosine table to cover the complete range, 2π, at an interval close enough to dispense with interpolation, and at the same time allowing for the ultimate definition of tidal elevations correct to 1/10 foot in the key to the problem. Consideration shows that if the elements of the table are defined by 4 decimal digits and if they are spaced at angular intervals of $\pi/1250$ then this is just sufficient to give the accuracy required. This being so there are two distinct advantages in selecting $\pi/5000$ as the unit of angular measure:

a) Using such a unit it is not necessary to test whether a constituent phase has exceeded 2π each time it is incremented by $\pi$ since such a condition would involve a "carry-over" to give an additional digit. This "carry-over" can be automatically prohibited by defining the length of the numerical field within the computer.

b) A 4 digit cosine table at intervals of $\pi/1250$ contains 2 x 5000 digits in the range 2π so that there is a simple relationship between the constituent phase angle, expressed in units of $\pi/5000$ and the address of its cosine within the computer.

To be precise, the angular unit which I use is of the form:

$$2 \text{DDDDD}$$

where DDDD is the 4 digit integral part of the angle expressed in terms of $\pi/5000$ and is always a multiple of 4. The senior digit is flagged to prevent "carry-over".

DDDDD is a 4 digit decimal part of the angle and is expressed in terms of $\pi/1250$.

These two parts are separated by the digit 0 initially set at zero and monitored throughout the computation so that when a "carry-over" appears in this location the quantity 39 is added to the field addressed by this separating digit.

Finally the cosine table is arranged so that the address of cosine zero is 20,000. The insertion of a leading digit 2 in the phase angle therefore ensures that 2 DDDD is the address of its own cosine.

This device ensures that hourly elevations can be produced at a rate of 8.5 mins per 30 constituent year.

4/ Mean Sea Level Reduction.

The calculation of mean sea level using a numerical filter (DOODSON'S X0) is a trivial task which is applied within card reading time. The program will treat any data irrespective of starting date or discontinuities and arrange the output, properly annotated, as to date and with appropriate monthly means calculated. A diagram shows the response function of the filter used.

As a final comment upon the 1620 programs it should be noted in reference to the schedule of running times which accompany this paper that a facility exists to store the programs upon discs which render them immediately accessible to the computer. This device effectively removes all time commitments under the heading "program input". Again under the heading "computation" are listed the duration of the intervals which are not used by input/output procedures. These entries will be seen to be very small indicating that the installation is being used at a highly efficient level.
THE TIDAL PROGRAMS WRITTEN FOR THE IBM 1620

The programs to which I refer are listed in tabular form in the attached diagram together with their requirements in storage capacity and computer time. This is a compatible family in that the output of one program can be used as the input of another and in fact there are facilities available which may not be immediately obvious from the diagram. The analysis program for example is converted into a device for the generation of input to the prediction program or for the generation of values of \( V \), \( u \) and \( f \) by simple operation of the console switches. The programs are designed, where possible, to have internal checking devices with appropriate error messages should these be required and moreover they do attempt to establish some degree of control over their operation by the generation of instructions for the benefit of the computer operator.

All are written in the basic language of the 1620, the Symbolic Programming System. Perhaps at this stage I might indicate very briefly the techniques involved under the various headings.

1/ Verification

Several references have been made in our discussions to the process of editing data. The technique adopted here is one of error detection rather than error correction and this takes the form of a simple smoothing check.

Since tidal observations are most smooth in a 25 hour sense the following condition is tested:

\[
T > \left| \frac{1}{8} \left( C_{t-1} - \frac{C_{t-2} + 4C_{t-1} + C_t}{5} - C_{t+1} \right) \right|
\]

Where \( T \) is a tolerance generally 2 or 3 \% of the tidal range and \( C_t \) is the tidal elevation at time \( t \) hours.

Due to atmospheric perturbations of sea level, which commonly have a duration of the order of a day, the above test alone is insufficient since it is inclined to reject many items of the tidal record which are in fact perfectly valid. Consequently if the above condition is not satisfied, a further test is applied before the particular item of data is recognised as an "error":

\[
T > \left| C_{t-1} - \frac{1}{8} \left( C_{t-2} + 4C_{t-1} + 4C_t + C_{t+1} \right) \right|
\]

If this second test were to be applied alone then again a large quantity of pseudo-error output would ensue because it has a tendency to fall near the turning points of the tide particularly where shallow-water distortions are present. In conjunction the two tests perform an efficient task and result in the verification of data within the time taken by the computer to read the cards involved.

2/ Analysis

On the 1620 I have programmed the traditional techniques of tidal analysis devised by DOODSON using linear combination processes. In this choice certain restrictions are implied and in particular it is necessary to have continuous data over a preset interval, which can be 358 days for one program or 32 days for a second. The philosophy of this choice, however, results from a recognition of the fact that DOODSON's techniques were developed for hand calculators so that the bulk of computation had necessarily to be kept to a minimum. If, therefore, this same technique can be programmed it must inevitably result in a process which is very economic in computer time. The disadvantage which must be overcome is that the technique does not lend itself easily to an automatic process so that there are considerable programming difficulties. The programs are cumbersome to write and in fact the 358 days analysis involves some 4000 instructions. Experience has shown that this selection was justified. The extraction of 60 harmonic constituents from a tidal record extending over one year can be accomplished in approximately 3 minutes and, as another contribution to this symposium will show, the quality of these results bear comparison with those from more sophisticated techniques.

3/ Prediction of hourly elevation

The bulk of calculation involved in the prediction of hourly elevations is such that a very careful consideration must be given to the techniques involved. As an illustration of this point, my first attempt to program this case on the 1620 involved most expensive use of computer time at the rate of 3.5 minutes per day's prediction. My most recent program will provide predictions for almost 6 months in the same time. The optimisation process depends upon the choice of a convenient unit of angular measure to fit the specific case.

The storing of a cosine table to cover the complete range, \( 2\pi \), at an interval close enough to dispense with interpolation, and at the same time allowing for the ultimate definition of tidal elevations correct to 1/10 foot is the key to the problem. Consideration shows that if the elements of the table are defined by 4 decimal digits and if they are spaced at angular intervals of \( \pi/1250 \), then this is just sufficient to give the accuracy required. This being so there are two distinct advantages in selecting \( \pi/5000 \) as the unit of angular measure:

a) Using such a unit it is not necessary to test whether a constituent phase has exceeded \( 2\pi \) each time it is incremented by \( \pi/5000 \) since such a condition would involve a "carry-over" to give an additional digit. This "carry-over" can be automatically prohibited by defining the length of the numerical field within the computer.

b) A 4 digit cosine table at intervals of \( \pi/1250 \) contains 2 \( \times \) 5000 digits in the range \( 2\pi \) so that there is a simple relationship between the constituent phase angle, expressed in units of \( \pi/5000 \) and the address of its cosine within the computer.

To be precise, the angular unit which I use is of the form

\[ 2 \pi \times \text{DOODSON} \]

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THE PREDICTION OF HIGH AND LOW WATER TIMES AND HEIGHTS

The routine production of tide table material in the form of high and low water data requires a more elaborate treatment than can be conveniently and efficiently handled on the IBM 1620, if a general program is desired. At the present time the Tidal Institute is required to prepare predictions for some 200 ports annually, and these represent a fair sample of most of the characteristic tidal forms which exist: semi-diurnal, diurnal and mixed-tides together with the extreme distortions encountered in shallow water. The problems encountered in a general program of this nature prohibit the use of many of the more rapidly converging devices for the sense of turning points so that M.T. MURRAY, who was responsible for this program, was forced to select an iterative process which in the first place seeks a straddle of a turning point by sensing for a change in the sign of the 1st derivative of the height curve, and then successively bisects this interval of time until the straddle is placed in time with an accuracy of 1 minute. The initial sensing interval is an input parameter which is normally 3 hours in the case of a semi-diurnal port and 1 hour in the case of a mixed-tide port. In this connection it should be noted that for several of the ports processed at the Tidal Institute the interval of time between successive turning points can range between 26 hours and 2 hours.

The program is written in machine language for an English Electric KDF9 computer and input and output is effected by punched tape. The general scheme of the program can be outlined by the following sequence of events:

a) The program reads a YEAR TAPE which contains the astronomical arguments (V, u & f) for the commencing day and the conclusion of the year to be predicted.

b) The program then reads a PORT TAPE which contains the amplitude and phase of each of 60 standard harmonic constituents. Where the amplitude of a constituent is zero this constituent is subsequently ignored.

c) The program then scans the PORT TAPE in order to ascertain whether any additional constituents are required for the particular port and reads appropriate amplitudes, phases and astronomical arguments if these are present. There is accommodation for 39 such constituents making 99 in all.

d) The program then proceeds to compute the basic predictions interpolating changes in u and f every 33 days.

e) The PORT TAPE is again scanned in order to ascertain whether the harmonic shallow-water treatment is required and if so proceeds to read the appropriate constants.

f) Depending upon e) the harmonic shallow water corrections to the times and heights of high and low water are computed and are applied to the basic predictions where upon the complete set of output data is stored within the computer.

g) These data are then rearranged in sequence so as to comply with tide table format and are then output on two punched tape units while the program returns to read a new PORT TAPE for the next case.

Predictions are normally run in sets of 20 and the computer time involved is approximately 2 minutes for a basic port and 2 1/2 minutes for a shallow water port.

A tape-operated typewriter of a special design is capable of accepting KDF9 output and of producing a typed version of the final predictions of photo-lithographic quality.

It is not possible to describe fully the facilities offered by this program in providing continuity checks from one year to the next for a particular port but I hope that I have described the system in such a way as to give a sufficiently detailed view of its main features.

The Institute is keenly interested in developing new processes which take more effective advantage of computer facilities. In particular a more rigorous and more flexible analysis program than has been described here is now in use and this will be the subject of a subsequent paper.

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<tr>
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<tbody>
<tr>
<td></td>
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<td>Program input</td>
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<tr>
<td>Metric to feet conversion program</td>
<td>2,570</td>
<td>2 secs</td>
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<tr>
<td></td>
<td>Data Verification</td>
<td>6,570</td>
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<td>Yearly Analysis I</td>
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</tr>
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<td>Generation of V, u &amp; f</td>
<td>31,676</td>
</tr>
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<td>20 day Analysis I</td>
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<td>1 min 30 secs</td>
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L'ANALYSE HARMONIQUE PAR LA MÉTHODE DES HEURES SPÉCIALES
À L'AIDE D'UN ORDINATEUR DE BUREAU

Louis ROUMEGOUX
Service Hydrographique de la Marine Française

Le Service Hydrographique dispose depuis 1963 d'une calculatrice électronique de bureau (1) qu'il utilise pour traiter, entre d'autres, les problèmes d'analyse harmonique de courbes de marée. Pour le moment, il continue à pratiquer la méthode des heures spéciales, largement diffusée dans le monde depuis les travaux de George HOWARD DARWIN.

Le principe de cette méthode est bien connu. À partir d'une certaine origine de temps, on effectue le relevé de la courbe de marée pour chaque heure spéciale, c'est-à-dire pour chaque \( \frac{34}{24} \) du jour spécial (ou de la période) de l'onde en cours de détermination, si \( \cos (\omega t - \beta) \) est l'expression analytique de cette onde, on recherche l'amplitude brute \( M \) et la phase à l'origine \( \beta \).

La contribution apportée par cette onde se retrouve en phase à la même heure spéciale, aux jours spéciaux successifs. Le relevé de ces hauteurs spéciales se poursuit pendant un nombre entier de jours spéciaux de l'onde, ce nombre étant choisi en fonction de la durée des observations.

On obtient enfin, par de simples moyennes, les ordonnées de la courbe particulière de l'onde, rapportée à l'instant origine et aux heures spéciales successives de 0 h à 23 h.

La méthode est simple dans son principe, mais d'une application relativement ardue, par suite des très nombreux relevés d'ordonnées, des totaux et des moyennes qu'elle requiert.

En principe, la recherche de chaque onde nécessitait un dépouillement particulier de la courbe de marée en hauteurs spéciales, mais on peut, heureusement, se limiter à l'effectuer sur la courbe, que des relevés de hauteurs horaires (temps moyen), à condition d'effectuer chacune de ces hauteurs horaires à l'heure spéciale de l'onde cherchée, la plus voisine de son heure temps moyen. Dans ces conditions d'ailleurs, il faut encore multiplier les ordonnées de la courbe de l'onde, décalant des moyennes, par un facteur d'augmentation légèrement supérieur à 1 dont l'expression est

\[
\frac{M}{2} \sin \frac{\omega t}{2}
\]

où \( \omega \) est la vitesse angulaire de l'onde recherchée.

Les différentes opérations qui viennent d'être résumées étaient délicates à exécuter, longues et fastidieuses malgré l'utilisation de formulaires, de grilles, et d'ingénieuses modalités de calcul.

Nous allons voir en revanche que le calcul électronique accepte avec beaucoup de souffrance cette méthode des hauteurs spéciales.

(1) Cab 500.
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(*) Cab 500, 149
Les hauteurs horaires successives sont relevées en cm, sur la courbe de marée et on les range dans une série de mémoires consécutives de la machine. Chaque hauteur peut être appelée par la suite, à l’aide de l’indication de l’heure temp moyen à laquelle elle corres- pond. Observons que pour traiter une année d’observations on est amené à faire appel de nombreuses fois à un peu moins de 9 000 hauteurs horaires. La durée de l’heure spéciale d’une onde déterminée (M2 par exemple) s’exprime, en heures temp moyen, par le nombre décimal 0,51754. Les heures spéciales successives de l’onde chérchée se forment en répétant par addition ce nombre décimal. Prenons pour exemple la 74ème heure spéciale de M2 ; elle correspond à l’instant temp moyen 380,213850, tandis que la 84ème correspond à l’instant 434,721000.

Dans le cas “74” la calculatrice appelle la hauteur temp moyen la plus voisine, qui se trouve dans la mémoire numérotée “390” (puisque la partie décimale de l’heure spéciale, 0,213850 est inférieure à 0,5).

Dans le cas “849” la calculatrice lera appel à la mémoire “435” (puisque la partie décimale 0,721000 est supérieure à 0,5).

On voit que la hauteur voulu est choisie automatiquement en comparant simplement à 0,5, la partie décimale de chaque heure spéciale (exprimée en temps moyen).

Il ne reste plus qu’à totaliser les hauteurs appelées pour chaque même heure spéciale, aux jours spéciaux successifs. Les 24 ordonnées de la courbe spéciale de l’onde cherchée, s’obtiennent en divisant ces totaux par le nombre des jours spéciaux. La détermination de l’amplitude brute (M) et de la phase à l’instant origine (p) s’obtiennent ensuite, en traitant ces 24 ordonnées par la méthode des moindres carrés.

Comme on le voit la classification méthode “des points doubles” (autre appellation, usitée en France, de la méthode des hautes spéciales) est particulièrement facile à rendre automatique.

Le calcul électronique permet également d’obtenir, par interpolation l’ordonnée de la courbe de marée correspondant exactement à chaque heure spéciale. Dans ce cas l’interpolation se fait au second ordre, à partir de l’ordonnée médiane d’une parabole, définie par trois hauteurs horaires consécutives de la courbe de marée. L’ordonnée médiane est toujours choisie à moins d’une demi-heure temp de l’heure spéciale.

On vérifie que, dès que le nombre de jours spéciaux dépasse quelques dizaines, il n’y a pratiquement pas de différence entre la résultat obtenu par interpolation ou par la méthode classique. Cette dernière méthode toutefois est sensiblement moins longue que l’autre, ce qui est à considérer dans le cas d’emploi d’un calculatrice de bureau.

La méthode d’analyse par les heures spéciales a le grand avantage de ne pas nécessiter le choix à priori de l’ensemble des ondes à rechercher. Ce choix n’est pas toujours facile à faire. Chaque onde est traitée dans cette méthode, indépendamment des autres et ce qui permet d’éviter l’analyse, à volonté, par étapes successives, en fonction des résultats déjà obtenus.

Cependant dans la recherche de chaque onde l’ordonnée de la courbe spéciale se trouve légèremment affectée du par la contribution apportée par quelques autres ondes, en particulier par celles dont l’amplitude est plus importante. Cette considération a conduit l’ingénieur en chef BIVINS à adopter pour l’analyse la méthode des constantes approchées qu’il avait imaginée dès 1958.

Dans un port donné, l’ordre de grandeur des constantes harmoniques des principales ondes est souvent connu. Dans le cas contraire, il suffit d’effectuer l’analyse sommaire d’une courte période d’observations et à partir des constantes harmoniques provisoires ainsi obtenues, de prédire les hauteurs horaires d’une courbe de marée approchée, couvrant toute l’étendue de la période des observations destinées à l’analyse. Enfin par différence on obtient une courbe de marée résiduelle (marée observée, marée prédite). Le calcul électronique se prête aisément à ces diverses opérations. C’est en définitive sur cette courbe résiduelle que va porter l’analyse précise des hautes spéciales. On observe que dans cette courbe toutes les ondes interviennent avec des amplitudes faibles et de valeurs comparables. C’est dire que, dans ce cas les ondes peuvent se traiter sans plus de difficultés indépendamment les unes des autres. Il ne reste plus qu’à recomposer les ondes qui avaient été choisies en valeur approchée, avec les ondes correspondantes de deuxième approximation, fournies par l’analyse précise.
Les hauteurs horaires successives sont relevées en cm, sur la courbe de marée et on les range dans une série de mémoires consécutives de la machine, Chaque hauteur peut être appelée par la suite, à l’aide de l’indication de l’heure temps moyen à laquelle elle correspond, Observons que pour traiter une année d’observations on est amené à faire appel de nombres très forts à un peu moins de 9 000 hauteurs horaires, La durée de l’heure spéciale d’une onde déterminée (M2 par exemple) s’exprime, en heures temps moyen, par le nombre décimal 0,51735. Les heures spéciales successives de l’onde cherchée se forment en répétant par addition ce nombre décimal, Prenons pour exemple la 784ème heure spéciale de M2 ; elle correspond à l’instant temps moyen 390,213560, tandis que la 840ème correspond à l’instant 454,721000.

Dans le cas "754" la calculatrice appelle la hauteur temps moyen la plus voisine, qui se trouve dans la mémoire numérotée "390" (puisque la partie décimale de l’heure spéciale, 0,213850 est inférieure à 0,5).

Dans le cas "849" la calculatrice fera appel à la mémoire "435" (puisque la partie décimale de l’heure spéciale 0,721000 est supérieure à 0,5).

On voit que la hauteur vouée est choisie automatiquement en comparant simplement à 0,5, la partie décimale de chaque heure spéciale (exprimée en temps moyen).

Il ne reste plus qu’à totaliser les hauteurs appelées pour chaque même heure spéciale, aux jours spéciaux successifs. Les 24 ordonnées de la courbe spéciale de l’onde cherchée s’obtiennent en divivant ces totaux par le nombre des jours spéciaux. La détermination de l’amplitude brute (M) et de la phase à l’instant origine (p) s’obtiennent ensuite, en traitant ces 24 ordonnées par la méthode des moindres carrés.

Comme on le voit la méthode "des points doubles" (autre appellation, usitée en France, de la méthode des heures spéciales) est particulièrement facile à rendre automatique.

Le calcul électronique permet également d’obtenir, par interpolation l’ordonnée de la courbe de marée correspondant exactement à chaque heure spéciale. Dans ce cas l’interpolation se fait au second ordre, à partir de l’ordonnée médiane d’une parabole, définie par trois hauteurs horaires consécutives de la courbe de marée. L’ordonnée médiane est toujours choisie à moins d’une demi-heure temps de l’heure spéciale.

On vérifie que, dès que le nombre de jours spéciaux dépasse quelques dizaines, il n’y a pratiquement pas de différence entre les résultats obtenus par interpolation ou par la méthode classique. Cette dernière méthode toutefois est sensiblement moins longue que l’autre, ce qui est à considérer dans le cas d’emploi d’une calculatrice de bureau.

La méthode d’analyse par les heures spéciales a le grand avantage de ne pas nécessiter le choix à priori de l’ensemble des ondes à rechercher. Ce choix n’est pas toujours facile à faire. Chaque onde est traité dans cette méthode, indépendamment des autres ce qui permet d’étendre l’analyse, à volonté, par étapes successives, en fonction des résultats déjà obtenus.

Cependant dans la recherche de chaque onde l’ordonnée de la courbe spéciale se trouvent légèrement affectées de la contribution apportée par les autres ondes, en particulier par celles dont l’amplitude est plus importante. Cette considération conduit l’Ingénieur en Chef HYRIX à adopter pour l’analyse la méthode des constantes approchées qu’il avait imaginée dès 1953.

Dans un port donné, l’ordre de grandeur des constantes harmoniques des principales ondes est souvent connu. Dans le cas contraire, il suffit d’effectuer l’analyse sommaire d’une courte période d’observations et à partir des constantes harmoniques provisoires ainsi obtenues, de prédire les hauteurs horaires d’une courbe de marée approchée, courant toute cette période des observations destinées à l’analyse. Ensuite par différences obtenir une courbe de marée résiduelle (marée observée, marée prédite). Le calcul électronique se prête aisément à ces diverses opérations. Il est en définitive sur cette courbe résiduelle que va porter l’analyse.

En considérant les hauteurs horaires des heures spéciales, on observe que dans cette courbe toutes les ondes intervien avec des amplitudes faibles et de valeurs comparables. C’est dire que, dans ce cas, les ondes peuvent se traiter sans mal doute indépendamment les unes des autres. Il ne reste plus qu’à recomposer les ondes qui avaient été choisies en valeur approchée, avec les ondes correspondantes de deuxième approximation, fournies par l’analyse précise.

Disons quelques mots sur la recherche des ondes à longue période par la méthode des heures spéciales. Cette recherche se fait sur une courbe de marée résiduelle, je veux dire, désarrasée de toutes les ondes d’amplitude importante : ce peut être, par exemple, celle qu’on utilise dans la méthode d’analyse par les constantes approchées.

La recherche d’une onde à longue période ne peut pas se faire directement sur cette courbe résiduelle, exactement comme celle d’une onde à courte période. En effet le nombre de jours spéciaux qu’on peut tirer d’un an d’observations de marée est très réduit, Il en est de même du nombre de hauteurs aux heures spéciales, à interroger dans la courbe de marée, en vue de la formation de la courbe moyenne.

Les erreurs d’observation ou les variations accidentelles du niveau moyen d’origine météorologique, viennent influer sur les résultats de l’analyse d’une façon d’autant plus sensible que l’amplitude des oscillations que l’on recherche est généralement assez faible. On en trouve confant, par suite, à substituer aux ordonnées horaires de la courbe à analyser, des valeurs moyennes de ces ordonnées prises sur des intervalles judicieusement choisis, par exemple :

- 12 h ou 1 demi-jour, pour les ondes semi-mensuelles et mensuelles
- 360 h ou 15 jours, pour les ondes semi-annuelles et annuelles.

Ainsi, toutes les hauteurs horaires de la courbe vont concourir à la détermination de chaque onde à longue période.

Il est facile de voir, dans le cas des ondes semi-mensuelles et mensuelles par exemple, que la valeur moyenne portant sur 12 hauteurs, de l’heure h à l’heure h + 11, n’est autre que la hauteur relative à l’heure h + 5,5 heures multipliée par un coefficient

\[ K = \frac{\sin \theta}{\sin \frac{\pi}{12}} \]

où \( \theta \) est la vitesse angulaire de l’onde cherchée. 

K est légèrement < 1.

En prenant, chaque jour, pour hauteur horaire à 0 h et à 12 h, les valeurs moyennes prises de 0 h à 11 h et 12 h à 23 h il y a donc lieu de tenir compte du coefficient K et du déphasage.

Sous cette réserve, on peut utiliser le programme général d’analyse d’une onde à courte période, à condition de prendre pour unité de temps, 12 h ou un demi-jour, ce qui donne une vitesse angulaire de :

\[ 12 \theta \text{ en degrés par demi-jour} \]

1 jour spécial de

30 \( \theta \) en demi-jours

une heure spéciale de

5 \( \theta \) en demi-jours

4 \( \theta \) en demi-jours

À la recherche de spéciaux sur lequel porte l’analyse demeurant inchangé,

La Section des Marées a mis au point un programme d’analyse entièrement automatique qui permet à la calculatrice de rechercher dans l’ordre :

- les ondes à courte période,
- les ondes semi-mensuelles et mensuelles,
- les ondes semi-annuelle et annuelle.

La machine reconnaît le type d’onde qui lui est proposé et elle effectue, suivant le cas, des moyennes de hauteurs horaires sur 12 heures ou sur 360 heures, avant d’entreprendre la recherche proprement dite des ondes de chaque groupe.
Calcul des facteurs $f$ et des phases $\psi + u$.

Pour parachever l’analyse, il reste encore à déterminer les constantes harmoniques usuelles, qui sont l’amplitude réduite $N = \frac{M}{T}$ la situation de l’onde $K$ en degrés rapportée au méridien local.

Ce nouveau calcul suppose la connaissance, pour chaque onde de deux fonctions :

- le facteur astronomique $f$ et la phase astronomique $\psi + u$.

On en recherche généralement les valeurs dans les tables de l’important ouvrage de Paul SCHUREMAN publié par l’United States Coast and Geodetic Survey. Bien qu’avec cet ouvrage ce travail final de réduction ne souffre pas de difficultés particulières, il demeure assez long et fastidieux, lorsqu’il faut l’étendre à une cinquantaine d’ondes, en sorte qu’il a paru plus sûr de faire feuilleter l’ouvrage de SCHUREMAN par la calculatrice électronique qui s’y prête bien volontiers.

On sait que les deux fonctions $f$ et $\psi + u$ se calculent à partir des cinq fonctions fondamentales $h$, $s$, $p$, $\eta$ et $N$ qui traduisent les mouvements moyens de la Lune et du Soleil. Ces fonctions fondamentales se calculent, à leur tour, par des formules linéaires, en fonction de la date, exprimée en siècles de 365,25 jours t.m., dans la chronologie du calendrier Julien.

La fonction $\psi + u$ de chaque onde, s’exprime par un polyème linéaire des fonctions fondamentales $h$, $s$, $p$ et $\eta$ de diverses sortes dépendant de $N$ et en quelques cas de $p$. Observons en passant que $\psi$ est calculé pour l’instant originel et que $u$ est relatif à l’instant milieu de la période d’observations soumises à l’analyse (ou de la période de prédiction). Quant à la fonction $f$ de chaque onde, son expression analytique est parfois un peu complexe à première vue, mais point compliquée à calculer électroniquement en partant de $N$ et éventuellement de $p$, pour l’instant milieu de la période.

En définitive, à condition de mettre en mémoire dans la calculatrice les coefficients des polyèmes linéaires caractéristiques de $\psi + u$ des différentes ondes, il ne reste plus, pour avoir les fonctions $f$ et $\psi + u$, qu’à prédire :

- la date et l’heure de début de la période
- la durée de cette période

et à appeler, par leurs numéros d’ordre, les différentes ondes pour lesquelles doit se faire le calcul.

On voit qu’une calculatrice de bureau d’études comme la Cab 500 peut mener à bien, sans difficultés, la mise des calculs nécessités par l’analyse harmonique d’une courte de marche s’étendant sur une durée de 1 an.

Le temps requis pour un semblable calcul peut être estimé à une trentaine d’heures de travail automatique. Ce temps est relativement important en raison du tout petit nombre de mémoires de calcul d’accès rapide dont on dispose.

Cependant il est bien commode d’avoir directement sous la main un robot, telle notre calculatrice, toujours prêt à accueillir de semblables problèmes et à les traiter de façon précise, dans un temps tellement meilleur que celui permis par les méthodes manuelles de jadis.

Ce temps était d’un bon mois, il ne faut pas l’oublier, et l’exécution d’une analyse harmonique, portant sur un an d’observations et une cinquantaine d’ondes était une performance austère.

**Observations sur la communication de M. Roumegoux**

M. ÉVRIELS, en ce qui concerne les "points doubles", je pense qu’il vaut mieux faire l’interpolation dans le cas de courtes durées d’observation. L’avantage de la recherche séparée des ondes, dans la méthode des heures spéciales, n’existe pas théoriquement mais seulement en pratique. La méthode des constantes approchées permet de contrôler dans la trajectoire des observations en éléments numériques. Le volume des données dans une analyse normale (I au) est grand, attention à ne pas choisir un ordinateur trop petit.

DE PATTULLO, il semble que la détermination de la vecteur différence pour chaque constituent ainsi que la méthode d’interpolation à l’aide de ses courbes soit utilisée. Bonne idée à proposer, certains signes de comparaison pourraient être donnés par l’expérience en verre, ce qui est fait par Physical Oceanographers. Chaque comparaison serait donc une base de comparaison et non de différences. Il y a certainement des résultats qui pourraient être obtenus de cette façon. Le point important est que l’on peut obtenir des résultats comparables de façon plus simple en utilisant des ordinateurs.
Calcul des facteurs $f$ et des phases $\varphi_i + u_i$.

Pour paraîtrice l'analyse, il reste encore à déterminer les constantes harmoniques usuélles, qui sont l'amplitude réduite $H + M$ à la situation de l'onde K en degrés rapportée au méridien local.

Ce nouveau calcul suppose la connaissance, pour chaque onde de deux fonctions :

- le facteur astronomique $f$ et la phase astronomique $\varphi_i + u_i$.

On en recherche généralement les valeurs dans les table de l'important ouvrage de Paul SCHUREMAN publié par l'United States Coast and Geodetic Survey. Bien qu'avec cet ouvrage ce travail final de réduction ne souffre pas de difficultés particulières, il demeure assez long et fastidieux, lorsqu'il faut l'étendre à une cinquantaine d'ondes, en sorte qu'il a paru plus sûr de faire feuilleter l'ouvrage de SCHUREMAN par la calculatrice électronique qui s'y prête bien volontiers.

On sait que les deux fonctions $f$ et $\varphi_i + u_i$ se calculent à partir des cinq fonctions fondamentales $h$, $s$, $p$, $\eta$ et $\rho$ qui traduisent les mouvements moyens de la Lune et du Soleil. Ces fonctions fondamentales se calculent, à leur tour, par des formules linéaires, en fonction de la date, exprimée en siècles de 365,25 jours t.m., dans la chronologie du calendrier Julien.

La fonction $\varphi_i + u_i$ de chaque onde, s'exprime par un polynôme linéaire des fonctions fondamentales $h$, $s$, $p$ et $\eta$ et de diverses autres dépendant de $n$ et en quelques cas de $p$. Observons en passant que $\varphi_i$ est calculé pour l'instant initial et que $u$ est relatif à l'instant milieu de la période d'observations soumises à l'analyse (ou de la période de prédiction). Quant à la fonction $f$ de chaque onde, son expression analytique est parfois un peu complexe à première vue, mais point compliquée à calculer électroniquement en partant de $H$ et éventuellement de $p$, pour l'instant milieu de la période.

En définitive, à condition de mettre en mémoire dans la calculatrice les coefficients des polynômes linéaires caractéristiques de $\varphi_i + u_i$ des différentes ondes, il ne reste plus, pour avoir les fonctions $f$ et $\varphi_i + u_i$, qu'à préciser :
- la date et l'heure de début de la période,
- la durée de cette période
et à appeler, par leurs numéros d'ordre, les différentes ondes pour lesquelles on désire le calcul.

On voit qu'une calculatrice de bureau d'études comme la Cub 500 peut mener à bien, sans difficulté, la masse des calculs nécessitée par l'analyse harmonique d'une courte de marche s'étendant sur une durée de 1 an.

Le temps requis pour un semblable calcul peut être estimé à une trentaine d'heures de travail automatique. Ce temps est relativement important en raison du tout petit nombre de mémoires de calcul d'accès rapide dont on dispose.

Cependant il est bien commode d'avoir directement sous la main un robot, telle notre calculatrice, toujours prêt à accueillir de semblables problèmes et à les traiter de façon précise, dans un temps tellement meilleur que celui permis par les méthodes manuelles de jadis.

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OBSERVATIONS SUR LA COMMUNICATION DE M. ROUMIGOUX

M. EVHIES. En ce qui concerne les "points doubles", je pense qu'il vaut mieux faire l'interpolation dans le cas de courbes d'ondes d'observation. L'avantage de la recherche séparée des ondes, dans la méthode des heures spéciales, n'existe pas théoriquement mais seulement en pratique. La méthode des constantes approchées permet le contrôle dans la traduction des observations en éléments numériques. Le volume des données dans une analyse normale (1 au) est grand, attention à ne pas choisir un ordinateur trop petit.

Dr. PATTULLO. Il semble que $2\pi$ dans les formules, on estime la détermination d'un vecteur différence pour chaque constante $h$ à la fois. Ce qui est une approximation, mais cela devient nécessaire lorsqu'on utilise une méthode de calcul harmonique. Cela permet de choisir des constantes de locomotion qui sont susceptibles de donner des résultats comparables. De plus, ceci est très intéressant pour les astronomes, qui peuvent utiliser la méthode pour obtenir des résultats comparables.

M. ROUMIGOUX. Il n'est effectivement pas raisonnable, dans le cas de l'Océanographie, de conserver l'analyse à partir d'une échelle de la planète, que l'on utilise, quand il est possible de l'ellipsoïde. En analysant une courbe différentielle, on ne recherchera pas que des termes périodiques d'amplitude faibles et de valeurs comparables. Il serait aussi intéressé de traduire les oscillations de fréquences voisines en séparées effectivement.

A. FRANCO. Je demande si, lorsqu'il y a une longue série d'observations à analyser et si on ne connaît pas les constantes pour un lieu voisin, on devra d'abord analyser une courte période, choisie dans la longue série pour pouvoir déterminer la manière différente à traiter par l'analyse finale.

M. ROUMIGOUX. Oui, certainement, il suffit de choisir une période d'observations de durée juste suffisante pour pouvoir extraire les constantes approchées des principales composantes (celles que l'on suppose à priori avoir les plus grandes amplitudes). Cette analyse préalable est d'ailleurs rapide.

M. HABICH. Concernant la comparaison du "$M$" et "$u"$, je pense qu'il est nécessaire de calculer $M$ et phase lag $\rho$ avec des coefficients de connaissance $h$ de la variation de la magnitude nodale variation. "$m$" et "$u$" peuvent être les constantes de la constante $h$ de la variation de la magnitude nodale. "$m$" et "$u$" doivent être calculés et acceptés de la même manière que les ondes qui sont définies ou qui doivent être prises en compte dans l'équilibre de la période "$t$" et "$u$" définis ainsi :

$$\int_0^T \left[ M \cos(\omega t - p - u) - M \cos(\omega t - p) - \tilde{f} \cos(t - u) \right] \, dt$$

Pour devenir un minimum, l'intervalle "$t$" doit être fixé sur le temps de calcul sur l'intervalle de calcul de l'intervalle de calcul de l'équilibre de la période "$t$" et "$u$", dans ce cas et appelé "$f$" et "$u$", elle-même. En principe, la même chose peut être utilisée pour distinguer dans l'analyse des ondes de plus de 15 jours ou de 20 jours, ce qui peut être réalisé par la méthode de décomposition des ondes à partir des données d'observations (15 jours ou 20 jours).
A METHOD OF TIDAL ANALYSIS BY USING EQUAL-INTERVAL TIDAL VALUES OF ABOUT ONE YEAR

Masomori MIYAZAKI
Japan Meteorological Agency

A few years ago (MIYAZAKI 1958), I proposed a method of tidal analysis which is convenient for the hand computation. The principle of this method is to make the 240-term harmonic analysis for the data of 355 days.

If the basic period of 355 days is taken, almost all tidal constituents are sufficiently approximated by its higher harmonics. And the 240 terms of 35.5 hourly interval are enough to separate about 60 constituents, which means that each of them is approximated by one of the different harmonics up to 120.

Of course, here we have a problem that a constituent does not always correspond to one of the original harmonics up to 120, but sometimes to one of the higher ones which can not be separated from the original ones. Therefore in this method, we are not based on the usual principle of the harmonic analysis in which these higher harmonics are regarded as noises. Some of you would perhaps point out the mathematical uncertainty of this method in this sense.

The author now intends to modify this method under the light of the present status that the high-speed electronic computers can be used for this purpose. This means to make use more terms than 240. If, for instance, the hourly data are used instead of 35.5 hourly ones, original harmonics will have the minimum period of two hours, and almost all constituents up to 1/12, diurnal ones will correspond to original harmonics. Provided that a sine table for about 2,000 values is prepared in this case, numbers of addition and multiplication will be about 17,000 and 4,000 per constituent, respectively. These are very easy tasks for a high-speed computer. I feel that the fixed-point operations would be more useful than the floating-point operations in many cases.

Now, figure 1 shows the phase speeds of tidal constituents and those of the corresponding harmonics. This table would show that each two are very close to each other. If we make use of the hourly data, the maximum resolutional harmonic is 4, 260, and all of the tabulated constituents are approximated by the original harmonics. Tabulated constituents are also approximated by the original harmonics if the data of every two hours are available.

Of course, the speed of each constituent is not exactly equal to that of the corresponding harmonics. Therefore, harmonic constants thus obtained should be slightly modified by the neighboring ones before converted to the corresponding constituent. Small phase and amplitude corrections are also necessary in order to obtain the final results. However, the computer time for these corrections are trivial against the main process of harmonic analysis.

REFERENCES

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Speeds of the constituents ($n_k$) and the corresponding harmonics ($n_h$) per mean solar hour.

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PRÉDICTION DE LA MARÉE AU SERVICE HYDROGRAPHIQUE FRANÇAIS

M. DARS
Ingénieur Hydrographe en Chef
Service Hydrographique de la Marine Française

La prédiction de la marée dans un certain nombre de ports du monde et la publication de ces prévisions constituent un aspect important des travaux du Bureau de calculs des Marées du Service Hydrographique de la Marine.

Le développement considérable des moyens de traitement de l'information et les possibilités offertes par le matériel mis au point par les constructeurs ont incité le Service Hydrographique à rendre aussi automatiques que possible l'élaboration et la présentation de l'annuaire des Marées.

Le problème se présente de manière différente selon que l'on considère le Tome I (Ports de France) et le Tome II (Ports d'Outre-mer) les formules utilisées pour la prédiction n'étant pas les mêmes. Nous les examinerons successivement.

I - PORTS DE FRANCE

1/ Calcul de la marée de Brest.

Toutes les prédictions du Tome I découlent de celles de la marée de Brest. La première partie du programme comprend donc le calcul de la marée de ce port.

La marée sur les côtes de France présente un caractère semi-diurne à peu près régulier. C'est pour cette raison que la formule de LAPLACE (1789) adaptée au calcul par l'Ingénieur Hydrographe CHAZALLON en 1839 représente de manière satisfaisante la marée à Brest et est toujours utilisée pour la prédiction.

Nous la rappellerons brièvement ci-dessous en indiquant les étapes successives du calcul.

Toute la méthode de prédiction repose sur le calcul de l'heure des pleines mers successives de Brest.

On admet que le terme diurne et le terme à longue période, en raison de leur faiblesse relativement au terme semi-diurne sont sans effet sur l'heure de la pleine mer. Il suffit donc de calculer cette dernière en annulant la dérivée par rapport au temps du terme semi-diurne.

On obtient ainsi l'équation

\[
\tan 2 \delta t = \frac{\sin 2 \omega}{\frac{27}{B} + \frac{3B'}{B} + \cos 2 \omega} = \tan 2p
\]

(1)

avec

\[\delta t \]: angle horaire de la lune
\[\omega - \omega': \delta t \]: différence des angles horaires du soleil et de la lune
\[B = 2,37734 \cos^2 \delta \]: coefficient lunaire
\[B' = 0,82154 \cos^2 \delta' \]: coefficient solaire

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L'équation (1) donne $a = p$ ; c'est une relation implicite puisque les éléments astronomiques doivent être pris eux-mêmes à l'heure de la pleine mer que l'on doit déduire ensuite de l'angle horaire.

Pratiquement on se contente de calculer les éléments astronomiques $i, l', d', d$ (d'où on déduit $B, B'$ et $u$) pour l'instant de passage de la lune au méridien de Greenwich. On calcule ensuite l'arc $p$ et l'heure de la pleine mer est donnée par la formule :

$$tm = Ho + 1,035p + 41 h 00 m, 4$$  

(2)

$tm$ : heure de la pleine mer de Brest en temps universel.

$Ho$ : heure de passage de la lune au méridien de Greenwich en temps universel.

$L'heure de la pleine mer est obtenue en faisant la moyenne des heures des plénums mers qui l'encadrent et en ajoutant 8 minutes au résultat, l'expérience ayant montré que la mer met à Brest en moyenne 16 minutes de plus à descendre qu'à monter.

La demi-amplitude de la marée est donnée par la formule

$$R = \sqrt{H^2 + B'^2 + 2 HB' \cos 2u}$$  

(3)

Compte tenu du décalage de l'heure de la PRM sur l'instant de passage de la lune au méridien, on est conduit à appliquer la valeur $R$ ainsi calculée à la base mer précédant immédiatement la pleine mer dont on vient de déterminer l'heure.

On obtient les demi-amplitudes des plénums mers en faisant la moyenne des demi-amplitudes des basses mers qui l'encadrent.

On ramène enfin ces hauteurs au niveau de référence de l'annuaire et des cartes, en leur ajoutant la cote du niveau moyen au-dessus de ce zéro, la marée diurne calculée par la formule harmonique en considérant les ondes $K_1, S_1, P_1, Q_1, Q_2$ et le terme à longue période.

En résumé, les données à introduire dans l'ordinateur sont les suivantes :

- Heures de passage de la lune au méridien de Greenwich pour toute l'année,
- Heures de passage du soleil au méridien de Greenwich pour toute l'année,
- Parallaxes de la lune et du soleil à 0 h TU et 12 h TU,
- Déclinaison de la lune pour l'heure ronde précédant le passage et variation horaire
- Déclinaison du soleil à 0 h TU.

Le programme comporte les séquences suivantes :

- Interpolation des éléments astronomiques pour les $H_0$ successifs
- Calcul des heures de plénums mers par la formule (2)
- Calcul des heures de basses mers
- Calcul des hauteurs de plénums mers
- Calcul des hauteurs de basses mers
- Calcul des coefficients de marée en faisant le quotient des demi-amplitudes des plénums mers semi-diurnes par l'unité de hauteur de Brest dont la valeur est 3,309 mètres.

2/ Calcul de la marée des autres ports,

Une fois obtenues les prédictions concernant Brest, il convient d'en déduire celles des seize autres ports de référence du littoral de la Manche et de l'Atlantique par emploi de la méthode dite des concordances.

Il ne s'agit plus d'un calcul à proprement parler mais de l'exploitation de quatre tableaux de corrélation, résultant d'observations systématiques prolongées effectuées simultanément à Brest et dans ces seize ports : deux de ces tableaux concernent les heures des plénums et des basses mers, deux autres les hauteurs des plénums et des basses mers.

L'exploitation de ces tableaux constitue une part importante du travail d'ensemble de l'ordinateur : c'est la deuxième partie du programme.

Dans le but d'éviter l'immobilisation d'un grand nombre de mémoires de la machine et de diminuer le temps de consultation de ces tableaux, ces derniers ont été transformés de la manière suivante : en remarquant que la différence entre un élément donné de la marée dans un port et l'élément correspondant à Brest ne varie que lentement dans toute l'étendue du tableau, on considère uniquement les variations de cette différence (et la valeur initiale de la différence) et on se garde en mémoire que les valeurs de l'élément de la marée à Brest, pour lesquelles une variation de cette différence se produit (à l'approche cherchée de la minute pour les heures et du demi-décimètre pour les hauteurs) et les valeurs des variations successives de cette différence.

3/ Tabulation des résultats,

Les calculs précédents sont effectués sur IBM 7094 et les résultats sortent sur bande magnétique. Celle-ci est réintroduite sur IBM 1401 qui réalise la dernière partie de l'élaboration automatisque du Tome I c'est-à-dire la tabulation finale des prédictions de marée sous leur forme définitive. Cette partie du programme ne comporte que des opérations logiques : raigissement des éléments prédits dans un ordre déterminé, adjonction d'un certain nombre de renseignements : jour de la semaine, quantième du mois, phase de la lune.

Les résultats sortent sous forme de tableaux à l'aide d'une imprimante à 600 lignes/mi- nute, avec une grande sécurité et une qualité de présentation très acceptable à condition de choisir spécialement le papier utilisé et de soigner l'encrage des caractères de l'imprimante.

Ceux tableaux, après superposition d'une grille sur pellicule transparente et réduction d'échelle, constituent, sans autre intermédiaire, le document envoyé à la photographie sur zinc en vue de l'impression de l'ouvrage.

Tout travail de recoup, avec les inévitables risques d'erreurs afférents, est ainsi évité et les opérations de vérification et de contrôle sont considérablement simplifiées. C'est là, avec le gain de temps, l'un des avantages les plus importants du calcul électronique dans ce domaine.

II - PORTS D'OUTRE-MER

Le Tome II de l'Annuaire des marées (Ports d'Outre-mer) concerne essentiellement les principaux ports des pays d'expression française. Un certain nombre de ces ports possèdent une marée de type mixte ou de forte inégalité diurne. La formule harmonique est particulièrement bien adaptée pour représenter la marée de ces ports et convient parfaitement pour le calcul électronique.

1/ Formule harmonique,

Cette expression donne la hauteur de la marée à un instant quelconque comme somme des effets d'un certain nombre de composantes $A_i$.

$$h = \sum \dot{A}_i f_i H_0 \cos \left[\omega_i t + (\varphi_i + \psi_i) - \beta_i\right]$$  

(1)

dans laquelle :

$A_i$ : hauteur du niveau moyen au-dessus du zéro de l'Annuaire
L'équation (1) donne \( M = p \); c'est une relation implicite puisque les éléments astronomiques doivent être pris eux-mêmes à l'heure de la pleine mer que l'on doit déduire ensuite de l'angle horaire.

Pratiquement on se contente de calculer les éléments astronomiques \( I, I', \delta, \delta' \) (d'où on déduit \( B, B' \) et \( \omega \)) pour l'instant de passage de la lune au méridien de Greenwich. On calcule ensuite l'arc \( p \) et l'heure de la pleine mer est donnée par la formule :

\[
\text{hm} = \text{Ho} + 1,035 \ p + 41 \ h \ 00 \ m, \ 4
\]

\[ \text{hm} : \text{heure de la pleine mer de Brest en temps universel} \]

\[ \text{Ho} : \text{heure de passage de la lune au méridien de Greenwich en temps universel} \]

L'heure de la pleine mer est obtenue en faisant la moyenne des heures des pleines mers qui l'encadrent et en ajoutant 8 minutes au résultat, l'expérience ayant montré que la mer monte à Brest en moyenne 16 minutes de plus à descendre qu'à monter.

La demi-amplitude de la marée est donnée par la formule

\[
R = \sqrt{B^2 + B'^2 + 2 BB' \cos 2\omega}
\]

Compte tenu du décalage de l'heure de la Ph M sur l'instant de passage de la lune au méridien, on est conduit à appliquer la valeur \( R \) ainsi calculée à la basse mer précédant immédiatement la pleine mer dont on vient de déterminer l'heure.

On obtient les demi-amplitudes des pleines mers en faisant la moyenne des demi-amplitudes des basses mers qui l'encadrent.

On ramène enfin ces hauteurs au niveau de référence de l'Annuaire et des cartes, en leur ajoutant la cote du niveau moyen au-dessus de ce zéro, la marée diurne calculée par la formule harmonique en considérant les ondes \( K, S, P, O, Q \) et le terme à longue période.

En résumé, les données à introduire dans l'ordinateur sont les suivantes :

- Heures de passage de la lune au méridien de Greenwich pour toute l'année,
- Heures de passage du soleil au méridien de Greenwich pour toute l'année,
- Parallaxes de la lune et du soleil à 0 h TU et 12 h TU
- Déclinaison de la lune pour l'heure rodon précédant le passage et variation horaire
- Déclinaison du soleil à 0 h TU

Le programme comporte les séquences suivantes :

- Interpolation des éléments astronomiques pour les \( H \) successifs
- Calcul des heures de pleines mers par la formule (2)
- Calcul des heures de basses mers
- Calcul des hautesurs de basses mers
- Calcul des hautesurs de pleines mers
- Calcul des coefficients de marée en faisant le quotient des demi-amplitudes des pleines mers semi-diurnes par l'unité de hauteur de Brest dont la valeur est 3,299 mètres.

2/ Calcul de la marée des autres ports,

Une fois obtenues les prédictions concernant Brest, il convient d'en déduire celles des seize autres ports de référence du littoral de la Manche et de l'Atlantique par emploi de la méthode dite des concordances.

Il ne s'agit plus d'un calcul à proprement parler mais de l'exploitation de quatre tableaux de corrélation, résultant d'observations systématiques prolongées effectuées simultanément à Brest et dans ces seize ports : deux de ces tableaux concernent les heures des pleines et des basses mers, deux autres les hauteurs des pleines et des basses mers.

L'exploitation de ces tableaux consiste une partie importante du travail d'ensemble de l'orédinateur : c'est la deuxième partie du programme.

Dans le but d'éviter l'immobilisation d'un grand nombre de mémoires de la machine et de diminuer le temps de consultation de ces tableaux, ces derniers ont été transformés de la manière suivante : en remarquant que la différence entre un élément donné de la marée dans un port et l'élément correspondant à Brest ne varie que lentement dans toute l'étendue du tableau, on considère uniquement les variations de cette différence (et la valeur initiale de la différence) et on se garde en mémoire que les valeurs de l'élément de la marée à Brest pour lesquelles une variation de cette différence se produit (à l'approximation cherchée de la minute pour la deuxième et du demi-décimètre pour les hauteurs) et les valeurs des variations successives de cette différence.

3/ Tabulation des résultats.

Les calculs précédents sont effectués sur IBM 7094 et les résultats sortis sur bande magnétique. Celle-ci est réintroduite sur IBM 1401 qui réalise la dernière partie de l'élaboration automatique du Tome I c'est-à-dire la tabulation finale des prédications de marée sous leur forme definitive. Cette partie du programme ne comporte que des opérations logiques : rangement des éléments prédits dans un ordre déterminé, adjonction d'un certain nombre de renseignements : jour de la semaine, quantième du mois, phase de la lune.

Les résultats sortent sous forme de tableaux à l'aide d'une imprimante à 600 lignes/min - une grande sécurité et une qualité de présentation très acceptable à condition de choisir spécialement le papier utilisé et de soigner l'encrage des caractères de l'imprimante.

Ces tableaux, après superposition d'une grille sur pellicule transparente et réduction d'échelle, constituent, sans autre intermédiaire, le document envoyé à la photogravure sur zinc en vue de l'impression de l'ouvrage.

Tout travail de recoupi, avec les inévitables risques d'erreurs afférents, est ainsi évité et les opérations de vérification et de contrôle sont considérablement simplifiées. C'est là, avec le gain de temps, l'un des avantages les plus importants du calcul électronique dans ce domaine.

II - PORTS D'OUTRE-MER

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1/ Formule harmonique.

Cette expression donne la hauteur de la marée à un instant quelconque comme somme des effets d'un certain nombre de composantes \( A_i \).

\[
h = A_0 + \sum I \ H_i \ \cos \left[ \phi_i + \left( \Phi_i + u \right) \right] - b_i
\]

dans laquelle :

\( A_0 \) : hauteur du niveau moyen au-dessus du zéro de l'Annuaire
Pour chaque argument \( x \), la table comporte 2 mémoires contenant l'une \( A_x \), l'autre \( B_x \) et \( C_x \) sont 86 mémoires en tout qui sont disposées de façon à être appelées par indice de saut sans perdre un tour de tambour soit 50 millisecondes à chaque fois.

La précision obtenue est de 2 unités de la sixième décimale ce qui est suffisant pour assurer la précision du centimètre sur chaque hauteur calculée.

3/ Anciennes méthodes de prédiction.

Autrefois la prédiction par la formule harmonique était effectuée à l'aide d'une machine analogique : le "Tide Predictor" de Lord Kelvin. On a commencé par déterminer directement les courbes tracées à l'aide de cet appareil puis on a utilisé des cadran à lecture directe.

L'opération se faisait alors en deux étapes:

a) Relevé des heures des pleines et des basses mers sur la "coubre dérivée" lorsque le style passe par le niveau moyen de cette courbe.

b) Lecture "au passage" des hauteurs des pleines et des basses mers sur la courbe de mardie proprement dite aux heures précédemment trouvées.

Une solution intermédiaire avant l'automatisation totale a été trouvée en 1961 de la façon suivante :

a) Relevé automatique des heures des pleines et des basses mers à l'aide d'un dispositif photographique actionné par le passage du style au niveau moyen de la courbe dérivée, ce dispositif (codeur à 1 000 points) fournit la valeur actuelle de l'angle de rotation de l'arbre principal du Tide Predictor en millièmes de tour, soit une précision de ± 1,4 minute de temps.

b) A l'aide du relevé précédent, perforé sur bande par un téléimprimante, calcul des hauteurs par la formule harmonique sur ordinateur.

4/ Procédé actuel de calcul.

La méthode précédente laisse subsister un des inconvénients majeurs du Tide Predictor qui est d'imposer à l'opérateur, le calcul de la somme des composantes de la marée à utiliser.

Aussi la mise en service d'un petit calculateur scientifique CAI 500, lent mais très souple et facile à programmer, à l'initié le Service Hydrographique a redonné la précision et la solution suivante a été finalement adoptée.

On calcule d'abord la suite des hauteurs horaires de la marée au centimètre près par la formule harmonique en utilisant pour chaque cosinus une interpolation parabolique dans une table de cosinus comme nous l'avons exposé plus haut.

Ces hauteurs horaires sont sorties sur ruban perforé. Elles ne sont pas publiées mais restent disponibles et peuvent être communiquées sur demande.

A partir de cette suite des hauteurs horaires, on obtient l'heure et la hauteur d'une pleine ou d'une basse mer en calculant les coordonnées du sommet de la parabole passant par les 3 hauteurs horaires les plus voisines de ce sommet.

Précisons ce calcul :

Soyons \( \gamma_1, \gamma_2, \gamma_3 \) trois hauteurs horaires consécutives correspondant aux heures \( t - 1, t, t + 1 \) (l'unité de temps est l'heure). Il y aura pleine mer ou basse mer s'il y a maximum ou minimum de la courbe de marée dans l'intervalle \( t - 0,5 ; t + 0,5 \).

La condition à remplir par les trois hauteurs sera donc :

\[ (\gamma_2 - \gamma_1)(\gamma_2 - \gamma_3) \geq 0 \]

ce qui peut s'écrire

\[ (\gamma_2 - \gamma_1)(\gamma_2 - \gamma_3) > 0 \]
Pour chaque argument $a$, la table comporte 2 mémoires contenant l'une $A_0$, l'autre $B_0$ et $C_0$ soient 86 mémoires en tout qui sont disposées de façon à être appelées par indice de saut sans perdre un tour de tambour soit 50 millisecondes à chaque fois.

La précision obtenue est de 2 unités de la sixième décimale ce qui est suffisant pour assurer la précision de centimètre sur chaque hauteur calculée.

3/ Anciennes méthodes de prédiction.

Autrefois la prédiction par la formule harmonique était effectuée à l'aide d'une machine analogique : le "Tide Predictor" de Lord Kelvin. On a commencé par déposer directement les courbes tracées à l'aide de cet appareil puis on a utilisé des cadrans à lecture directe.

L'opération se faisait alors en deux étapes.

a) Relévé des heures des pleines et des basses mers sur la "courbe dérivée" lorsque le style passe par le niveau moyen de cette courbe.

b) Lecture "au passage" des hauteurs des pleines et des basses mers sur la courbe de marée proprement dite aux heures précédemment trouvées.

Une solution intermédiaire avant l'automatisation totale a été trouvée en 1961 de la façon suivante :

a) Relévé automatique des heures des pleines et des basses mers à l'aide d'un dispositif photo-électrique actionné par le passage du style au niveau moyen de la courbe dérivée, ce dispositif (coûteur à 1 000 pointu) fournit la valeur actuelle de l'angle de rotation de l'arbre principal du Tide Predictor en millièmes de tour, soit une précision de ± 1,4 minute de temps.

b) A l'aide du relevé précédent, perforé sur bande par un télémultipriseur, calcul des hauteurs par la formule harmonique sur ordinateur.

4/ Procédé actuel de calcul.

La méthode précédente laisse subsister un des inconvénients majeurs du Tide Predictor qui est d'imposer à l'opérateur les composantes de la marée à utiliser.

Ainsi la mise en service d'un petit calculateur scientifique CABS 500, lent mais très souple et facile à programmer, a incité le Service Hydrographique à reprendre le problème et la solution suivante a été finalement adoptée.

On calcule d'abord la suite des hauteurs horaires de la marée au centimètre près par la formule harmonique en utilisant pour chaque composante une interpolation parabolique dans une table de cosinus comme nous l'avons exposé plus haut.

Ces hauteurs horaires sont sorties sur ruban perforé. Elles ne sont pas publiées mais restent disponibles et peuvent être communiquées sur demande.

A partir de cette suite des hauteurs horaires, on obtient l'heure et la hauteur d'une pleine ou d'une base en calculant les coordonnées du sommet de la parabole passant par ces 3 hauteurs horaires les plus voisines de ce sommet.

Précisons ce calcul :

Soient $Y_1$, $Y_2$, $Y_3$ trois hauteurs horaires consécutives correspondant aux heures $t-1$, $t$, $t+1$ (l'unité de temps est l'heure). Il y aura pleine mer ou base mer s'il y a maximum ou minimum de la courbe de marée dans l'intervalle $t - 0,5 ; t + 0,5$.

La condition à remplir par les trois hauteurs sera donc :

\[
(Y_2 - Y_1) (Y_3 - Y_2) > 0
\]

ce qui peut s'écrire

\[
\frac{Y_2 - Y_1}{Y_3 - Y_2} > 0
\]
Le programme distingue s'il s'agit d'une pleine mer ou d'une basse mer ; pour éviter de calculer deux fois la même marée : le programme ne détecte pas le cas où Yi = Yi.

Au voisinage de cet extrémité, on assimile la courbe de marée à une parabole déterminée par trois points et on obtient l'heure ti et la hauteur yi du sommet par les formules :

\[ t_i = 1 + 0,5 \frac{Y_i - Y_j}{Y_j + Y_i - 2 Y_k} \]
\[ \delta_i = \delta_j - \frac{1}{8} \frac{(Y_i - Y_j)^2}{Y_j + Y_i - 2 Y_k} \]  

(5)  
(6)

Le déroulement du programme est alors le suivant :

a) Calcul des constantes de prédicteurs :

\[ b_i = f_i H_i \]
\[ \beta_i = \sqrt{\delta_i + u_i^0} - h_i \]

b) Calcul des hauteurs horaires pour toute l'année avec sortie sur ruban perforé et rangement en mémoire pour la suite du calcul.

c) Calcul des heures et hauteurs des pleines et des basses mers par couple de deux mois avec arrondi à la minute et au demi-décamètre ou au demi-décamètre.

La machine explore systématiquement la suite des hauteurs horaires, chaque fois qu'elle rencontre 3 hauteurs satisfaisant à la condition (4) elle calcule les formules (5) et (6).

d) À la fin de chaque couple, sortie des résultats sur ruban perforé que l'on peut tabuler sous la forme définitive de présentation de l'Annuaire exactement comme pour le Tome 1.

Le sous-programme de tabulation est assez complexe : en effet, les éléments de la marée sont calculés chronologiquement ce qui correspond aux colonnes de l'Annuaire et l'impression des tableaux à envoi à la photogravure doit être faite par lignes. De plus il faut y adjoindre un certain nombre de renseignements tels que :

- quantième du mois
- jour de la semaine
- phases de la lune,

5/ Cas particuliers,

Un certain nombre d'anomalies peuvent se présenter au cours de l'exploration systématique de la suite des hauteurs horaires.

a) Au voisinage d'un extrémité de la courbe de marée on peut avoir deux hauteurs horaires (ou plus dans le cas de la tension du plein) consécutives égales.

Soit par exemple la suite : 281, 296, 296, 278.

La machine va calculer normalement l'heure et la hauteur de la pleine mer avec les 3 hauteurs : Yi = 281, Yi = 296, Yi = 296, Yi = 295.

Ensuite elle va tester la suite : Yi = 296, Yi = 296 et Yi = 278.

Elle va trouver Yi = Yi et l'inégalité (4) serait vérifiée également. Mais le programme est prévu pour calculer la marée dans le cas où Yi = Yi et passer au test concernant les 3 hauteurs suivantes,

b) Si le cas d'une marée demi-diurne à peu près régulière ne présente pas de difficultés particulières, le cas d'une marée mixte est plus complexe : lors du passage du régime diurne au régime demi-diurne, il peut se produire un "pli" dans la courbe de marée quand vont apparaître la pleine mer inférieure et la basse mer supérieure ce qui se traduira par l'approximation de trois hauteurs consécutives ou plus de même valeur à l'approximation du centième.

Exemple : soit la suite de hauteurs horaires

128, 130, 130, 130, 130, 130, 130 et 130.

La condition (4) va donner une pleine mer entre les 3 premières hauteurs et la calculer normalement puis elle ne va pas fonctionner pour la basse mer se trouvant entre les hauteurs 3 et 4 puisqu'elle ne retient pas le cas où Yi = Yi. On va donc se retrouver une nouvelle fois avec la pleine mer supérieure normale : donc avec deux pleines mers consécutives.

Une sécurité a été prévue dans le programme qui arrête la machine lorsqu'elle trouve deux pleines mers (ou deux basses mers) consécutives. Il ne reste plus qu'à retrouver en mémoire les hauteurs horaires égales grâce à des compteurs prévus à cet effet, à les modifier pour supprimer cette anomalie : par exemple où on remplace par les valeurs suivantes :

128, 130, 130, 129, 132, 132, 132, etc.

et on reprend le calcul normalement.

La réduction de cette anomalie peut être faite par une autre méthode quand on prévoit que le régime de la marée risque de provoquer la présence relativement fréquente de ces anomalies. Cette méthode consiste à calculer les hauteurs horaires avec une décimale supplémentaire, c'est-à-dire au millième : ce qui diminue considérablement la probabilité d'avoir plusieurs hauteurs horaires consécutives égales. Bien entendu le sous-programme d'anomalie est modifié en conséquence.

c) Un autre cas particulier peut se présenter, notamment dans le cas de portes à marées mixtes en morte-eau durant par exemple, il peut arriver que la condition (4), une fois rattachées les anomalies du paragraphe précédent, fournissent plus de 4 marées par jour dont certaines seront évidemment très rapprochées. Il faut alors satisfaire à deux conditions :

- avoir un maximum de 4 marées dans la journée maximum imposé par la présentation de l'Annuaire,
- respecter l'alternance pleine mer - basse mer.

La machine s'arrête automatiquement dès qu'elle trouve une cinquième marée dans la même journée et imprime :

- l'heure ti de la marée en cours de calcul
- les 4 heures des marées du jour
- l'heure de la dernière marée du jour précédent (on trouve zéro s'il y a moins de 4 marées le jour précédent).

Au vu de ces heures, trois solutions sont possibles :

- reporter cette cinquième marée au jour suivant lorsque ti est très voisin de 24 heures. Il suffit de remplacer ti par zéro et de faire progresser d'une unité le compteur de jours,
- reporter deux marées de la journée en cours quand on ne trouve deux très rapprochées. Il suffit de modifier les mémoires correspondantes en les décalant éventuellement,
- reporter la première marée de la journée en cours au jour précédent lorsque cette marée est très proche de 0 heure et qu'il y a moins de 4 marées le jour précédent. Il faut remplacer ti par zéro et décaler les 4 marées de la journée en cours,
Le programme distingue s'il s'agit d'une pleine mer ou d'une basse mer ; pour éviter de calculer deux fois la même marée : le programme ne détecte pas le cas où $Y_2 = Y_3$.

Au voisinage de cet extréme, on assimile la courbe de marée à une parabole déterminée par trois points et on obtient l'heure $t_1$ et la hauteur $h_1$ du sommet par les formules :

$$t_1 = 1 + 0,5 \frac{Y_1 - Y_3}{Y_2 + Y_3 - 2Y_2}$$

$$h_1 = Y_1 - \frac{1}{8} (Y_2 - Y_3)^2$$

Le déroulement du programme est alors le suivant :

a) Calcul des constantes de prédications :

$$H_i = h_i$$

$$p_i = (Y_0 + a_i) h_i - b_i$$

b) Calcul des hauteurs horaires pour toute l'année avec sortie sur ruban perforé et rangement en mémoire pour la suite du calcul,

c) Calcul des heures et hauteurs des pleines et des basses mers par couple de deux mois avec arrondi à la minute et au demi-décimètre ou au demi-décimètre.

La machine explore systématiquement la suite des hauteurs horaires, chaque fois qu'elle rencontre 3 hauteurs satisfaisant à la condition (4) elle calcule les formules (5) et (6),

d) À la fin de chaque couple, sortie des résultats sur ruban perforé que l'on peut tabuler sous la forme définitive de présentation de l'Annuaire exactement comme pour le Tome 1.

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La machine va calculer normalement l'heure et la hauteur de la pleine mer avec les 3 hauteurs : $Y_1 = 281$, $Y_2 = 296$, $Y_3 = 295$.

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Elle va trouver $Y_1 = Y_3$ et l'inégalité (4) serait vérifiée également. Mais le programme est prémé concernant de marée dans le cas où $Y_1 = Y_3$ et passer au test concernant les 3 hauteurs suivantes,

b) Si le cas d'une marée semi-durasse à peu près régulière ne présente pas de difficultés particulières, le cas d'une marée mixte est plus complexe : lors du passage du régime durasse au régime semi-durasse, il peut se produire un "pli" dans la courbe de marée quand vont apparaitre la pleine mer inférieure et la basse mer supérieure ce qui se traduira par l'apparition de trois hauteurs consécutives ou plus de même valeur à l'approximation du centième.

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$$128, 130, 130, 130, 132, 139, etc.$$  

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$$128, 130, 130, 130, 132, 139, etc.$$  

et on reprend le calcul normalisé.

La réduction de cette anomalie peut être faite par une autre méthode quand on prouve que le régime de la marée risque de provoquer la présence relativement fréquente de ces anomalies. Cette méthode consiste à calculer les hauteurs horaires avec une décimal supplémentaire, c'est-à-dire au millième : ce qui diminue considérablement la probabilité d'avoir plusieurs hauteurs horaires consécutives égales. Bien entendu le sous-programme d'anomalie est modifié en conséquence.

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- avoir un maximum de 4 marées dans la journée maximum imposé par la présentation de l'Annuaire
- respecter l'alternance pleine mer - basse mer.

La machine s'arrête automatiquement dès qu'elle trouve une cinquième marée dans la même journée et imprime :

- l'heure $t_1$ de la marée en cours de calcul
- les 4 heures des marées du jour
- l'heure de la dernière marée du jour précédent (on trouve zéro s'il y a moins de 4 marées le jour précédent).

Au vu de ces heures, trois solutions sont possibles :

- reporter cette cinquième marée au jour suivant lorsque $t_3$ est très voisin de 24 heures. Il suffit de remplacer $t_4$ par zéro et de faire progresser d'une unité le compteur de jours
- supprimer deux marées de la journée en cours quand on en trouve deux très rapprochées. Il suffit de modifier les mémoires correspondantes en les décalant éventuellement.
- reporter la première marée de la journée en cours au jour précédent lorsque cette marée est très proche de 0 heure et qu'il y a moins de 4 marées le jour précédent. Il faut remplacer $t_4$ par zéro et décaler les 4 marées de la journée en cours.
Dr. ROSENBURG. Where the hourly heights are not required it is often best to use the derived curve to determine the turning points on the tidal curve by iterative methods.

M. ROUMDEDOUX. L'intersection de la courbe dérivée avec la droite moyenne est mal définie quand l'heure de la Pleine Mer ou de la Basse Mer est elle-même mal définie. Les difficultés de conduite du calcul de contrôle subsistent dans la méthode de la courbe dérivée comme dans celle de la parabole tangente quand la forme de la courbe de marée est compliquée.

Dr. LEBON. I would like to make this small point in connection with M. ROUMDEDOUX's remarks. Where the intersection of the derivative with the zero line is poorly defined, this, in fact, means that the turning point in the elevation curve is also poorly defined so that for all practical purposes, this makes no difference.

M. EYRIES. Au Service Hydrographique Français, nous avons pendant un temps utilisé la courbe dérivée, mais il est aussi compliqué pour la machine de déterminer son ordonnée nulle que de déterminer les extrêmes sur la courbe elle-même; la machine le fait par approximations successives et c'est le genre de calcul qu'elle fait le plus lentement, il est probable que la méthode que vous employez est plus convenable pour votre machine; la notre est plus "digestible" par notre machine.

MECHANICAL AND ELECTRONIC DATA PROCESSING AS DEVELOPED BY THE TIDES, CURRENTS AND WATER LEVEL SECTION, CANADIAN HYDROGRAPHIC SERVICE

G. DOHLER

Hydrographic Service - Canada

INTRODUCTION

Automation may be divided into two parts:

a) complete automated production processes;

b) intensive rationalized processes for certain aspects in the production.

With the increasing use of data-handling machines and electronic computers, it has become necessary to replace traditional thinking and well-established procedures with new ideas to achieve automation. This does not mean that yesterday's approach is old today and will be obsolete tomorrow.

The most important factors in automation are:

1/ the acceleration of the production process;

2/ the accumulation of the data;

3/ the disposition of the human data-handling force carrying out non-routine functions such as regulation and coordination within the flow of the data production;

4/ the continuity between work phases;

5/ the checking and correction of the data which is the last and perhaps most complicated stage.

With respect to the acceleration of the production process it is important to have error-free transmission or flow of data, while for the disposition of the data-handling force greater responsibility, better judgement and research-mindedness are a prerequisite.

During the next few years there may be a danger that the development of new machines may outpace the training of staff to operate or to utilize them to the best advantage.

THE DATA-PROCESSING SYSTEM OF THE TIDES, CURRENTS AND WATER LEVEL SECTION OF THE CANADIAN HYDROGRAPHIC SERVICE

While data processing is considered as a series of planned actions to achieve certain results, a data-processing system is a combination of procedures and devices by which these results are obtained. Figure 1 illustrates the procedures presently adopted by this service for Tide and Water Level data.

Punched cards have been selected as the machine-readable medium, and computations as well as publications are prepared with I.B.M. data-processing equipment.

A large technical staff would be necessary to tabulate the hourly heights and high and low water times and heights from over one hundred gauge records if an analogue-to-digital device were not available.
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A large technical staff would be necessary to tabulate the hourly heights and high and low water times and heights from over one hundred gauge records if an analogue-to-digital device were not available.
In the figure, the flowchart represents the process flow of a system. The operations are connected with arrows indicating the direction of data flow. The diagram includes various nodes and branches, each representing a specific step or function in the process. The flowchart is used to illustrate the sequence of operations, decision points, and data or information flow within the system. The diagram is a visual tool for understanding complex processes and identifying inefficiencies or areas for improvement.
DETAILS OF THE DEVICE

Punched cards are prepared from the analogue recording with a semi-automatic scaling, tabulating and printing device (figure 2).

The device, which is loaded with a paper chart, is basically a recorder unit from a strip chart gauge. It is possible to operate this unit either in 10 and 20 millimetres steps, representing one hour of time on the chart, or continuously in forward or backward directions. The pointer is moved either by a cursor or by a hand wheel to follow the analogue line drawn on the chart by the gauge. The spindle is fitted with an inverse tracing head to allow readout of levels otherwise exceeding the width of the paper.

The encoder is coupled to the left end of the spindle of the scaling device. The shaft rotation of the spindle is converted into an electrically simulated digital form containing thousands, hundreds, tens and units. As a check on the encoder, a counter is geared to the drive shaft.

The data from the chart, now converted to a digital form, are fed to a Coleman data processor. This data processor (No. 1) is wired for serial and parallel output. The serial output is required for the IBM Keypunch (No. 2) and the parallel output for the digital indicator (No. 3). The data processor contains the relays associated with each digit of the encoder (digitizer) plus logic-control relays. Its function is to register and store the analogue value in its digital form.
A keypunch control unit (No. 4) accepts the digital data from the data processor and processes them for transmission to the IBM 027 (No. 2).

The IBM 027 (Card Proof Punch) combines card punching with the simultaneous preparation of an adding-machine tape (No. 5). At the end of 24 readings, a total is automatically printed on the tape and the accumulator cleared.

**DATA FLOW**

Figure 1A is a block diagram of the flow of the basic data and the programs or steps required to carry out certain computations and/or listings.

The data from the scaling device are checked by the computer and the results are tabulated and plotted (figure 3).

![Figure 3: Plotting](image)

Tabulations for intermediate publications are prepared from the hourly heights after the daily totals. Daily and monthly mean listings which include the maximum and minimum values recorded throughout each month, are prepared by the IBM 407 accounting machine. These listings come in blocks of up to twelve stations throughout the period of one year and are available on request (figure 5). At the end of the year, up to twelve months of data are listed station by station and used in the preparation of our annual water level book, which also contains high and low water observations, the frequency distribution of daily mean values in non-tidal waters, and information about each gauge station.

![Figure 4: Tabulation of hourly heights](image)
A keypunch control unit (No. 4) accepts the digital data from the data processor and processes them for transmission to the IBM 027 (No. 2).

The IBM 027 (Card Proof Punch) combines card punching with the simultaneous preparation of an adding-machine tape (No. 5). At the end of 24 readings, a total is automatically printed on the tape and the accumulator cleared.

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Figure 4: Tabulation of hourly heights.
Figure 5: Tabulation of mean levels.

Figure 7: Card system.

Figure 8: Card system.
Hourly gauge heights abstracted by means other than the aforementioned scaling device must be listed on form 501 (figure 6) for key punching. Afterwards the punch cards can be used for computations. High and low water values are computed from hourly readings at stations where the tide is semi-diurnal and there is no appreciable shallow-water effect. The accuracy is within plus or minus five minutes in time and one hundredth of a foot in height. At all the other stations this information is abstracted in a similar manner to the hourly heights.

TIDAL AND WATER LEVEL COMPUTATIONS

As mentioned previously, punch cards are used as the machine-sensible medium and several card formats are employed (figures 7, 8, 9).

Harmonic analyses of periods between 15 and 365 days are made using the hourly height cards No. 1 and 2.

High and low water listings, either from predictions or observations, are made using cards No. 90 and 91.

Daily mean frequency distributions are computed using card No. 5.

Lunntidal intervals are computed from the constituent cards and thereafter the tidal differences can be calculated.

The plotter, on line with the computer, is utilized to present data for specific purposes in a manner which formerly required draughting skill and many hours of work.

Tidal predictions are listed on preprinted forms by the "on-line" printer of the computer. Bold station identification and monthly headings are added by utilizing a plastic overlay which is re-used each year. The listings are then photographed directly for use in the printing of the tide tables. (figure 10).

A start has been made to punch data prior to 1962 for the benefit of scientists and engineers studying mean sea level, the elevation or subsidence of land masses, the design of mathematical models, and new forecasting and prediction techniques for tides and water levels.

SUMMARY

Prior to the introduction of electronic data-processing techniques, most of our analyses and predictions were done by the Liverpool Tidal Institute. It was not until very recently that we have taken over this task. No additional staff was required, but some re-assignment of duties became necessary.

We realize that not all the agencies using our water and tide level data have access to electronic computers. We therefore microfilm all our data at the end of each year. Prints may be obtained at a reasonable cost, and a special ordering form is available for that purpose. Microfilming, of course, also provides safe and convenient storage. A start has been made to transfer punch cards for the years 1962 and 1963 to magnetic tape.

We have introduced automation within the Tides, Currents and Water Levels sections, but its success depends largely on the foresight and imagination of the computer programmer and unit heads, the cooperation of the machine operators, and our ability to keep ahead of the ever-increasing demands for computer-acceptable data and changes in equipment.

OBSERVATIONS SUR LA COMMUNICATION DU Dr DOHLLER

Dr ROSSITER. Are the computer predictions subjected to any checking process once they are printed?

Dr DOHLLER. Yes, either by inspection of the results as predicted and by plotting in line with the computer.

M. FYHRS. Je felicite vivement le Dr DOHLLER et le Canada pour la mise en place d'un tel reseau de marégraphes et le son appareil à son exploitation.
Tidal predictions are listed on preprinted forms by the "on-line" printer of the computer. Bold station identification and monthly headings are added by utilizing a plastic overlay which is re-used each year. The listings are then photographed directly for use in the printing of the tide tables. (Figure 10).

A start has been made to punch data prior to 1962 for the benefit of scientists and engineers studying mean sea level, the elevation or subsidence of land masses, the design of mathematical models, and new forecasting and prediction techniques for tides and water levels.

SUMMARY

Prior to the introduction of electronic data-processing techniques, most of our analyses and predictions were done by the Liverpool Tidal Institute. It was not until very recently that we have taken over this task. No additional staff was required, but some re-assignment of duties became necessary.

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## EVALUATION TESTS OF TIDAL ANALYTICAL PROCESSES

B. D. ZETLER  
U.S. Coast and Geodetic Survey U.S.A.;  
and  
G. W. LENNON  
University of Liverpool, Tidal Institute and Observatory (England)

### ABSTRACT

Tests have been conducted on three analytical processes and the results have been examined by statistical and power spectral techniques. The various one-year series of hourly height processes comprised (a) two sets of real observations containing a major contribution from long period noise, (b) a set of real observations containing tidal or semi-diurnal noise, and (c) a set of synthetic (noise-free) data. For real data, the residuals (observed minus predicted) were evaluated for total energy (variance) and/or energy per frequency band. The stability of derived constants for one station was tested by prediction with constants from the analysis of the previous year's data. For synthetic data, the results from analysis were compared with the input constants.

### INTRODUCTION

We wish to stress very strongly that this paper constitutes only an interim report. There are many processes that have not yet been made and therefore our results are not necessarily final conclusions.

Why did we decide upon this line of investigation? There are many answers and we list the most important:

1. The computer has opened the door to new approaches to analysis and it is necessary to objectively evaluate these.

2. There have been analytical evaluations of tidal techniques made by others. By and large, these have been with synthetic noise-free data. The paper, "Super-resolution of Tides" by MUNK and HASSELMANN stresses the importance of the signal to noise ratio in solving for tidal lines close together in frequency. Therefore, we agree to select constants from data that have significant noise at different portions of the frequency range.

3. To a large extent, different analysis techniques have evolved in various countries and little effort has been expended in objective comparisons. Our paper is a modest start in this direction.

4. Finally, and possibly the most important, we were aware of Dr. MUNK'S revolutionary approach to tide predictions and decided it would be well to develop a yardstick by which his results could be compared.

Our first efforts were directed to obtaining the variance of the residual data when predictions were subtracted from the observations and then normalized in the sense that the algebraic sum was made equal to zero. It very quickly became obvious that a more refined procedure was needed as the small differences between total variances looked insignificant. It was decided to calculate the power spectra of the residuals and to compare residual energy at various frequencies.
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### Figure 10

176

177
In considering the degree of resolution for our power spectrum analysis, we were concerned primarily with separating the species, minimizing the computer effort, and having sufficient degrees of freedom to have reasonable confidence in the results. As much as the data floundered, the Nyquist frequency in 12 cycles per day, using 100 lags in the autocorrelations, we get a "Gelman" of .12 cph which essentially separates the species by about eight values, a satisfactory separation. The degrees of freedom in a Tukey analysis is roughly 2N/M, where N is the number of data points and M is the number of lags used in the autocorrelations. For a year of data, we have about 17,000/100, or 170 degrees of freedom, a rather conservative and satisfactory number.

We note that Drs. MUNK and CARTWRIGHT have used a different concept in calculating residual energy in particular narrow frequency bands by summing the energy for each harmonic within these bands. We have no quarrel with this technique provided that some tests are conducted at higher frequencies, possibly 4 to 10 cph, to insure that they are matching the data for those stations having significant energy in these frequency ranges. Thus, possibly the most important aspect of this paper is that it invites a discussion of the criteria for evaluating analysis procedures.

THE DATA

Figure 1 shows a statistical comparison of the tests completed so far, Atlantic City was chosen as a station having a high noise level primarily at low frequencies. It is directly on the open coast of the United States and the shallow water fetch is very short. Hence, the nonlinear interaction terms are very small. However the station is wide open to the effect of all storms on the North Atlantic. One expects therefore that most of the noise will be produced by storms having effective periods of greater than a day.

Residuals from analysis of tidal data (energy in feet^4)

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Figure 1

The first set of data shows the result for three different analysis methods using 1939 Atlantic City observations, The first of the methods described in C, G, S, Publication 98, was done for only 14 constituents. Except for stencill smearing, this method has not been modified for computer usage and requires a considerable number of man hours, particularly if many constituents are sought. The Coast and Geodetic Survey recently adopted the Harris-Pare-Cummings least squares program for the analysis of a year of data, This, as of now, includes only the 37 constituents traditionally sought by the Coast Survey but the program is readily modified to include additional constituents if they appear to be significant. The DOODSON analysis gave values for 59 constituents, the 60th being calculated and found insignificant.

The variance of the residual is calculated in the least squares analysis program and therefore a power spectrum analysis was not made of the 1939 residuals. Subsequent tests indicated a possible small deviation from true value for the variance of the residuals, and the .3196 shown may be slightly inaccurate.

In any case, the DOODSON analysis gave the best results and thus the question was raised as to whether this was due to the additional constituents determined by this method. This is discussed at a later point in this text.

The second comparison shows the results using 1939 harmonic constants to predict for 1940, thus testing the stability of the derived constants, 1940 appears to have been somewhat less noisy than 1939, this being based on observed energy and total residual energy. Again the DOODSON analysis gave better results, essentially the entire improvement coming at two cycles per day.

Similar results were obtained in analyzing Swansea, a station with less noise in the low frequencies and more in the tidal frequencies.

Tests were also done with synthetic (predicted) data for Port Gladstone, These tidal constants for both analysis methods matched the input data well, most differences being small, These remain to be predicted and subtracted from the synthetic set and the variances of the residuals obtained. Possibly we will also do power spectrum analysis of the two sets of residuals.

Figure 2 shows a plot of daily sea level at Atlantic City, Essentially sea level is bouncing up and down like a rubber ball, the extreme range being comparable to the mean tidal range, There does not appear to be a systematic pattern to the high frequency oscillations; only Sa and Sr are computed.

Figure 3 shows the spectral results for Atlantic City (1940) which are almost equal except immediately at 2 cph, The continuous line is from DOODSON residuals and the dotted line from least squares. Note that the vertical scale is logarithmic and that about 98% of the energy is concentrated in the low frequencies.

In figure 4 the results are somewhat different. To begin with, the residual energy for Swansea is not concentrated to the same extent in the low frequencies, The two curves vary comparably again at 2 cph, but there is a larger anomaly near the lunar sixth diurnal term. In this region of the spectrum, the additional compound terms appear to have given the DOODSON method a substantial advantage and this indicates to the Coast Survey the need for additional constituents for such a tidal regime.

The gap at 2 cph is more perplexing, DOODSON uses six more semi-diurnal constituents than the Coast Survey, However, when the energies of these six are added, using E x 1/2 s^4 where s is the amplitude, the observed differences between both Atlantic City and Swansea is significantly greater than the sum of the energies for these six constituents, This would appear to demonstrate the superior quality of the DOODSON method and makes one wonder about the accuracy of a least square procedure which is designed to minimize the residual variance, the parameter being used in these tests for our evaluations.

We reserve judgment however because the summation of energy is a tricky affair. For example, if we have a pure cosine curve with an amplitude of 10, its energy is 50, if we subtract a curve of the same speed and initial phase but with an amplitude of 9, we get a residual curve with amplitude of 1 and energy of 0.5, Thus, with a curve of energy 40.5, we
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FUTURE PLANS

As a minimum, we plan to do comparable analysis using the MURRAY least square program, to do again the demonstrated analyses expanding the Coast Survey program to include some additional constituents, and to calculate the residual energies for Port Gladstone.

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Dr. LENNON. I have just searched my papers for the names of 6 semi-diurnal in question: these are QG, MSN, OP, MKs, MSN, KS and so include some at least of the triple terms which Dr. CARTWRIGHT has in mind.

M. FRANCO. On voudrait bien maintenant que la méthode de DOODSON pour la durée d'une année d'observations donne des résultats comparables à la méthode des moindres carrés, En fait les différences, en faveur de la méthode de DOODSON peuvent provenir du plus grand nombre de composantes semi-diurnes et sixièmes diurnes qui y sont considérées. Mais il ne faut pas oublier que, sauf pour la séparation des espèces, c'est-à-dire pour la séparation des composantes de même indice, obtenue par la méthode journalière, la méthode de DOODSON est une méthode d'approximation de la méthode des moindres carrés, appliquée en succession. En fait, les multiplicateurs journaliers, comme les multiplicateurs mensuels, sont les indices les plus proches que l'on obtient en arrondissant le double des coefficients et des alphas, résultant de l'application directive de la méthode des moindres carrés. Dans ces conditions il ne nous semble pas donnant que les résultats soient semblables. Malgré la classification que nous a donné M. GODIN de la méthode de DOODSON, nous pouvons assurer que toutes les opérations que nous y trouvons sont bien basiques mais qu'il faut de temps pour les comprendre d'après les ouvrages originaux de son auteur. En réalité, la recherche des multiplicateurs pour le procédé journalier est très difficile et j'en suis l'auteur, mais si on utilise la méthode algébrique d'Aragoel, cette recherche n'est pas difficile.

L'EMPLOI DES FILTRES NUMÉRIQUES

DANS L'ÉTUDE DES FLUCTUATIONS LENTES DU NIVEAU DES EAUX

AU PORT DE MONTRÉAL

G. GODIN

Division des Recherches en Oceanographie

Département des Mines et Relevés Techniques Ottawa, Canada

Résumé: L'application d'un filtre numérique passe-bas aux données sur le niveau des eaux à Montréal démontre l'existence d'une marée de longue période à cet endroit. L'application de ce même filtre aux données des autres ports en aval permet de reconnaitre les ondes longues qui se propagent jusqu'à Montréal et d'en suivre le mouvement.

L'EMPLOI DES FILTRES NUMÉRIQUES DANS L'ÉTUDE DES FLUCTUATIONS LENTES DU NIVEAU DES EAUX AU PORT DE MONTRÉAL

M. JOBERT (Marées Terrestres, Bulletin d'Informations No, 37, septembre 1964) a porté à notre attention le concept d'un filtre numérique et la possibilité de son emploi dans l'analyse des marées.

D'autre parti dans la partie est du Canada, à la suite d'une série d'années de sécheresse, les niveaux des grands lacs ont baissé considérablement. Puisque leur débit d'écoulement contrôle en grande partie le niveau du fleuve St-Laurent, on a éprouvé au cours des dernières saisons de navigation de grandes difficultés à maintenir le niveau des eaux au port de Montréal à une hauteur tolérable. Il y a donc intérêt à étudier les facteurs qui contrôlent ces niveaux.

Il y a deux de ces facteurs qui ont retenu notre attention:

1/ la marée

2/ les tempêtes dans le golfe St-Laurent.

La forte marée semi-diurne se propage le long du fleuve jusqu'à Grondines et Dechassons; elle s'affaiblit rapidement en amont de ces endroits. Mais ce type de marée ne se rend pas jusqu'à Montréal, la marée de longue période qui est amplifiée entre Pointe au Père et Québec, se manifeste à Montréal par une variation lente des niveaux. Ceci d'ailleurs est en accord avec l'hydrodynamique puisqu'une onde de longue période sera beaucoup moins amortie qu'une onde de courte période le long d'un canal à friction constante.

À Montréal, une masse d'oscillations de courtes périodes se superpose sur cette marée de longue période et ces oscillations n'ont pas d'intérêt immédiat. Une analyse traditionnelle des données horaires aurait peu de chance de donner de bons résultats à cause de la faible amplification des oscillations à étudier et de l'ampleur du bruit superposé. De plus, au cours de l'hiver, des embâcles étouffent cette marée et les données durant ces intervalles ne sont pas utilisables.

L'application d'un filtre passe-bas aux données horaires sur le niveau des eaux à Montréal semble donc tout-à-fait naturel. Tout d'abord ce filtre nous débarasse d'à peu près tout ce qui n'est pas marée, d'une façon strictement mathématique. De plus, il nous révèle dans toute leur pureté ces ondes lentes qui nous intéressent. En appliquant ce filtre non seulement aux données de Montréal, mais aussi à celles des autres stations d'observation qui s'éche- lonnent le long du fleuve, on peut obtenir une idée du changement d'amplitude et de phase de ces ondes. L'attache graphique de l'ordinateur IBM 1620 rend ce genre de comparaison très facile.
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**Dr. CARTWRIGHT.** I quite agree with the principle of this sort of analysis. We also found at Newlyn that neglect of a few small constituents caused a surprising increase in residual energy. Those neglected were in the terms which we had ignored, but are included in DOODSON's list which we used for the harmonic analysis. The increased residual at 3 and 6 C.P.D., shown by M. ZETTLER suggest that the T.S.C.G. method also neglected triple constituents. Is this so?

**Dr. LEVY.** I have just been searching my papers for the names of 6 semi-diurnals in question; these are Qb, Mn, Q5, Mn5, Mn51, Ks and so include some at least of the triple terms which Dr. CARTWRIGHT has in mind.

**M. FRANK.** On suit très bien maintenant que la méthode de DOODSON a duré d'une année d'observations donne des résultats comparables à la méthode des moindres carrés. En fait les différences, en faveur de la méthode de DOODSON peuvent provenir du plus grand nombre de composantes semi-diurnes et sixièmes diurnes qui y sont considérées. Mais il ne faut pas oublier que, sauf pour la séparation des espèces, c'est-à-dire pour la séparation des composantes de même indice, obtenue par la méthode journalière, la méthode de DOODSON est une approximation de la méthode des moindres carrés, appliquée en succession. En fait, les multiplicateurs journaliers, comme les multiplicateurs mensuels sont les indices les plus proches que l'on obtient en arrondissant le double des coefficients et des axes, résultant de l'application directe de la méthode des moindres carrés. Dans ces conditions il ne nous semble pas donnant que les résultats soient semblables. Malgré la classification des dites données de M. GODIN de la méthode de DOODSON, nous pouvons assurer que toutes les opérations que nous y trouvons sont bien basiques mais qu'il faut de temps pour les comprendre d'après les ouvrages originaux de son auteur. En réalité, la recherche des multiplicateurs pour le procédé journalier est très difficile et J'oublie l'auteur, mais si on utilise la méthode algébrique d'Aragone, cette recherche n'est pas difficile.

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**L'EMPLOI DES FILTRES NUMÉRIQUES DANS L'ÉTUDE DES FLUCTUATIONS LENTES DU NIVEAU DES EAUX AU PORT DE MONTRÉAL**

**G. GODIN**

_Département des Minés et Recherches Techniques Ottawa, Canada_

Résumé: L'application d'un filtre numérique passe-bas aux données sur le niveau des eaux à Montréal démontre l'existence d'une marée de longue période à cet endroit. L'application de ce même filtre aux données des autres ports en aval permet de reconstituer les ondes longues qui se propagent jusqu'à Montréal et d'en suivre le mouvement.

**L'EMPLOI DES FILTRES NUMÉRIQUES DANS L'ÉTUDE DES FLUCTUATIONS LENTES DU NIVEAU DES EAUX AU PORT DE MONTRÉAL**

M. JOBERT (Marées Terrestres, Bulletin d'Informations No, 37, septembre 1964) a porté à notre attention le concept d'un filtre numérique et la possibilité de son emploi dans l'analyse des marées.

D'autre part dans la partie est du Canada, à la suite d'une série d'années de sécheresse, les niveaux des grands lacs ont baissé considérablement. Puisque leur débit d'écoulement contrôle en grande partie le niveau du fleuve St-Laurent, on a éprouvé au cours des dernières saisons de navigation de grandes difficultés à maintenir le niveau des eaux au port de Montréal à une hauteur tolérable. Il y a donc intérêt à étudier les facteurs qui contrôlent ces niveaux.

Il y a deux de ces facteurs qui ont retenu notre attention:

1/ la marée

2/ les tempêtes dans le golfe St-Laurent.

La forte marée semi-diurne se propage le long du fleuve jusqu'à Grandes et Dechassons; elle s'affaiblit rapidement en amont de ces endroits. Mais si ce type de marée ne se rend pas jusqu'à Montréal, la marée de longue période qui est amplifiée entre Jointe au Père et Québec, se manifeste à Montréal par une variation lente des niveaux. Ceci d'ailleurs est en accord avec l'hydrodynamique puisque l'une de longue période sera beaucoup moins amortie qu'une onde de courte période le long d'un canal à fréquence constante.

À Montréal, une masse d'oscillations de courte période se superposent sur cette marée de longue période et ces oscillations n'ont pas d'intérêt immédiat. Une analyse traditionnelle des données horaires aurait peu de chance de donner de bons résultats à cause de la faible amplitude des oscillations à étudier et de l'ampleur du bruit superposé. De plus, au cours de l'hiver, des embâcles étroits cette marée et les données durant ces intervalles ne sont pas utilisables.

L'application d'un filtre passe-bas aux données horaires sur le niveau des eaux à Montréal semble donc tout-à-fait naturel. Tout d'abord ce filtre nous débarrasse d'à peu près tout ce qui n'est pas marée, d'une façon strictement mathématique. De plus, il nous révèle dans toute leur pureté ces ondes lentes qui nous intéressent. En appliquant ce filtre non seulement aux données de Montréal, mais aussi à celles des autres stations d'observation qui s'échelonnent le long du fleuve, on peut obtenir une idée du changement d'amplitude et de phase de ces ondes. L'attachement graphique de l'ordinateur IBM 1620 rend ce genre de comparaisons très facile.
La figure 1 montre l'effet d'un filtre passe-bas sur une marée pure synthétisée à partir des huit ondes composantes principales. Le filtre passe-bas de JOBERT laisse passer une bonne partie de la marée diurne et même un peu de la marée semi-diurne. Le filtre passe-bas qui laisse passer une bande de fréquence deux fois moins large que celle admise par JOBERT coupe ces deux types de marées et ne laisse passer que le niveau moyen et Mf.

La figure 2 montre un exemple des données horaires pour Montréal, Québec, Trois-Rivières et Pointe au Père.

La figure 3 montre les mêmes données filtrées. On peut noter la progression de ce genre de fluctuation le long du fleuve et leur remarquable ampleur à Québec et Trois-Rivières.

Nous remarquons dans le graphique certaines fluctuations qui ne sont probablement pas des marées. Elles semblent dues en partie aux tempêtes qui occasionnellement sevissent sur le golfe.

Il n'y a donc pas de doute qu'un filtre numérique peut être de grande utilité dans le travail que nous avons entrepris.

Une fois filtrées, les données peuvent ensuite être analysées. Un stage préliminaire nécessaire est l'identification des ondes composantes. Une analyse du filtrat de Pointe au Père par périodogramme va être utilisée dans ce but. Par après on recherchera ces ondes dans les filtrats des autres stations. Le résidu de l'analyse devrait nous donner les fluctuations de tempête dont la corrélation avec les passages cycloniques pourra être étudiée.

Il y a fort peu d'espoir de modifier la marée de longue période par des travaux d'aménagement dans le fleuve, mais la connaissance de son amplitude et de sa phase pourront aider au contrôle rationnel du niveau des eaux à Montréal.

**OBSERVATIONS SUR LA COMMUNICATION DU Dr GODIN**

Mr. HYMES, Quelle est la nature du filtre numérique ?

Dr GODIN, C'est un filtre classique (fonction représentée par échelon- unité).

Dr ROBIDOUX. Can Dr GODIN comment on the origin of the large fortnightly tide at the head of the gulf of St-Lawrence ? If these are due to shallow water tides and fresh water flow, it would appear that the prediction of water levels in the Gulf could be effectively achieved by a method which has not yet been mentioned at this symposium. I infer to the solution of hydrodynamical equations using digital computers, a numerical model of the Gulf, with oceanic tidal input at the mouth and fresh water flow at the head.
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THE TIDAL INSTITUTE FLEXIBLE ANALYSIS PROGRAM

J. R. ROSSITER
Liverpool University - G.B.

1/ In designing the Tidal Institute Flexible analysis method, the following requirements were borne in mind:

i) To accept any span of hourly height data, as opposed to a fixed period such as a month or year. Introducing $f$ and $u$ as constants for blocks of data not exceeding two months permits the analysis of data ranging from a few days to any number of years, though in practice it is most convenient to use a maximum of one year.

ii) To accept incomplete data without the need for interpolation.

iii) To incorporate data checking and correcting processes in the main body of the program, thereby avoiding the need to read data into the computer twice.

iv) To produce results for any number and selection of constituents compatible with the span of data; for deep water ports and those with a small tidal range, less than thirty constituents are needed, but for shallow water ports and those with a large range more than sixty are needed.

The number and choice of constituents to be sought requires some experience in tidal analysis. In theory, i.e., using theoretical data, only $2n$ consecutive observations are needed to determine the amplitude and phase for $n$ constituents. In practice, however, as observed data are rarely accurate to 0.1 ft, or 2 cm, it is safest to assume that any pair of harmonic constituents cannot be adequately separated in less than their synodic period, i.e.,

$$\frac{2\pi}{\Delta t}$$ hours,

This condition, fundamental to tidal analysis, is of prime importance when using the method. It can only be relaxed by assuming some relationship between the amplitudes and phase lags of the constituents concerned, and then only if $\Delta t$ is small (< 3° per mean solar hour). Such a relationship may be deduced from a knowledge of constituents in the equilibrium tide, but a better estimate of the relationship may be deduced from a knowledge of the true constituents at a neighbouring station. Thus we have

v) To use information additional to the observed data. One example has been given. Another arises when $S_A$ and $S_B$ are well known regionally, often from mean sea level analysis. Inserting $H$ and $g$ for these constituents into the program obviates the need to analyse them, and improves the efficiency of the analysis proper.

2/ Requirements (i), (ii), (iv) and (v) very largely determine the method of analysis to be used. The problem is to find $H_n$, the harmonic constants for $N$ constituents, from the equations

$$\zeta_n = \sum_{n=0}^{N} x_n g_n$$ (1)
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$$2 \pi T < \frac{1}{5} \text{ hours}$$

This condition, fundamental to tidal analysis, is of prime importance when using the method. It can only be relaxed by assuming some relationship between the amplitudes and phase lags of the constituents concerned, and then only if $\phi$ is small (< 3° per mean solar hour). Such a relationship may be deduced from a knowledge of constituents in the equilibrium tide, but a better estimate of the relationship may be deduced from a knowledge of the true constituents at a neighbouring station. Thus we have

v) To use information additional to the observed data. One example has been given. Another arises when $S_a$ and $S_e$ are well known regionally, often from mean sea level analysis. Inserting $H$ and $g$ for these constituents into the program obtains the need to analyze for them, and improves the efficiency of the analysis proper.

2/ Requirements (i), (ii), (iv) and (v) very largely determine the method of analysis to be used. The problem is to find $H_a$, $q_a$, the harmonic constants for $N$ constituents, from the equations:

$$\zeta = \sum_{n=1}^{2n} x_n \sin nt$$

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where
\[ x_\theta = z_\theta \] (Mean sea level) \[ \theta = 1 \]
\[ x_{u1} = \frac{\xi_1}{12} \cos(\theta_1 + u_1 - \eta_1) \]
\[ x_{u2} = -\frac{\xi_2}{12} \sin(\theta_1 + u_1 - \eta_1) \]
\[ \eta_{p1} = \cos \phi_1 \]
\[ \eta_{p2} = \sin \phi_1 \]

This result from the application of the least squares principle. Seeking a least squares solution leads to the matrix equation

\[ Fx = Q \]

where
\[ F \theta = \Sigma \eta \phi \] and \[ Q \theta = \Sigma \frac{z_\theta}{12} \eta \phi \].

The required flexibility means that a once for all inversion of the matrix \( F \) is not possible, and some other type of solution is necessary. MURRAY, the author of the method, uses either the Gauss-Seidel iterative procedure or the pivotal condensation method. At one stage in the program a test is made on the matrix components to determine whether the Gauss-Seidel method converges; if \( a_{ii} \) is an element of \( F \), the test is that

\[ a_{ii} > \sum_j a_{ij} \]

If the span of data exceeds the synodic periods of all pairs of components sought, this is assured. If not, the pivotal condensation method, which always converges, is used.

Additional features of the program are that it ends (optionally) with the prediction of hourly heights for the period analyzed (using the harmonic constants produced), computes the differences between observed and predicted, and prepares a frequency distribution of the differences.

3/ The versatility of TIFAM is illustrated by the following cases which have been run on the English Electric KD9S computer.

1/ 13 hours of synthesized \( S_0 \) and \( S_2 \) to 5 significant figures.

Errors in \( H \) : 0.4% in \( g \) : 0.01.

2/ 24 hours of synthesized \( S_0 \), \( K_1 \), \( S_2 \), \( M_1 \), \( M_2 \) and \( M_4 \) to 5 significant figures.

Errors as for 1/.

3/ 30 days (720 hours) of synthesized \( T_8 \) and \( K_2 \) (0.04/hr, speed difference) to 3 significant figures, with \( T_8 \) relative to \( K_2 \).

Errors in \( H \) : 0.5% in \( g \) : 0.08.

4/ 2 \times 30 days, separated by 30 days, of 3/.

Errors as for 3/.

5/ Atlantic City tests.

The above test cases only check the computer program. For a general test on the efficiency of the method, the attached table gives a selection of harmonic constants derived from hourly height data at Atlantic City. Pairs of constituents within brackets have synodic periods of 365.2 days. Two points are worthy of note. The first is that, even by ignoring 7 constituents, good results (columns (b)) can be obtained from 6 months data. Secondly, by assuming a knowledge of the amplitudes and phase lags of \( S_1 \) relative to \( H_1 \), and \( T_2 \) relative to \( H_0 \), and by assuming \( S_0 \) and \( M_0 \) known, the nearby constituents are appreciably improved (column(c) compared with columns (b)).

### Comparison of flexible analysis results for 369 days and 181 days, Atlantic City, 1939.

<table>
<thead>
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<th></th>
<th>(a)</th>
<th>(b)</th>
<th>(c)</th>
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<td></td>
<td>( H )</td>
<td>( g )</td>
<td>( H )</td>
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<td>( M_0 )</td>
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<td>353.8</td>
<td>0.014</td>
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(a) 369 days commencing 1 January : 61 standard constituents.
(b) 181 days commencing 1 January : 61 standard constituents less \( S_0 \), \( M_0 \), \( S_0 \), \( q_1 \), \( T_2 \) and \( R_4 \).
(c) As (b), but with \( S_0 \) and \( M_0 \) given, \( S_0 \) relative to \( H_1 \), \( T_2 \) relative to \( H_2 \).
where

\[ x_n = Z_n \text{ (Mean sea level)} \]
\[ \theta_n = 1 \]
\[ \alpha_{11} = \cos \phi \]
\[ \alpha_{12} = \sin \phi \]
\[ \alpha_{21} = -\cos \phi \]
\[ \alpha_{22} = \sin \phi \]

This result from the application of the least squares principle. Seeking a least squares solution leads to the matrix equation

\[ F X = Q \]

where

\[ F_n = E \eta_1 \eta_1 \] and \[ Q_n = E \zeta_n \eta_1. \]

The required flexibility means that a once for all inversion of the matrix \( F \) is not possible, and some other type of solution is necessary. MURRAY, the author of the method, uses either the Gauss-Seidel iterative procedure or the pivotal condensation method. At one stage in the program a test is made on the matrix components to determine whether the Gauss-Seidel method converges; if \( \alpha_{ij} \) is an element of \( F \), the test is that

\[ \alpha_{ij} > \sum_{k=1}^{n} \alpha_{ik}. \]

If the span of data exceeds the synodic periods of all pairs of components sought, this is assured. If not, the pivotal condensation method, which always converges, is used.

Additional features of the program are that it ends (optionally) with the prediction of hourly heights for the period analyzed (using the harmonic constants produced), computes the differences between observed and predicted, and prepares a frequency distribution of the differences.

3/ The versatility of TIFAM is illustrated by the following cases which have been run on the English Electric KDF9 computer,

1/ 13 hours of synthesized \( S_2 \) and \( S_8 \) to 5 significant figures.

Errors in \( H \) : 0.04 %, in \( g \) : 0.01.

2/ 24 hours of synthesized \( S_2 \), \( K_1 \), \( S_7 \), \( M_2 \), \( MS_4 \) and \( M_4 \) to 5 significant figures.

Errors as for 1/.

3/ 30 days (720 hours) of synthesized \( T_k \) and \( K_2 \) (0.04/hr, speed difference) to 3 significant figures, with \( T_2 \) relative to \( K_2 \).

Errors in \( H \) : 0.5 %, in \( g \) : 0.08.

4/ 2 x 30 days, separated by 30 days, of 3/.

Errors as for 3/.

5/ Atlantic City tests.

The above test cases only check the computer program. For a general test on the efficiency of the method, the attached table gives a selection of harmonic constants derived from hourly height data at Atlantic City. Pairs of constituents within brackets have synodic periods of 365.2 days. Two points are worthy of note. The first is that, even by ignoring 7 constituents, good results (columns (b)) can be obtained from 6 months data. Secondly, by assuming a knowledge of the amplitudes and phase lags of \( S_1 \) relative to \( H_1 \), and \( T_2 \) relative to \( S_8 \), and by assuming \( S_6 \) and \( MF \) known, the nearby constituents are appreciably improved (column (c) compared with columns (b)).

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4/ A program of this type is extremely valuable for the analysis of experimental observations from, for example, off-shore tide gauges. If reliable harmonic constants are known from a permanent neighbouring, land-based gauge, with the exception of shallow-water constituents (most of which in any case have synodic periods of less than a day) many relationships between constituents adjacent in the spectrum can be used to give the maximum number of harmonic constituents from as little as three or four days' data.

5/ Disadvantages of the method so far encountered are functions solely of the computer configuration available to us. The pivotal condensation method of solving the matrix equation requires a large internal storage for efficient operation; our present computer does not possess sufficient storage, and the ALGOL translation unfortunately does not permit the economic use of the magnetic tape storage facilities. Until an additional module of core storage is provided, we are therefore not using the second method of solution, but confining our work to cases we know will be handled by the Gauss-Seidel process.

Moreover, the solution of a 122 x 122 matrix, when it uses magnetic tape stations, makes the program lengthy to run. This objection, we hope, will also be removed when the additional core storage is available. For smaller matrices, i.e. when seeking fewer constituents, the running time is very reasonable. For example, six constituents from twenty-four hours of data required only six seconds computing time.

The third difficulty we have experienced arises from the need to use paper tape input for this particular machine. Here again, the intention to provide card input facilities will shortly produce more favourable operating conditions.

Comparison of flexible analysis results for 369 days and 181 days, Atlantic City, 1939.

(a) 181 days commencing 1 January : 61 standard constituents.

(b) 181 days commencing 1 January : 61 standard constituents less \( S_6 \), \( MS \), \( T_2 \), \( T_4 \) and \( R_4 \).

(c) As (b), but with \( S_8 \) and \( M_4 \) given, \( S_8 \) relative to \( P_1 \), \( T_2 \) relative to \( S_2 \).

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OBSERVATIONS SUR LA COMMUNICATION DU Dr ROSSITER

M. FRANCO. Je demande à M. ZETTLER de nous préciser si la méthode développée aux États-Unis a la même souplesse que celle qui vient de décrire le Dr ROSSITER.

Dr ZETTLER. Non, le programme américain n'a pas autant de souplesse.

M. EYNES. La méthode des constantes approchées permet automatiquement de combler les lacunes dans les observations ; la "mêlée différence" telle qu'elle a été définie par M. ROUSSOUL est alors évidemment nulle.

Dr ROSSITER. C'est un autre moyen de résoudre ce problème.

Dr HAHNICH. The harmonic analysis of the tides according to the method of least squares is always in principal the same task. For this reason we have only one program which covers all our different kinds of analysis. Chosen a set of constituents \( \eta, \ldots, \gamma \) (as long as we only deal with tides which are not much disturbed by shallow water effects we use the rules pointed out by Dr ROSSITER ; for shallow water there given some advice by Dr HAUSCHELBAECH, but experience is still needed) ; given \( N + 1 \) observations at equidistant time steps ; calculate the coefficients of the matrix. When there are no gaps, the coefficients are calculated by formulas, as soon as there are gaps, they are calculated by summing.

The normal equations are solved by iteration. If the iteration does not converge fast enough, there is a break in the program and we use others methods. As long as the constituents are chosen according to the rules given by Dr ROSSITER there is no harm to solve the normal equations by iteration. We used this program to analyse 15 days observations taken at 5 minutes, to analyse 369 days hourly observations, and to analyse 19 years observations of high and low water ; in the last case the \( \gamma \) are the increase of argument of the constituents in 5 minutes, in the second case at 1 Solar hour, and in the last case at one linear day. A test of the method and a modification was the analysis of 15 days of tidal streams. The stream itself was fairly weak ; the amplitude of largest tide (of the M\(_1\) ) was less than 12 cm/sec ; the observations were disturbed by strong west and east wind. The velocity went up to more than 30 cm/sec. In order to "clean" the harmonic coefficients from the meteorological effects or whatever it was, we introduced polynomials and observed the decreased of the mean deviation ; with polynomials of about degree 12, the mean deviation was minimised ; polynomials of high degree may interact with the tides.

M. FRANCO. Je demande au Dr ROSSITER si le fait d'abandonner des observations dans la méthode d'analyse ne détruit pas la symétrie et par conséquent n'entraîne pas la possibilité de former deux systèmes indépendants (un pour \( H \cos \mu \), l'autre pour \( H \sin \mu \).

Dr ROSSITER. MURRAY'S method does not use a central time origin, and there is therefore no need to dispense with observations to create symmetry in the data.

Dr ZETTLER. The least square analysis program of the Coast and Geodetic Survey orders the constituents by magnitude on the theory that this helps the convergence of the solution. I would appreciate comments from others as the uniqueness of this aspect.

Dr GODIN. There is no advantage to the solution in such an ordering procedure.

Dr HAHNICH. Taking constituents into account which disturb one another in the matrix of the normal equations in a small square large values will be near the diagonal, perhaps of the same order as the members in the diagonal. By linear transformation of these disturbing constituents the matrix can be improved so that the convergence of iteration will be accelerated.

Dr ROSSITER. The analysis I have described does not consciously make any attempt to solve in descending order of constituent amplitudes. The order of solution effectively depends upon the method of solving the matrix equation ; some methods require operations to be performed on the largest terms in a given column, to ensure convergence in the solution, a situation somewhat similar to that described by M. ZETTLER.

OBSERVATIONS SUR LA COMMUNICATION DU Dr ROSSERT

À FRANCO. Je demande à M. ZETTLER de nous préciser si la méthode développée aux États-Unis a la même souplesse que celle que vient de décrire le Dr ROSSERT.

Dr ZETTLER. Non, le programme américain n'a pas autant de souplesse.

M. EVREUX. La méthode des constantes approchées permet automatiquement de combler les lacunes dans les observations ; la "mâture différence" telle qu'elle a été définie par M. BOUMÉGOU est alors évidemment niée.

Dr ROSSERT. C'est un autre moyen de résoudre ce problème.

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TIDAL SPECTROSCOPY AND PREDICTION

Walter MUNK
Scripps Institution of Oceanography
and
D. E. CARTWRIGHT
National Institute of Oceanography

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a) Gravitational potential
b) Radiational function
c) Expansion in Greenwich coordinates
d) Solar orbital constants
e) Lunar orbital constants

APPENDIX B : SAMPLING DISTRIBUTIONS AND CONFIDENCE LIMITS

Nineteen years of hourly tide-readings at Honolulu, Hawaii, and Newlyn, England, are analyzed without astronomical prejudice as to what frequencies are present, and what are not, thus allowing for background noise. The method consists of generating various input functions \( \zeta(t) \) for the same time interval as the recorded tide \( \zeta(t) \), and of determining the associated lag weights \( w \) in the convolutions

\[
\zeta(t) = \sum_{i} \sum_{j} w_{ij} c_{ij} (t - t_{ij}) + \sum_{i} \sum_{j} w_{ij} c_{ij} (t - t_{ij}) c_{ij} (t - t_{ij}) + \ldots
\]

by the condition \( \zeta(t) - \zeta(t)^2 \) minimum. The two expansions represent linear and bilinear processes; the Fourier transforms of \( w \) for any chosen \( i \) (or \( j \)) are the linear (or bilinear) admittances.

Input functions are the (time variable) spherical harmonics of the gravitational potential and of radiant flux on the EARTH'S surface; these functions are numerically generated hour-by-hour, directly from KEPLER'S laws and the known orbital constants of Moon and Sun, without time-harmonic expansions (unlike the harmonic method of KELVIN-DARWIN-DOODSON). The radiative input is required to predict non-gravitational tides, and it allows for the essential distinction that the Earth is opaque to radiation and transparent to gravitation.

The input functions are confined to bands, centered at 0, 1, 2, ..., cycles-per-day, which occupy roughly one-fourth the frequency space (less for radiational inputs) at the -60 db level. Within these bands the admittances turn out (with one exception) to be reasonably smooth, as expected. Subsequently we force the admittance to be smooth by truncating the expansion in \( s \), subject to this "credo of smoothness" the overlapping gravitational, radiational and non-linear admittances can be disentangled. The procedure is thus a result of computing the lag weights by inverting a correlation matrix of input functions, and the admittance by subsequent Fourier inversion, allowing for the varying uncertainties in the tidal components. The residual record, \( \zeta(t) - \zeta(t) \), is associated with the irregular oscillations induced by winds and atmospheric pressure. The residual spectrum smoothly fills the space between the bands centered on 0, 1, 2 cycles per day and rises sharply toward zero frequency, reflecting a similar pattern in the meteorological spectra. The residual spectrum rises into cusp-like peaks about each of the strong spectral lines, as might be expected from nonlinear interaction between a low-frequency modulation of "tidal carrier frequencies." Detailed analyses fail to confirm this hypothesis. Another unexplained feature is a slight 2 cycles-per-year "jitter" in the admittances, as if declinational splitting was not accurately taken into account in the gravitational theory.

The foregoing equation serves as a basis for a tide prediction which differs from the harmonic method now in use. The convolution formalism explicitly separates astronomical inputs from oceanographic responses, with Newtonian-Keplerian mechanics fully taken into account (in the harmonic method, N-K mechanics serves only to identify principal tidal frequencies). Moreover, the convolution method leads to a systematic expansion for weak nonlinearities. At Honolulu the convolution method gives better prediction with fewer station constants. Using linear and bilinear terms the two methods do equally well at Newlyn; trilinear convolutions should give significant improvement. But in all events the expected reduction in variance will be small compared to the low frequency residuals. To reduce these we have tried a Wiener-type self prediction with past values of the recorded tide as input function. The residual variance can be reduced by 50 % for a prediction time of 40 days at Honolulu, but only 4 days at Newlyn where the effect of local weather (storm tides) is severe. Hence the convolution method should be generalized to include as additional input functions some pertinent meteorological variables as well as tide records at other stations.

I - INTRODUCTION

Since the days of Darwin, tide records have been analyzed for the amplitude and phase of those particular periods which had been previously observed in the orbital motion of the Moon and Sun. The assumption was implicit that low frequencies were not of importance, although some workers had speculated that they were not. The desire for increased precision if only a sufficient number of tidal periods were included. In the language of "stationary time series" this is equivalent to the assertion that the tidal line spectrum exists all by itself, rather than being superimposed on a "noisy" spectrum. But noise-free processes do not occur (except in the literature on tidal phenomena).

In tide records the continuum is associated largely with irregular oscillations due to wind and pressure. By 1960 the continuum had been successfully measured from the high frequencies associated with sea and swell down to frequencies as low as 8 cycles per day (cph), only two octaves removed from the semi-diurnal tides (2 cph). The spectrum was found to rise with increasing frequency; extrapolation into tidal frequencies suggested that if one were to analyze tides by line without preconceived ideas as to what periods were present and what periods were not, then one would obtain just about as much mean-square amplitude at nonlinear periods as had been found for some of the weaker spectral lines. This suggests that there has now been confirmed. It means that the weaker lines are hopelessly contaminated by noise, and this accounts for some of the inconsistencies from one year to the next.

In addition to the underlying noise spectrum, two other difficulties arise in the analysis of tide records: (i) some daily and seasonal components of the tides are not the result of gravitational processes, and (ii) nonlinear processes in shallower water lead to interaction between tidal components. Spectral lines associated with (i) and (ii) overlap somewhat with the gravitational tide spectrum, and it is awkward to disentangle the processes.
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II - CONCERNING TIDE PREDICTION

a) Nonharmonic method

The connection between Moon and tides is so obvious that long before the formulation of any theory quite satisfactory rule-of-thumb predictions of tide were made and published. Tide tables constructed by undivulgued methods were considered as painful family possessions and passed on from father to son, The Liverpool Tide Tables published by a clergyman named Holden carried this to its highest perfection.

Starting 1831, Sir John LUBBOCK initiated what has been called the nonharmonic method of tide prediction. Tides at a given port are represented by certain tidal elements: time and height of high water, low water. These are related to astronomical observables: the age of the Moon (reckoned from new Moon), the declination, and the parallax of Moon and Sun. The prediction is made by successive approximations. For example:

- Height of high water = mean height above datum + correction for age + correction for declination + correction for parallax + diurnal inequality.

The corrections were derived by successive regressions between observed tidal and orbital elements using long series of data. Tables produced in this manner were quite successful for ports with predominantly semi-diurnal tides (as in England). According to Whewell (1837) some mistakes in these as first published (mistakes unimportant as to the theoretical value of the work) served to show the jealousy of the practical tide table calculators, by the acrimony with which the overights were dwelt upon; but in a very few years, the tables thus produced by an open and scientific process were more exact than those which resulted from any of the secrets; and thus practice was brought into its proper subordination to theory.

b) Harmonic method

The harmonic method of tide analysis was developed by Lord KELVIN and Sir George DARWIN starting 1867, following an earlier suggestion by LAPLACE along similar lines. The tidal height at a given port is predicted according to

$$\hat{H}(t) = \sum_i C_i \cos(2\pi t + \phi_i)$$  \hspace{1cm} (2.1)

summed over a set of denumerable frequencies $\hat{f}_i$. The coefficients $C_i, \phi_i$ are found by harmonic analysis of the tide record $H(t)$ for frequencies $\hat{f}_i$ specified in advance (to eight sigla-

 $$\hat{f}_i = f_i + n_1 f_1 + n_2 f_2 + \ldots + n_k f_k$$

where

$$f_i = 1 \text{ day is the period of the Earth's rotation (relative to Sun)}$$

$$f_i = 1 \text{ month is the period of the Moon's orbital motion}$$

$$f_i = 1 \text{ year is the period of the Sun's orbital motion}$$

$$f_i \approx 8.85 \text{ years is the period of lunar perigee}$$

$$f_i \approx 18.61 \text{ years is the period of regression of lunar nodes}$$

$$f_i \approx 20,900 \text{ years is the period of solar perigee}$$

We can ignore lower frequencies arising from planetary perturbations. The set of numbers

$$\{a_k, a_1, a_2, a_3, a_4, a_5\}$$

completely defines the frequency $\hat{f}_i$. Because of the predominant effect of the Moon, there is some convenience in referring to a lunar day of $1/f_1 = 0.035$ solar days, the relation being $f_1 = f_1^* - f_2^* - f_3^*$. The resulting set

$$\{a_0, a_1, a_2, a_3, a_4, a_5\} = \{a_0, a_1 + a_2, a_3, a_4, a_5\}$$

(2.3)

will be called the "DOODSON number"$^3$. $a_0 = 0$, $1, 2$ refers to frequencies near 0, 1, 2 cycles per lunar day) and are called the long period, diurnal and semi-diurnal species, respectively. $a_4, a_5$ is called the group-number, and $(a_0, a_1, a_2)$ the constituent-number. Neighboring groups differ in frequency by 1 rpm (cycle per month); neighboring constituents by 1 cpy (cycle per year). In the language of spectroscopy the tidal spectra show four orders of splitting: monthly splitting, a fine structure due to yearly splitting, a hyperfine structure from lunar perigee and regresional splitting, and a further splitting associated with solar perigee.

In principle, prediction by the harmonic method could be performed without any recourse to Keplerian and Newtonian mechanics, without regard even as to whether any given term is lunar or solar. All that needs to be done is to analyze a long record into all possible DOODSON numbers, starting with zero values and proceeding until the computed amplitudes are below some desired limit. For a limiting amplitude of $10^4$ times the largest term, about 400 terms are required, and in no instance does $a_5$ exceed 6. But this is not a practical procedure. The largest terms are chosen once and for all on the basis of potential theory. If the sea surface coincided with a potential surface, the departure from mean sea level (the equilibrium height) would be given by

$$V = \frac{GM}{r} - \frac{V}{\bar{r}}$$

(3.4)

where $V(t)$ is the gravitational potential due to the Moon or Sun (mass M) whose center of mass is at a distance $r(t)$ from the point of observation F, $G$ is the gravitational constant, $g$ local gravity, and $V$ a suitable reference potential. Let $R(t)$ designate the Earth's radius, $R(t)$ the distance between the centers of Earth and M, and $\theta(t) = a/r$ the parallax. It can be shown (see Appendix A) that

$$p = R(1 + \theta^2 - 2 \cos \theta)$$

(2.5)

$^3$ DOODSON (1923) used $a_4, a_5 + 5, a_6 + 5, a_7 + 5$ to assure positive indices.
We decided to take a new look at the tide records, without astronomical prejudice and freely allowing for the presence of noise. Modern methods of time series analysis seemed appropriate. Experimental analyses with very long series are readily carried out with high-speed digital computers. Continuous hourly readings for half a century or longer are available for more than a dozen ports (comprising some 10 observations) and constitute a unique geophysical record.

As a result of our analyses we are led to propose a method of tide prediction which differs somewhat from the classical method now universally accepted. The proposed method leads to slightly greater precision with a lesser number of tidal constants; there are other advantages. It can be said that we are here attempting to improve the one geophysical prediction that works tolerably well already; to this charge we plead guilty. But predicting and learning are in a sense orthogonal, and the most interesting effects are those that cause most trouble with forecasting: the continuous, the nongravitational tides, and the nonlinear interactions. Nearly all of the tidal energy is in narrow clusters centered at frequencies of 0, 1 and 2 cycles per day. For prediction this is of great advantage. For exploration one would have preferred a broad band excitation, such as tidal forces arising from a nearby supernova.

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$$\Delta h(t) = \sum \psi_i \cos(2\pi t + \varphi_i)$$

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summed over a set of denumerable frequencies $\psi_i$. The coefficients $\psi_i, \varphi_i$ are found by harmonic analysis of the tide record $\Delta h(t)$ for frequencies $\psi_i$ specified in advance (to eight sigi-
where $\alpha$ is the local zenith angle of $M$. For any point $P$

$$\alpha = \alpha(\theta, \phi)$$

(2.6)

where $\theta(t)$ is the polar angle, and $L(t)$ the (terrestrial) longitude of $M$. The distance, polar angle and longitude of $M$ can now be expressed in terms of sines and cosines of six fundamental arguments,

$$R_C, R_P, L_p, R_P, L_{DP}, L_{DP} = \text{functions of } (3 \times 2)^{1/2} \theta, \phi, R_C, R_P, R_P, R_P$$

(2.7)

whose frequencies $f_p, f_{DP}, \ldots, f_{DP}$ are the basic frequencies in equation (2). Here $t$ is Greenwich time, $\phi$ and $\phi_0$ are the mean ecliptic longitudes, $\phi_C$ and $\phi_P$ the longitudes of peregrine, and $\phi_P$ the longitude of the Moon's nodes. The arguments $R_C, R_P, \ldots, R_P$ are usually given in the form

$$A + BT + CT^2$$

(2.8)

where $T$ is in JULIAN centuries since January 1, 1900, $A$ is the phase in 1900, $B = 2 \pi f_0$ denotes the frequency in cycles per century, and $CT^2$ is a small correction arising from planetary perturbations and tidal frictions.

A systematic development of equations (4) to (8) leads to the trigonometric expansion

$$\frac{\nu}{\beta} = \sum_{n=0}^{\infty} \cos \left(2 \pi f_n t + \phi_n \right) f_n = \alpha_n + \beta_n + \gamma_n + \ldots$$

which associates any DOODSON number $(\alpha_n, \beta_n, \gamma_n, \ldots)$ with an amplitude $\alpha$ and phase $\phi$ consistent with potential theory. In this way the frequencies of the important terms are selected.

DOODSON considered $R_C, R_P, R_P, R_P$ as sensibly constant over any one year, and expanded in terms of $t, h_C, h_0$ only: the frequencies are then fully defined by the constituent numbers $(\alpha_n, \beta_n, \gamma_n)$. DOODSON's expansion included 39 terms, none smaller than $10^3$ times the largest amplitude, and all within the limits $|\alpha| < 3$. This development was universally accepted by 1883, and led to substantial improvements over JUBBINS'S method. But it was found that when all Darrufian constituents were removed, the residual tides still showed significant components. Consequently DOODSON carried out the expansion into some 400 terms exceeding $10^4$ times the amplitude of the largest term and assigning each term with the complete DOODSON number.

In the application of the harmonic method one encounters an awkward situation which is not corrected by the more complete expansion of DOODSON'S. At most tide stations records are available for only a few years or less, and it is not practical to resolve terms whose frequencies differ by less than 1 cycle. We are back to constituents $(\alpha_n, \beta_n, \gamma_n)$ of the DARWIN type. The accepted procedure is to replace equation (1) by

$$\frac{\zeta(t)}{\beta} = \sum_{n=0}^{\infty} \cos \left(2 \pi f_n t + \phi_n \right)$$

(2.9)

where the factors $f_n(t), \phi_n(t)$ are taken as constant over any one year, but varied from year to year with the nodal period of 18.61 years. The $f, \phi$-factors were introduced by DARWIN and are tabulated in manuals on tide prediction. The 21,000 year variation (frequency $f_0$) is ignored.

The use of slowly varying amplitudes and phases implies that the DARWIN-DOODSON method is not, strictly speaking, a harmonic method.

(c) The convolution method

The complexity of the DARWIN-DOODSON expansion is, in a sense, an artifact arising from an insistence of expressing the tides as sums of harmonic functions of time. With $n, p, \beta$ as given functions of time, it is nowadays relatively simple to evaluate equations (4) to (8) numerically and obtain a computer-generated time series $V(t)$. Suppose that hourly values $V(t)$ have been computed for a given port. One could attempt a prediction for time $t$ as a weighted sum of past and future values of the potential,

$$\hat{V}(t) = \sum w(s) V(t - s)$$

(2.10)

with the weights $w$ determined so that the prediction error $\zeta(t) - \hat{V}(t)$ is a minimum in the least-square sense. This formulation has the defect that it assumes predictions for any port to depend only on the equilibrium tide at this port. One would do better to evaluate the equilibrium tide at a grid of points surrounding the port, thus obtaining $V_i(t), V_j(t), \ldots$, and then predicting according to

$$\hat{V}(t) = \sum w_i(s) V_i(t - s) + \sum w_j(s) V_j(t - s) + \ldots$$

Alternatively one could expand $V(t)$ in the vicinity of the port $(\theta_0, \lambda_0)$ in terms of a Taylor series

$$V + \frac{\partial V}{\partial \theta} \theta_0 + \frac{\partial V}{\partial \lambda} \lambda_0 + \ldots$$

numerically evaluate $V(t)$ and its spatial derivatives $V_i, V_j, \ldots$ at position $(\theta_0, \lambda_0)$, and attempt a prediction according to

$$\hat{V}(t) = \sum w_i(s) V_i(t - s) + \sum w_j(s) V_j(t - s) + \sum w_k(s) V_k(t - s) + \ldots$$

Our scheme is to expand $V$ in spherical harmonics,

$$V(\theta, \lambda, t) = \sum_{n=0}^{\infty} \sum_{m=0}^{n} [a_n^m(t) \zeta_n^m(\theta, \lambda) + b_n^m(t) \zeta_n^m(\theta, \lambda)]$$

(2.11)

and compute the coefficients $a_n^m(t)$ and $b_n^m(t)$ for the desired time interval. The convergence of the spherical harmonics is rapid and just a few terms $m, n$ will do. The prediction formalism is then

$$\hat{V}(t) = \sum_{n=0}^{\infty} \sum_{m=0}^{n} \bar{a}_n^m(s) \zeta_n^m(t - s) + \bar{b}_n^m(s) \zeta_n^m(t - s)$$

(2.12)

The prediction weights $\bar{a}_n^m(s) = a_n^m(t) + \gamma_0^m(s)$ are determined by least-square methods, and tabulated for each port (these take the place of the tabulated coefficients $C_{n, \beta}$ in the harmonic method). For each year the global tide function $\zeta_n^m(t) = a_n^m(t) + b_n^m(t)$ is computed and the tides then predicted by forming weighted sums of $C$ using the weights $w$ appropriate to each port. The spectra of the numerically-generated time series $c(t)$ have all the complexity of the DARWIN-DOODSON expansion; but there is no need for carrying out this expansion, as the series $c(t)$ serves as direct input into the convolution prediction. There is no need to set a lower bound on spectral lines; all lines are taken into account in an optimum sense. There is no need for the $f, \phi$-factors, for the nodal variation (and even the 21,000 year variation) is already built into $c(t)$. In this way the convolution method makes explicit and general what the harmonic method does anyway -- in the process of applying the $f, \phi$-factors. The convolution method leads to a more systematic procedure, better adapted to computer use. The convolution formalism is readily extended (as we shall see) to include nonlinear, and perhaps even meteorological effects.
where \( \phi(t) \) is the local zenith angle of \( M \). For any point \( P \)

\[ a = a(Z, \lambda) \]  

(2.6)

where \( Z(t) \) is the polar angle, and \( L(t) \) the (terrestrial) longitude of \( M \). The distance, polar angle and longitude of \( M \) can now be expressed in terms of sines and cosines of six fundamental arguments,

\[ R_C, Z_C, L_C, R_P, Z_P, L_P = \text{functions of } (3 \times 1^\circ/24 \ H, b_C, b_P, \lambda_C, \lambda_P, \lambda_P) \]  

(2.7)

whose frequencies \( \omega_1, \omega_2, ..., \omega_6 \) are the basic frequencies in equation (2). Here \( t \) is Greenwich time, \( b_C \) and \( b_P \) are the mean ecliptic longitudes, \( \phi_C \) and \( \phi_P \) the longitudes of perigee, and \( \lambda_C \) the longitude of the Moon's nodes. The arguments \( b_C, b_P \) in terms usually given in the form

\[ A + BT + CT^2 \]  

(2.8)

where \( T \) is in JULIAN centuries since January 1, 1900. \( A \) is the phase in 1900, \( B = 2 \pi f_0 \) denotes the frequency in cycles per century, and \( C \) is a small correction arising from planetary perturbations and tidal friction.

A systematic development of equations (4) to (8) leads to the trigonometric expansion

\[ \frac{V}{e} = \sum_1^6 \omega_i \cos \left( 2\pi f_i t + \phi_i \right) \]  

(2.9)

which associates any DOODSON number \( (a_1, a_2, a_3, a_4, a_5, a_6) \) with an amplitude \( \omega \) and phase \( \phi \) consistent with potential theory, in this way the frequencies of the important terms are selected.

DOODSON considered \( R_C, Z_C, L_C \) as sensibly constant over any one year, and expanded in terms of time, \( b_C, b_P \), only; the frequencies are then fully defined by the constituent numbers \( (a_1, a_2, a_3) \). DOODSON's expansion included 39 terms, none smaller than 10^4 times the largest amplitude, and all within the limits \[ |a| < 3 \]. This development was universally accepted by 1883, and led to substantial improvements over LUNDBROCK'S method. But it was found that all Darwinian constituents were removed, the residual tides still showed significant components. Consequently DOODSON carried out the expansion into some 400 terms exceeding 10^9 times the amplitude of the largest term and associating each term with the complete DOODSON number.

In the application of the harmonic method one encounters an awkward situation which is not corrected by the more complete expansion of DOODSON'S. At most tide stations records are available for only a few years or less, and it is not practical \(^*\) to resolve terms whose frequencies differ by less than 1 copy. We are back to constituents \( (a_1, a_2, a_3) \) of the DARWIN type. The accepted procedure is to replace equation (1) by

\[ \frac{V(t)}{e} = \sum_1^6 \left( \omega_i + \alpha_i + \uplambda_i \right) \]  

(2.10)

where the factors \( f_i(t), \alpha_i(t), \uplambda_i(t) \) are taken as constant over any one year, but varied from year to year with the nodal period of 18.61 years. The \( f, \alpha \)-factors were introduced by DARWIN and are tabulated in manuals on tide prediction. The 21,000 year variation (frequency \( f_0 \)) is ignored. The use of slowly varying amplitudes and phases implies that the DARWIN-DOODSON method is not, strictly speaking, a harmonic method.

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\(^*\) But DOODSON'S statement that 12 years of record are required to separate regresstonal terms is incompletes. Any 4 known values of \( \phi \) will provide 4 equations to solve for the amplitudes and phases of each \( f, \alpha \), \& \( \uplambda \) regardless of the frequency separation. In an ideal record, 4 hourly values would resolve regresstonal splitting. A statement concerning the required length of record has to take into account the underlying noise spectrum and, when this is done, the situation is not much more better than stated by DOODSON (MUNK and HASSELMANN, 1964).

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**c) The convolution method**

The complexity of the DARWIN-DOODSON expansion is, in a sense, an artifice arising from an insistence of expressing the tides as sums of harmonic functions of time. With \( n \) as given functions of time, it is nowadays relatively simple to evaluate equations (4) to (8) numerically and obtain a computer-generated time series \( \phi(t) \). Suppose that hourly values \( \phi(t) \) have been computed for a given port. One could attempt a prediction for time \( t \) as a weighted sum of past and future values of the potential.

\[ \frac{\phi(t)}{e} = \sum_{-\infty}^{\infty} w(s) \phi(t - s) \]  

(2.11)

with the weights \( w \) determined so that the prediction error \( \phi(t) - \frac{\phi(t)}{e} \) is a minimum in the least-square sense. This formulation has the defect that it assumes predictions for any port to depend only on the equilibrium tide at this port. One would do better to evaluate the equilibrium tide at a grid of points surrounding the port, thus obtaining \( \phi(t), \phi(t), ..., \) and then predicting according to

\[ \phi(t) = \sum_{-\infty}^{\infty} w(s) \phi(t - s) \sum_{-\infty}^{\infty} w(s) \phi(t - s) + ... \]  

Alternatively one could expand \( \phi(t) \) in the vicinity of the port \( (\theta_0, \lambda_0) \) in terms of a Taylor series

\[ \phi(t) = \sum_{-\infty}^{\infty} w(s) \phi(t - s) \sum_{-\infty}^{\infty} w(s) \phi(t - s) + ... \]  

numerically evaluate \( \phi(t) \) and its spatial derivatives \( \partial \phi \), \( \partial \phi \) at position \( \theta_0, \lambda_0 \), and attempt a prediction according to

\[ \frac{\phi(t)}{e} = \sum_{-\infty}^{\infty} [a_i(t) \frac{\partial \phi}{\partial \theta} + b_i(t) \frac{\partial \phi}{\partial \lambda}] \]  

(2.12)

and compute the coefficients \( a_i(t) \) and \( b_i(t) \) for the desired time interval. The convergence of the spherical harmonics is rapid and just a few terms \( m, n \) will do. The prediction formalism is then

\[ \phi(t) = \sum_{-\infty}^{\infty} w_i(t) \phi(t - s) + u_i(t) \]  

(2.13)

The prediction weights \( w_i(t) = a_i(t) + u_i(t) \) are determined by least-square methods, and tabulated for each port (these take the place of the tabulated coefficients \( C \), \( \phi \), in the harmonic method). For each year the global tide function \( \phi(t) = a_i(t) + b_i(t) \) is computed and the tides then predicted by forming weighted sums of \( \phi \) using the weights \( w \) appropriate to each port. The spectra of the numerically-generated time series \( \phi(t) \) have all the complexity of the DARWIN-DOODSON expansion; but there is no need for carrying out this expansion, as the series \( \phi(t) \) serves as direct input into the convolution prediction. There is no need to set a lower bound on spectral lines; all lines are taken into account in an optimum sense. There is no need for the \( f, \alpha \)-factors, for the nodal variation (and even the 21,000 year variation) is already built into \( \phi(t) \). In this way the convolution method makes explicit and general what the harmonic method does anyway - in the process of applying the \( f, \alpha \)-factors. The convolution method leads to a more systematic procedure, better adapted to computer use. The convolution formalism is readily extended (as we shall see) to include nonlinear, and perhaps even meteorological effects.
The harmonic and convolution methods are closely related. As the lower bound on spectral lines is reduced, and as the number of spherical harmonics and lags are increased, the results of the two methods rapidly approach one another. It should be stated at the outset that under ordinary circumstances the improvement in the accuracy of tide prediction by the use of the convolution method is slight. But there is an advantage in introducing Kepler-Newtonian mechanics from the very start, and the prediction formalism (12) makes the separation of astronomy from oceanography more explicit than does equation (11). In the sense that the convolution method does not involve a time-harmonic expansion (it involves only a spherical-harmonic expansion), it is a move back toward the nonharmonic method of LUBBOCK.

III - INPUT FUNCTIONS

We have written a computer program for generating the coefficients \( c_\ell^m \) (i) arising from lunar gravitational, solar gravitational, and radiational effects, for any specified values of \( m, n, \) and for any prescribed start time, time interval and end time. The numerical scheme follows a series of steps already outlined in equations (2.4) to (2.8). The derivation of the gravitational potential does not differ much from that given by DOODSON (or for that matter by DAIWEN), but there are some innovations. The normalization of spherical harmonics is adopted from quantum mechanics, as this leads to the most symmetrical expressions for \( c_\ell^m (t) \). We have included some third-order terms in the lunar eccentricity, and allowed for planetary perturbations of the solar, eccentricity. Numerical values concerning the Sun-Moon-Earth system have been revised. Details are found in the Appendix. The use of the convolution method carries some obligation to include a complete set of realistic input functions. We must do something about such features in the tide records that cannot be accounted by the gravitational tide-producing forces. For example, sea level responds to surface pressure associated with atmospheric tides. At temperate latitudes, maximum pressure occurs around 10 a.m. and 10 p.m., local time, minimum pressure at 4 a.m. and 4 p.m., with amplitudes of the order of 1 mb (gravitational forces on the atmosphere can account for only 1% of this variation). There are also irregular day-to-day pressure fluctuations, possibly by many millibars, associated with "weather". In the former case we have a process with a discrete (or line) spectrum whose frequencies overlap the gravitational line spectrum, in the latter case the process has a continuous (or noisy) spectrum which underlies the discrete spectrum. Similarly the land-and-sea breezes regime is associated with a line spectrum, principally at 1 and 2 cph, as well as a continuum, Storm tides are an extreme case of the latter type. Sea level responds to both the regular and irregular pressure variations nearly as an inverted barometer, down 1 cm for a pressure increment of 1 millibar (MUNK and BULLARD, 1963).

September sea level typically exceeds March sea level by 30 cm in the northern hemisphere (PATULLO et al., 1955). Tidal effects and atmospheric pressure effects can only account for 10% of the seasonal oscillation. Local changes in the specific volume of the water associated with water transport, solar radiation, back radiation, evaporation, etc., appear to be responsible. The total mass of the water column plus air column remains sensibly constant, so that a pressure recorder on the sea bottom would hardly sense the seasonal fluctuation in sea level, In addition there are irregular variations from year to year. Again we may refer to the discrete spectrum (principally at 1 and 2 cph) and the continuum.

We distinguish between three input functions, with the following spectral properties:

(i) The known gravitational line spectrum (linear and solar)

(ii) The unknown non-gravitational line spectrum

(iii) The unknown non-gravitational continuum.

(ii) and (iii) are ultimately the result of radiational processes, but at different stages of "orderliness", In a highly dissipative atmosphere strongly coupled by nonlinear processes, the line spectrum arising from the daily and seasonal variation in solar radiation is no longer discernible in the "weather" and in the long-term variations, The prediction problem associated with (iii) will require special consideration.

We need an input function to model (ii). The function must, in some vital way, be related to the daily pressure and wind variations and to the seasonal changes in ocean temperature, and yet avoid the need for detailed solution of these complicated processes. For a trial input we define the radiational function

\[ a = \text{R}_0 \cos \alpha \text{ in day time, } 0 < \alpha < \pi \]

\[ a = 0 \text{ in night time, } \frac{1}{2} \pi < \alpha < \pi \]

which varies with the radiant energy falling on a unit surface in a unit time. Expanded in spherical harmonics (Appendix A),

\[ \frac{\Delta R}{R_0} = \text{R}_0 \left\{ \frac{\text{M}_0}{\text{M}} \sum_{\ell=1}^{\infty} \frac{2n+1}{(2n+1)(2n+2)} \sum_{m=1}^{2n+1} \left[ \begin{array}{c} \ell \\ n \end{array} \right] \frac{\text{P}_n^m(\cos \alpha)}{\text{P}_n^m(\cos \beta)} \right\} 

\]

where \( \text{R}_0 \) and \( \text{P}_n^m \) can now be expressed in terms of the fundamental orbital constants, just as was done for the gravitational potential.

\[ \text{V} = \text{M}_0 \sum_{\ell=1}^{\infty} \text{M}_0 \sum_{m=1}^{2n+1} \frac{1}{(2n+1)(2n+2)} \frac{\text{R}_0}{\text{R}} \frac{\text{P}_n^m(\cos \alpha)}{\text{P}_n^m(\cos \beta)} \text{P}_n^m(\alpha) \]

The essential distinction between gravitational and radiational inputs is that the Earth is transparent to gravity and opaque to radiation and this is contained in the formulation of the input functions. The "clipped" day and night distribution of the radiation function is much richer in higher harmonics than the gravitational function, as observed. One may hope that the seasonal modulation of the daily pressures and winds is properly modeled by the radiation function, so that the fine structure in the input spectra near 1, 2, ... cph is in the proper proportion.

IV - LINEAR ANALYSIS

a) The physical system

Before analyzing an actual record of sea level, we must first outline the main physical concepts involved in a linear regime, and the corresponding processes required to analyze them. Non-linear perturbations will be considered in section 7. We concentrate attention on the coefficients \( a_\ell^m (t) \) of the gravitational potential \( V_\ell (t; \lambda ; \tau) \) (equation 2.11). The complex input potential and the recorded tide can be represented, for any \( m, n, \) by

\[ G(t) = \int_{-\infty}^{\infty} \text{c}(t) e^{i\omega t} dt, \quad H(t) = \int_{-\infty}^{\infty} \text{z}(t) e^{i\omega t} dt \]

respectively. The same quantities are involved in the "impulse response" relation,

\[ \tilde{z}(t) = \text{real part} \left\{ \int_{-\infty}^{\infty} c(t - \tau) w(\tau) d\tau \right\} \]

where \( c + a \cdot b \) is the complex conjugate of \( c \), and \( w(t) = \text{w}(t) + \text{i}\text{v}(t) \) is the sea level response following an instantaneous value \( c \) at time \( t = 0 \). The Fourier transform of the impulse response is the "admittance".
The harmonic and convolution methods are closely related. As the lower bound on spectral lines is reduced, and as the number of spherical harmonics and lags are increased, the results of the two methods rapidly approach one another. It should be stated at the outset that under ordinary circumstances the improvement in the accuracy of tide prediction by the use of the convolution method is slight. But there is an advantage in introducing Kepler-Newtonian mechanics from the very start, and the prediction formalism (12) makes the separation of astronomy from oceanography more explicit than does equation (11). In the sense that the convolution method does not involve a time-harmonic expansion (it involves only a spherical-harmonic expansion), it is a move back toward the nonharmonic method of LUBBROCK.

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(i) and (ii) are ultimately the result of radiational processes, but at different stages of "orderliness". In a highly dissipative atmosphere strongly coupled by nonlinear processes, the line spectrum arising from the daily and seasonal variations in solar radiation is no longer discernible in the "weather" and in the long-term variations, the prediction problem associated with (iii) will require special consideration.

We need an input function to model (ii). The function must, in some vital way, be related to the daily pressure and wind variations and to the seasonal changes in ocean temperature, and yet avoid the need for detailed solution of these complicated processes. For a trial input we define the radiational function

\[ R(\theta, \phi) = S R(\theta, \phi) \cos \alpha \text{ in day time, } 0 < \alpha < \pi \]

\[ = 0 \text{ in night time, } \frac{1}{2} \alpha < \pi < 2 \pi \]

which varies with the radiant energy falling on a unit surface in a unit time. Expanded in spherical harmonics (Appendix A),

\[ R(\theta, \phi) = \sum_{l=1}^{l} \sum_{m=-l}^{l} \left[ \frac{2l+1}{4\pi} \right] \left( \frac{\cos \theta}{\sin \theta} \right)^{2l+1} \left( \frac{\cos \phi}{\sin \theta} \right)^{2l+1} \left( \frac{\cos \phi}{\sin \theta} \right)^{2l+1} P_l^m(\cos \theta) \]

where $R_0$ and $P_l^m(\cos \theta)$ can now be expressed in terms of the fundamental orbital constants, just as was done for the gravitational potential.

\[ \frac{\partial R}{\partial t} = \sum_{l=1}^{l} \sum_{m=-l}^{l} \left[ \frac{2l+1}{4\pi} \right] \left( \frac{\cos \theta}{\sin \theta} \right)^{2l+1} \left( \frac{\cos \phi}{\sin \theta} \right)^{2l+1} \left( \frac{\cos \phi}{\sin \theta} \right)^{2l+1} P_l^m(\cos \theta) \]

The essential distinction between gravitational and radiational inputs is that the Earth is transparent to gravity and opaque to radiation and this is contained in the formulation of the input functions. The "clipped" day and night distribution of the radiation function is much richer in higher harmonics than the gravitational function, as observed. One may hope that the seasonal modulation of the daily pressures and winds is properly modeled by the radiation function, so that the fine structure in the input spectra near 1, 2, ..., c.p.d. is in the proper proportion.

IV - LINEAR ANALYSIS

a) The physical system

Before analyzing an actual record of sea level, we must first outline the main physical concepts involved in a linear regime, and the corresponding processes required to analyze them. Non-linear perturbations will be considered in section 7. We concentrate attention on the coefficients $A_n^k(t)$, $B_n^k(t)$ of the gravitational potential $V(0, \lambda; t)$ (equation 2.11). The complex input potential and the recorded tide can be represented, for any m, n, by

\[ G(t) = \int_{-\infty}^{\infty} c(t) e^{i\omega t} dt, \quad H(t) = \int_{-\infty}^{\infty} \xi(t) e^{i\omega t} dt, \]

respectively. The same quantities are involved in the "impulse response" relation,

\[ \xi(t) = \text{real part of} \int_{-\infty}^{\infty} c(t - \tau) w(\tau) d\tau, \]

where $c = a - i = \text{complex conjugate of } c$, and $w(t) = w(t) + i w(t)$ is the sea level response following an instantaneous value $c$ at time $t = 0$. The Fourier transform of the impulse response is the "admittance".
to which corresponds an admittance function

$$Z(f) = Z_0 + \sum_{k} \left( \frac{4}{\pi} \right) \frac{k^{3}}{k} \exp(-2\pi f/k) U_{k}.$$  

(4.10)

At the discrete frequencies $f = f/355$ cph, $Z(f)$ equals precisely $Z_0$ is equation (4.8).

An important parameter is the coherence

$$\gamma^2 = \frac{C_H^2}{E_H^2 E_i^2}$$

(4.11)

which is positively biased (to remove the bias, see Appendix B). We shall use $\gamma^2$ in a significant way to separate the sea level energy $E_H^2$ into two parts, namely

"coherent energy" $= \gamma^2 E_H^2 = Z_0^2 E_i^2$

(4.12)

which is the sea level energy in the appropriate frequency band that may be directly ascribed to the tidal effect concerned, and

"non-coherent energy" $= (1 - \gamma^2) E_H^2$

(4.13)

which is effectively noise energy, but may contain energy coherent with other input functions.

c) Multiple input functions

The stronger lines in $Z(f)$ are separated from those in $Z_0$ by multiples of the perigee frequency $f_0$ and nodal frequency $f_n$ (see DOODSON'S 1921 schedules (*). In an analysis for tides of degree 2 over 19 x 355 days = 2,087 perigee cycles = 0.992 nodal cycles, the relative phase $\phi_k$ of third degree tides pass uniformly through nearly integral multiples of $2\pi$. Consequently the energies of $E_0^2$ and $E_k^2$ are separable; in the analysis for $F_0^2$, the $F_k^2$ tides appear as non-coherent energy.

But there are important cases where the frequencies of tides of distinct origin are strictly identical, and therefore not separable in the above manner: (i) in mixed semi-diurnal and diurnal tides: (ii) in the mixed gravitational and radiational lines that occur in narrow bands, about 4 cph wide, of groups (2 0), (1 1), and (2 3) (***); We shall refer to "disguised" linear and radiational line.

The spectral estimates $H_k$ of sea level $\zeta$, for any doublet may be written

$$H_k = Z(f) G_k + Z_0(f) G_0 + N_k.$$  

(4.14)

(*) We shall not consider non-identical frequencies which merely differ by a very small quantity, such as $f_0$ or $f_n = -\lambda_k$, since in all such cases listed by DOODSON at least one of the amplitudes involved is negligibly small.

(**) Non-linearities in the Keplerian laws and Newtonian mechanics produce sum and difference frequencies that are responsible for the complexity in the "linear" input functions (e.g., the DOODSON numbers, section 5b), Hydrodynamic non-linearities inherent in the Navier-Stokes equation tend to additional splitting and produces some overlapping frequencies. The peculiarity of the situation arises then from the circumstance that the Kepler-Newton non-linearities are in series with the Navier-Stokes non-linearities.

(*** Group (2 0) contains the radiational inputs $\omega_0^2 \omega_0^2 + \lambda_0 \omega_0^2$, plus a negligible gravitational input $\omega_0^2$. Group (1 1) has a triplet input $\omega_1^2 \lambda_1^2 \omega_1^2$. Group (2 3) contains $\gamma_3^2 \lambda_3^2$.}
embracing an "amplitude response" $|Z(f)|$ and a phase lead arg $Z(f)$, Relations analogous to (4.1 - 4.4) are fundamental to a wide variety of linear noise-free physical systems.

A common approach is to estimate values of $G(f)$ and $H(f)$ of appropriate frequencies by spectral analysis of $\xi(t)$ and $\eta(t)$ respectively, and then form $Z(f) = H(f)/G(f)$. This method suffers from neglect of the "noise" which is inevitably present in $H$. In that event (4.4) is no longer a valid definition of $Z$. An estimate of $Z$ is obtained by (4.3), or by cross-spectral analysis.

b) Noise-free estimates for discrete sampling

For sampling at discrete intervals, spectral estimates $G_n, H_n$ of $a_n(t)$ and $\xi(t)$ were formed according to

$$ G_n = \sum_{t=1}^{\infty} \xi(t) a(t) \exp(2 \pi n / T)$$

$$ H_n = \sum_{t=1}^{\infty} \eta(t) \xi(t) \exp(2 \pi n / T)$$

where the summations are from $t = -3556$ to $+3556$ at intervals $\Delta t$, $n = 356 / \Delta t$, and $\xi(t)$ is the "cosine taper" function $1 - \cos(\pi t / T)$, inserted for rapid convergence of "side-band" effects. In most cases $a(t)$ and $\xi(t)$ have both previously been "smoothed" by a low-pass filter with cut-off at $6$ cph, so that three-hourly values ($\Delta t = 1/8$ day, $n = 3846$) are used in (4.5) without trouble from "aliasing". The period of 3556 days is close to 13 lunar months and to 1 year, so that all the tidal constituents fall centrally within the filters corresponding to integral values of $r$ (CARTWRIGHT and CALDON, 1963). Note that the spectrum of $b(t)$ is not required, since it is identical, with the spectrum of $a(t)$ with a phase change of $\pi/2$.

From (4.5) we form estimates of the input energy spectrum $E_1^b$, the output energy spectrum $E_2^b$, and the cross-spectrum $C_{12}^b$, namely

$$ E_1^b = \left( \frac{1}{C_{12}^b} \right) \sum_{t=1}^{\infty} A_n(t) B_n(t) \exp(-2 \pi n / T)$$

$$ E_2^b = \left( \frac{1}{C_{12}^b} \right) \sum_{t=1}^{\infty} A_n(t) B_n(t) \exp(-2 \pi n / T)$$

$$ C_{12}^b = \left( \frac{1}{\sqrt{E_1^b E_2^b}} \right) \sum_{t=1}^{\infty} A_n(t) B_n(t) \exp(-2 \pi n / T)$$

where the ensemble averages are taken for consecutive values of $r$, or, more accurately, by averaging quantities for the same $r$ derived from analyses at different epochs. Since the input energy is noise-free, we may express the cross-spectrum as

$$ C_{12}^b = \left( \frac{1}{\sqrt{E_1^b E_2^b}} \right) \sum_{t=1}^{\infty} A_n(t) B_n(t) \exp(-2 \pi n / T)$$

where $N_n$ is the noise element in $C_{12}^b$ and $\phi_n$ its phase relative to $C_{12}^b$. The phase of the noise is random (*) so that the second term in the ensemble average is negligibly small. The admittance estimate

$$ Z_n = \frac{X_n + jY_n}{C_{12}^b} = \frac{A_n(t) B_n(t)}{C_{12}^b}$$

is not biased by noise effects, whereas $H_n/G_n$ is always greater than $|Z_n|$ by a proportion dependent on the noise : signal ratio, no matter how extensive the ensemble averaging. The sampling variability of the admittance estimate (4.8) is discussed in Appendix B.

A convolution similar in form to the impulse response relation (4.2), but expressed in discrete time intervals, is

$$ \xi(t) = \sum_{t=1}^{\infty} \xi(t - t_0) W_n$$

(*) We then include in the "noise" such tidal components that are incoherent with the particular spherical harmonic under consideration (see section 4c).

\begin{align*}
Z(f) &= \frac{w_0}{(1 + \delta f) W_0} \exp(-2\pi f f_{cc}) \\
Z(f) &= \frac{G(f)}{H(f)}
\end{align*}

At the discrete frequencies $f = r/355$ cph, $Z(f)$ equals precisely $Z_n$ is equation (4.8).

An important parameter is the coherence

$$ \gamma_n^2 = \frac{C_{12}^b}{E_1^b E_2^b}$$

which is positively biased (to remove the bias, see Appendix B). We shall use $\gamma_n^2$ in a significant way to separate the sea level energy $E_1^b$ into two parts, namely

$$ "coherent energy" = \gamma_n^2 E_1^b = Z_n^2 E_1^b$$

$$ "non-coherent energy" = (1 - \gamma_n^2) E_1^b$$

which is effectively noise energy, but may contain energy coherent with other input functions.

c) Multiple input functions

The stronger lines in $a_n(t)$ are separated from those in $a_n(t)$ by multiples of the perigee frequency $f_4$ and nodal frequency $f_0$ (see DOODSON'S 1921 schedules (*)). In an analysis for tides of degree $2$ over $19 \times 355$ days $= 2,087$ perigee cycles $\times 0.992$ nodal cycles, the relative phase $\phi_n$ of third degree tides pass uniformly through nearly integral multiples of $2 \pi$. Consequently the energies of $P_1^b$ and $P_2^b$ are separable ; in the analysis for $P_1^b$, the $P_2^b$ tides appear as non-coherent energy.

But there are important cases where the frequencies of tides of distinct origin are strictly identical, and therefore not separable in the above manner : (i) in mixed semi-diurnal gravitational tides, principally $K_1$ and $O_1$ (but there is no point in separating linear from solar gravitational effects) ; (ii) in mixed linear and shallow-water tides ($\ast$) ; and (iii) in the mixed gravitational and radiational lines that occur in narrow bands, about 4 cph wide, of groups (2 0), (1 1), and (2 2) (**). We shall refer to "disguised" linear and radiational lines.

The spectral estimates $H_n$ of sea level $Z$ for any doublet may be written

$$ H_n = Z(f) G_n = Z_n^2(t) G_n^2 + N_n$$

(*) We shall not consider non-identical frequencies which merely differ by a very small quantity, such as $f_4$ or $f_0 - f_0$, since in all such cases listed by DOODSON at least one of the amplitudes involved is negligibly small.

(**) Non-linearities in the Keplerian laws and Newtonian mechanics produce sum and difference frequencies that are responsible for the complexity in the "linear" input functions (e.g., the DOODSON numbers, section 5b). Hydrodynamic non-linearities inherent in the Navier-Stokes equation tend to additional splitting and produces some overlapping frequencies. The peculiarities of the situation arises then from the circumstance that the Kepler-Newton non-linearities are in series with the Navier-Stokes non-linearities.

\textbf{Group (2 0)} contains the radiational inputs $a_n^2, a_n^4, a_n^6$ plus a negligible gravitational input $a_n^2$. Group (1 1) has a triplet input $\gamma_n^2, \gamma_n^4, \gamma_n^6$. Group (2 2) contains $\gamma_n^2, \gamma_n^4, \gamma_n^6$. 202

\textbf{203}
where $G_1$ and $G_2^*$ are the respective input spectra, and $Z^*$ and $Z'$ are the conjugates of the associated admittances, $(Z^*, G_1^*)$ cannot be classed with the noise $N_f$ as in (7), because its phase relative to $G_1$ is constant. (14) cannot be solved for both $Z$ and $Z'$, even when the noise is reduced by ensemble averaging. The six equations for the cross-spectral elements $G_1^*, H, G_2^*, H$ are inter-dependent, and reducible to two independent equations for the unknowns $X, Y, X', Y'$. We therefore need a least-squares solution for two or more consecutive values of $r$ for which the admittances $Z$ and $Z'$ can be regarded as constant, or slowly varying. Strictly even the least-squares solution is possible only if the ratios $G_1/G_2$ vary with $r$, but in practice this seems to be the case. The equations involved are straightforward.

d) Time resolution and Nyquist time

It is possible that an arbitrary sequence of time intervals $\tau_i$ could be chosen along with the coefficients $g_{21}, g_{31}$ in (9) to optimize the correspondence of (10) to a given natural system. However, there appears to be no simple rule for choosing such a sequence, apart from systematic trial and error, and after some experiments which proved tedious and unrewarding we decided to restrict the analysis to arithmetic sequences only.

With $\tau_i = \tau_{11}$, the formalism becomes more elegant, and one may think of the convolution (9) as equivalent to fitting a Fourier series (10) to the actual admittance $Z(f)$. The Fourier series has periodicity $1/\delta t$, which is of course unrealizable, but quite acceptable in practice provided $1/\delta t$ is greater than twice ($^2$) the bandwidth $P$ within which the spectrum $G(f)$ of the spherical harmonic is confined. The effective bandwidth for the gravitational potentials $P_1$ and $P_2$ is a little less than 0.3 cph, which suggests a "time resolution" $\delta t = 1.7$ days approximately. Some numerical tests for Honolulu showed $\delta t = 2$ days to be a good compromise, and this interval was adopted.

This choice of an optimum "time resolution" $\delta t = 1/2\Delta t$ is somewhat analogous to the choice of frequency resolution $\delta f = 1/2\Delta f$ for a spectrum of a time series of duration $\Delta t$. We also have an analogue to the "Nyquist frequency" $f_{\text{Nyquist}} = 1/2\Delta f$, which is the highest frequency which can be determined unambiguously from sampling of time intervals $\Delta t$. In our case, the spectrum $G(f)$, and therefore information about $Z(f)$, is concentrated in narrow groups separated by $2\Delta f = 1$ cpm, so we have a "Nyquist lag" $\tau_{\text{Nyquist}} = 1/(2\Delta f) = 0.5$ months, as the largest value of $\Delta t$ we could meaningfully consider.

In summary, the bandwidth of the tidal species, and within each species the group intervals associated with first order splitting, suggest lag intervals of 2 days up to some maximum lag of 14 days.

e) Credibility of smoothness

The greatest time increment we actually allowed was 6 days. This choice depends on the belief that the admittances of the ocean to tidal forces are fairly smooth, and should not require periodicities in $Z(f)$ below 1/6 cph. If the admittances were more wiggly, then consecutive groups, separated by 0.34 cph, would have virtually independent admittances, and the classical harmonic method would be equally suitable. However, our results, especially those for Honolulu, do seem to confirm a reasonable degree of smoothness consistent with the adopted limit.

The relative advantage of the convolution method over the harmonic method increases with the smoothness of the admittance and complexity of the spectral input. In the limiting case of a continuum input it is the only possible method; for unresolved lines it is the only practical one. But the advantages may be there even for a resolved line spectrum. At the same time the imposed smoothness of admittance does impose upon the user the responsibility of having generated realistic input functions, and in this sense the convolution method requires more care than the harmonic method.

\footnote{1/2\delta t = 1 \times P$ is theoretically possible, but would entail a discontinuity near the limits of the frequency band, making for slow convergence of the series.}

\footnote{The $f, u$-factors (section 2b) are a case in point.}

\section*{f) Realizability of admittance}

Equation (10) restricts the admittance $Z(f) = X(f) + jY(f)$ to orthogonality in $X$ and $Y$. This restriction has been dropped by permitting negative as well as positive values of $r$. For then

\[ \begin{align*}
\frac{\bar{Z}(f)}{\tau_i} & = \sum_{n=-\infty}^{\infty} \left[ u_n a(t - n\Delta t) + v_n b(t - n\Delta t) \right] \\
X(f) & = u_0 + \sum_{n=1}^{\infty} \left[ (u_n + u_{-n}) \cos(2\pi f n\Delta t) + (v_n - v_{-n}) \sin(2\pi f n\Delta t) \right] \\
Y(f) & = v_0 + \sum_{n=1}^{\infty} \left[ (u_n - u_{-n}) \sin(2\pi f n\Delta t) - (v_n + v_{-n}) \cos(2\pi f n\Delta t) \right]
\end{align*} \]

(4.15)

(4.16)

This permits $2S + 1$ degrees of freedom as compared to $S + 1$, without exceeding the limits discussed in sections (d) and (e).

The procedure apparently violates causality, the prediction (15) depending on both future and past values of the input functions. An equivalent statement is that the admittance (16) is not physically realizable, but since our knowledge is limited to a few narrow frequency bands, we are under no obligation to fit $Z(f)$ over all frequencies. In fact, the periodic representation of $Z(f)$ inevitably makes (16) invalid outside the known bands. (Restriction to positive lags would lead to an admittance (14) which is realizable, but still not applicable outside the bands because of the imposed periodicity.)

g) The weight matrix and prediction variance

In the last three sections we described the criteria governing the time interval and range of the linear convolutions used. It remains to comment on the computations involved in deriving the weights $P_{ij}, q_{ij}$ for a particular sea level record.

The predicted sea level is given by

\[ \Delta \zeta(t) = \sum_{i=1}^{S} \sum_{j=1}^{S} \left[ (u_i^* \cdot a_j^* \Delta t) - |u_i|^2 \right] + \text{radiational terms} + \ldots \]

(4.17)

with the weights determined by the condition that the time average

\[ \sigma^2 = \frac{1}{\bar{\zeta}(t) - \bar{\zeta}(t)^3} \]

be a minimum. We can use the general notation

\[ \Delta \zeta(t) = \sum_{i=1}^{S} w_i \cdot c_i \]

with the understanding that $w_i$ designates any of the weights $(u_i)^2$, or $(v_i)^2$, and $c_i$ the associated lagged input $a_i^* \Delta t - |u_i|^2$, or $b_i^* \Delta t - |v_i|^2$, whatever the value of $n, a, z$, and whatever the source of the terms, gravitational, radiational, or otherwise. The weights $w_i$ are found by solving the matrix of linear equations

\[ [M_{ij}] \cdot [w_i] = [b_j], \]

(4.18)

where

\[ M_{ij} = \langle c_i, c_j \rangle, \quad R_i = \langle c_i, \zeta \rangle \]

(4.19)

Mean values are removed from $\Delta \zeta(t)$ and $c_i(t)$ when necessary.
where $G$ and $G^*$ are the respective input spectra, and $Z$ and $Z^*$ are the conjugates of the associated admittances, $Z^*$ cannot be classed with the noise $N$, as in (1), because its phase relative to $G$ is constant. (14) cannot be solved for both $Z$ and $Z^*$, even when the noise is reduced by ensemble averaging. The six equations for the cross-spectral elements $G_{1}^{*}$ $H$, $G_{2}^{*}$ $H$, $G_{1}^{*}$ $G_{2}^{*}$ are inter-dependent, and reducible to two independent equations for the four unknowns $X$, $Y$, $X'$, $Y'$. We therefore used a least-squares solution for two or more consecutive values of $r$, for which the admittances $Z$ and $Z^*$ can be regarded as constant, or slowly varying. Strictly even the least-squares solution is possible only if the ratios $G_{1}/G_{2}$ vary with $r$, but in practice this seems to be the case. The equations involved are straightforward.

d) Time resolution and Nyquist time

It is possible that an arbitrary sequence of time intervals $t_{i}$ could be chosen along with the coefficients $p_{r} q_{s}$ in (9) to optimize the correspondence of (10) to a given natural system.

However, there appears to be no simple rule for choosing such a sequence. Apart from systematic trial and error, and after some experiments which proved tedious and unrewarding we decided to restrict the analysis to arithmetic sequences only.

With $t_{i} = 20/3$, the formalism becomes more elegant, and one may think of the convolution (9) as equivalent to fitting a Fourier series (10) to the actual admittance $Z(t)$. The Fourier series has periodicity $1/10$ in $t$, which is of course unrealistic, but quite acceptable in practice provided $1/10$ is greater than twice the bandwidth $F$ within which the spectrum $G(t)$ of the spherical harmonic is confined. The effective bandwidth for the gravitational potentials $P_{0}$ and $P_{4}$ is a little less than 0.3 cps, which suggests a "time resolution" $\delta t = 1.7$ days approximately. Some numerical tests for Honolulu showed $\delta t = 2$ days to be a good compromise, and this interval was adopted.

This choice of an optimum "time resolution" $\delta t = 1/20$ is somewhat analogous to the choice of frequency resolution $\delta f = 1/100$ (or $1/20$) for a spectrum of a time series of duration $T$. We also have an analogue to the "Nyquist frequency" $f_{y} = 1/(20 t_{i})$, which is the highest frequency which can be determined unambiguously from sampling of time intervals $t_{i}$. In our case, the spectrum $G(t)$, and therefore information about $Z(t)$, is concentrated in narrow groups separated by $1/20$ in $t$, so we have a "Nyquist lag" $1/20 = 1.0$ months, as the largest value of $\delta t$ we could meaningfully consider.

In summary, the band width of the tidal species, and within each species the group interval associated with first order splitting, suggest lag intervals of 2 days up to some maximum lag of 14 days.

e) Credo of smoothness

The greatest time increment we actually allowed was 6 days. This choice depends on the belief that the admittances of the ocean to tidal forces are fairly smooth, and should not require periodicities in $Z(t)$ below 1 day. If the admittances were more wiggly, then consecutive groups, separated by 0.04 day, would have virtually independent admittances, and the classical harmonic method would be equally suitable. However, our results, especially those for Honolulu, do seem to confirm a reasonable degree of smoothness consistent with the adopted limit.

The relative advantage of the convolution method over the harmonic method increases with the smoothness of the admittance and complexity of the spectral input. In the limiting case of a continuum input it is the only possible method; for unresolved lines it is the only practical one. But the advantages may be there even for a resolved line spectrum. At the same time the imposed smoothness of admittance does impose upon the user the responsibility of having generated realistic input functions, and in this sense the convolution method requires more care than the harmonic method.
V. RESULTS FOR HONOLULU

a) The observations

We chose Honolulu for our first analysis, it is an oceanic island relatively free from shallow water effects. About fifty years of hourly readings were available and these have been carefully edited and stored on magnetic tape (*). Tides of diurnal and semi-diurnal species are of about equal intensity. We analyzed most thoroughly the 20 x 355 days period 1938 July 25 to 1938 January 1 ; in the very low frequency range we used all available 52 years, 1905 to 1958. Two short gaps, 11 days in 1950 and 13 days in 1953, and a larger gap of 50 days in 1942, were filled by conventional harmonic prediction with linear trends. The hourly series were smoothed by a low-pass filter with cut-off at 4 cpm, and thinned 3 : 1 to give 8 readings per day. This reduced the bulk of the subsequent computations. The resulting "Nyquist" frequency of 4 cpm is quite high enough, because tidal frequencies above 3 cpm are negligible.

b) The spectrum of cpm resolution

The heights of the columns in the upper two panels of figure 1 refer to the spectra, \( R^2 \) and \( R_0^2 \), of equilibrium \( {}^2 \langle n \rangle \) and recorded sea level, the two time series were subjected to identical filtering processes, and to ensemble averaging over 10 consecutive periods of 355 days (section 4b), leading to estimates of the constituents \( (a, b, c) \) at 1 cpm resolution. The plotted group estimates \( (a, b, c) \) at 1 cpm resolution were obtained by forming the averages

\[
\langle a, b, c \rangle = \frac{1}{10} \sum_{i=1}^{10} a_i, b_i, c_i
\]

over 7 adjoining constituents.

In the equilibrium spectra the significant energy \(-10^4 \text{ cm}^2\) or more) is contained in the groups \((0, 1)\) to \((6, 7)\), \((7, 8)\) to \((1 + 4)\) and \((2 - 4)\) to \((2 + 4)\) with energy gaps between these groups. In the recorded spectra the same groups are prominent, but there are no gaps between. Rather, a plateau of roughly \(10^3 \text{ cm}^2\) per group is attained, corresponding to an energy density of 1 cm²/cpm. Presumably this is the non-tidal continuum which underlies the tidal line spectrum (MUNK and BULLARD, 1963).

The hypothesis can be tested by considering separately the coherent and non-coherent parts of the recorded energy,

\[
\gamma^2 R_0^2 \quad \text{and} \quad (1 - \gamma^2) R_0^2
\]

corresponding to the filled and unfilled portions of the columns. Panea 2 and 3 contain the same information, the two displays being necessary because of the logarithmic scale. In columns containing largely coherent energy the non-coherent portion is perceptible only in panel 3, and

References

(*) The methods are described in "A User's Guide to ROMM: a system of programs for the analysis of time series" by BULLARD, OCLERAY, MUNK and MILLER, Institute of Geophysics and Planetary Physics, University of California, La Jolla, April 1964 (unpublished).

(**) In section 5 we use equilibrium level

\[
\xi_m^2(t) \xi_m^2(\lambda, \varphi, \theta = 65^\circ 42', \lambda = 202^\prime 13',
\]

rather than the gravitational potential \( a_m^2(t) \) for the input function in equation (5.3). For a fixed m, the two functions differ only by a constant amplitude factor and phase shift. The physical meaning of equilibrium level is somewhat clearer and there is some advantage in referring to \( \xi_m^2 \), provided the station does not lie close to a nodal cotitude of one of the spherical harmonics. The discussion of prediction variance (section 6) is based on inputs \( \xi_m^2(t) \), \( a_m^2(t) \).
For long records the mean covariances between different tidal species are zero and the matrices may be split into separate blocks, one for the spherical harmonics contributing to each species m, and inverted separately. The minimum value of $c^2$ is then

$$c^2 = c^2(0) > c^2 - c^2 - 2c^2 - 2c^2 - \ldots$$

We refer to

$$c^2 = \sum_{m} w^m R^m,$$

(4.20)

as the "prediction variance" for species m, with the meaning that the convolution prediction reduces the variance of residual (observed minus predicted) sea level by $c^2$. It is not in general the same as the variance of the prediction itself, which is

$$\langle w^m c^2 \rangle^2.$$

As in spectral analysis, 355 days is a good period for summing the cross-products, and 19 x 355 days is considerably better. In order to compare prediction variances for different combinations of variables, it was found important to compute the mean products of lagged quantities with high precision. For example, if $c_1 = a_1^2(0)$, $c_2 = a_2^2(0)$, $c_3 = a_3^2(0)$, and three hourly values are summed for 355 days, then

$$M_{12} = \sum_{i=1}^{355} a_{1i} a_{2i},$$

and $M_{13}$ may not be assumed equal to $M_{12}$ as in conventional time series analysis. In comparison of prediction variances arrived at by slightly different schemes it was found desirable to form the summed products (19) by integer-arithmetic. The computers used, CDC 1604 and 3600, permit storage of integers up to 15 significant figures.

b) Admittance by sequential and lumped analysis

In the following discussion of tidal observations we have followed two distinct (but closely related) methods of estimating admittance, In the sequential scheme we first perform the cross-spectral analysis between the observed sea level $\zeta(t)$ and the input function $f(t)$ to obtain the ensemble-averaged admittances $Z_{ij}$ (equation 4.8). Smoothed curves are fitted to the point estimates of $D_{ij}$ by least squares, with weights inversely proportional to the sample variances. The second degree harmonics of the gravitational potential multiplied by these smoothed admittances is then subtracted vectorially from the sea-level spectra to form residual spectra. These residuals are then compared with $c^2(t)$ in exactly the same fashion and new residuals formed. These in turn are compared to radiational inputs, etc. The method of multiple input (section 4c) was used when required.

An alternate scheme is to perform a lumped analysis involving all selected input functions in the manner discussed in section 4g). This yields the weight matrix from which the admittances are computed by the appropriate Fourier transforms (equation 4.16). The coherence at different parts of the spectrum is automatically taken into account. Multiple inputs are separated subject to the criterion of smoothness.

The sequential scheme has certain pedagogical advantages, but the lumped analysis is more straightforward and precise and does not impose conditions concerning the relative magnitudes of successive input functions.

V - RESULTS FOR HONOLULU

a) The observations

We chose Honolulu for our first analysis. It is an oceanic island relatively free from shallow water effects. About fifty years of hourly readings were available and these have been carefully edited and stored on magnetic tape (*). Tides of diurnal and semi-diurnal species are of about equal intensity. We analyzed most thoroughly the 20 x 355 days period 1938 July 25 to 1938 January 1, in the very low frequency range we used all available 52 years, 1905 to 1957. Two short gaps, 11 days in 1950 and 13 days in 1933, and a larger gap of 50 days in 1942, were filled by conventional harmonic prediction with linear trends. The hourly series were smoothed by a low-pass filter with cut-off at 4 cpm, and thinned 3:1 to give 8 readings per day. This reduced the bulk of the subsequent computations. The resulting "Nyquist" frequency of 4 cpm is quite high enough, because tidal frequencies above 3 cpm are negligible.

b) The spectrum of cpm resolution

The heights of the columns in the upper two panels of figure 1 refer to the spectra, $E_n^2$ and $E_n^2$, of equilibrium $m/(n = 2)$ and recorded sea level. The two time series were subjected to identical filtering processes, and to ensemble averaging over 19 consecutive periods of 355 days (section 4b), leading to estimates of the constituents $(a_0, a_n)$ at 1 cpy resolution. The plotted group estimates $(a_0, a_n)$ at 1 cpm resolution were obtained by forming the averages

$$\langle a_0 \rangle = a_0 a_n$$

over 7 adjoining constituents.

In the equilibrium spectra the significant energy ($\sim 10^5$ cm$^2$ or more) is contained in the groups (0 1) to (0 2), (1 4) to (1 4) and (2 4) to (2 4) with energy gaps between these groups. In the recorded spectra the same groups are prominent, but there are no gaps between. Rather, a plateau of roughly 10$^4$ cm$^2$ per group is attained, corresponding to an energy density of 1 cm$^2$/cpd. Presumably this is the non-tidal continuum which underlies the tidal line spectrum (MUNK and BULLARD, 1963).

The hypothesis can be tested by considering separately the coherent and non-coherent parts of the recorded energy,

$$E_n^2 = \gamma^2 E_n^2$$

and

$$E_n^2 = (1 - \gamma^2) E_n^2,$$

corresponding to the filled and unfilled portions of the columns.Panels 2 and 3 contain the same information, the two displays being necessitated by the logarithmic scale. In columns containing largely coherent energy the non-coherent portion is perceptible only in panel 3,
vice versa. It will be seen that the coherent recorded energy corresponds closely to equilibrium energy. Non-coherent energy fills the frequency space between prominent groups; furthermore it can be traced along the groups, and there it is found to peak, particularly with respect to the group (2 0) which contains the strong $M_2$ constituent. This is the "tidal cusp", discussed by MUNGE, ZETLIER, and GHIOVES (1965). However, as we shall see later, their simple explanation in terms of interaction with the low frequency continuum does not fit the facts. The relatively large fraction of non-coherent energy in the solar groups (1 1) and (2 2) will be ascribed to radiational processes.

The non-coherent energy contributes a very small fraction to the total energy in species 1 and 2. The reverse situation applies for species 0. The anomalous peak and trough in the continuum at about 0.5 cpd has been attributed by M.S. LONGUEU-HIGGINS (1965) to a cut-off effect in the theory of planetary waves.

The circles in the lower two panels of Figure 1 show hte admittance $Y_1$, $Y_2$, and [Z$_j$], $\text{arg}(Z_j)$, derived from (4.8). The confidence limits (Appendix B) illustrate the inestimability of tidal estimates whenever the energy is below continuum level. The admittances are obviously far from constant within any tidal species, as already known from published tables of phase lag (essentially the same as $\text{arg}(Z_j)$), but the variation in admittance is sufficiently smooth to be simulated by the convolution process discussed earlier. In fact, the plotted curves represent the admittances from a lumped analysis subject to the crede of smoothness; input parameters are given in Table 2, section 6.

The admittance circles for the semi-solar groups (1 1) and (2 2) were derived by a special treatment. Since the strongest lines in the groups contain multiple input functions from non-separable gravitational and radiational potentials, the admittance estimates used for the linear groups are not applicable. However, there are some non-trivial purely lunar lines in these solar groups which modulate the lunar parts of $K_1$ and $K_2$ at the nodal cycle of 18.6 years, the most important being (1 0 0 1) and (2 2 0 0 1). The energy at these lines is certainly not affected by radiation. We isolated these lines and some lesser neighboring lines by analyzing the modulations of the $K_1$ and $K_2$ constituents over 19 successive years, deriving admittances from their cross spectral components as before. The resultant admittances, as plotted at the appropriate frequencies, left a very low residue of "noise". They fit well into the general trend suggested by the neighboring lunar zones.

c) The spectrum at cpy resolution

Figures 2, 3, and 4 show the same spectra and admittances at greater resolution. Here we see the tidal "constituents" named by DARWIN and other constituents of lesser importance, usually unnumbered or allocated to non-linear "over-tides". In figure 2, the constituent labelled MS is the linear lunar input at (0 2 -2), not the $M_2$ or $S_2$, non-linear interaction at the same frequency. The coherent energies at 1 and 2 cpy (Sa and Sb) were not plotted because they include radiational energy and give anomalous admittances. Separation of gravitational and radiational energy is possible by the method of multiple inputs. A better procedure is to lump analysis on which the plotted curves are based. These curves approach a gravitational admittance of $Z = 0.92$ at zero frequency. The reliability of this limit is about the same as that of the estimate at 1 cpy, namely $\pm 0.24$ at the 95% confidence level. The corresponding static limit in terms of the customary Love Number notation is $1 + k = h + l = 1 + 0.290 - 0.59 = 0.70$.

Cusps in the continuum are more prominent at the present resolution, particularly those centered on $M_2$, $K_2$, and $K_1$. Significant departures from the smoothed admittance (as produced by the lumped analysis) can be associated with pairs of constituents separated by 2 cpy (decili- nautical splitting); for example, between (2 2 0 2) and (2 2 2 2), and between (2 1 1 0) and (2 1 1 2). The effects of these anomalies on the overall results of the present study are slight, but they are worthy of further consideration.

The admittance points for the $K_1$ constituent are those derived from the nodal modulations. The small anomaly in the admittance for $P_2$ is an indication of radiation effects; the anomaly could be removed by the method of multiple inputs. Other indications of radiation are the prominent non-coherent residuals of $S_2$ and $S_1$. It is interesting to note that the phase lead at $S_2$ is slightly less than that at $M_2$, making Honolulu one of the relatively rare places (even among oceanic islands) where the "age of the tide" is not positive.

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vice versa. It will be seen that the coherent recorded energy corresponds closely to equilibrium energy. Non-coherent energy fills the frequency space between prominent groups; furthermore it can be traced around the groups, and there it is found to peak, particularly with respect to the group (2 0) which contains the strong M\textsubscript{s} constituent. This is the "tidal cuff", discussed by MUNGE, ZETLER, and GHOVESS (1965). However, as we shall see later, their simple explanation in terms of interaction with the low frequency continuum does not fit the facts. The relatively large fraction of non-coherent energy in the solar groups (1 1) and (2 2) will be ascribed to radiational processes.

The non-coherent energy contributes a very small fraction to the total energy in species 1 and 2. The reverse situation applies for species 0. The anomalous peak and trough in the continuum at about 0.5 cpm has been attributed by M. S. LONGFORD-HIGGINS (1965) to a cut-off effect in the theory of planetary waves.

The circles in the lower two panels of figure 1 show the admittance $X_1$, $Y_1$, and $Z_1$, $\arg(Z_1)$, derived from (4.8). The confidence limits (Appendix B) illustrate the instability of tidal estimates whenever the energy is below continuum level. The admittances are obviously far from constant within any tidal species, as already known from published tables of phase lag (essentially the same as $\arg(Z_1)$), but the variation in admittance is sufficiently smooth to be simulated by the convolution process discussed earlier. In fact, the plotted curves represent the admittances from a lumped analysis subject to the credo of smoothness; input parameters are given in table 2, section 6.

The admittance circles for the bi-nuclear groups (1 1) and (2 2) were derived by a special treatment. Since the strongest lines in the groups contain multiple input functions from non-separable gravitational and radiational potentials, the admittance estimates used for the linear groups are not applicable. However, there are some non-trivial purely linear lines in these solar groups which modulate the lunar parts of $K_1$ and $K_2$ at the nodal cycle of 18.6 years, the most important being (1 1 0 0 1) and (2 2 0 0 1). The energy at these lines is certainly not affected by radiation. We isolated these, and some lesser neighboring lines by analyzing the modulations of the $K_1$ and $K_2$ constituents over 19 successive years, deriving admittances from their cross spectral components as before. The resultant admittances, as plotted at the appropriate frequencies, left a very low residue of "noise". They fit well into the general trend suggested by the neighboring lunar zones.

c) The spectrum at cpm resolution

Figures 2, 3, and 4 show the same spectra and admittances at greater resolution. Here we see the tidal "constituents" named by DARWIN and other constituents of lesser importance, usually unconfirmed or allocated to non-linear "over-tides". In figure 2, the constituent labelled MSF is the linear lunar input at (0 2 -2), not the $M_s - M_p$ non-linear interaction at the same frequency. The coherent energies at 1 and 2 cpm (Sa and Sb) were not plotted because they include radiational energy and give anomalous admittances. Separation of gravitational and radiational energy is possible by the method of multiple inputs. A better procedure is the lumped analysis on which the plotted curves are based. These curves approach a gravitational admittance of $Z = 0.82$ at zero frequency. The reliability of this limit is about the same as that of the estimate at 1 cpm, namely ±0.24 at the 95% confidence level. The corresponding static limit in terms of the customary Xylophone notation is $1+1.4\times0.29 = 0.59 < 0.70$.

Cuffs in the continuum are more prominent at the present resolution, particularly those centered on $M_s$, $K_2$ and $S_2$. Significant departures from the smooth admittance (as produced by the lumped analysis) can be associated with pairs of constituents separated by 2 cpm (decennial splitting); for example, between (2 2 0 2) and (2 2 -2) and between (2 1 0 0) and (2 1 -2). The effects of these anomalies on the overall result of the present study are slight, but they are worthy of further consideration.

The admittance points for the $K_2$ constituent are those derived from the nodal modulations. The small anomaly in the admittance for $P_1$ is an indication of radiation effects; the anomaly could be removed by the method of multiple inputs. Other indications of radiation are the prominent non-coherent residuals of $S_2$ and $S_4$. It is interesting to note that the phase lead at $S_4$ is slightly less than that at $S_2$, making Honolulu one of the relatively rare places (even among oceanic islands) where the "age of the tide" is not positive.

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Figure 2: Honolulu tide spectra at 1 cpy resolution for the low frequency spherical harmonic $P_8^0$. Some "DARWIN" symbols and "DOUG'SON" constituent numbers are included in the upper panel. Legend is otherwise as in figure 1.

Figure 3: Honolulu tide spectra at 1 cpy resolution for the diurnal spherical harmonic $P_1^1$.
Figure 2: Honolulu tide spectra at 1 cpy resolution for the low frequency spherical harmonic $P_2^0$. Some "DARWIN" symbols and "DOUGOR" constituent numbers are included in the upper panel. Legend is otherwise as in figure 1.

Figure 3: Honolulu tide spectra at 1 cpy resolution for the diurnal spherical harmonic $P_1^1$. 

The spectrum at nodal resolution

Figure 5 was derived from two overlapping 37-year records, and represents the highest spectral resolution we attempted (1). We show only the results for species \( N \). Each column has one degree of freedom, as opposed to 19 in figures 2, 3 and 4, and 247 in figure 1. The noise level at the low frequencies is too great for any of the tidal lines to stand out except the (mainly radiational) annual and semi-annual constituents. Three adjoining lines centered at 0.85 cpy are probably associated with the 14 month "pole-tide" due to the wobble of the Earth (Huibrich and Munk, 1959), but their energy can scarcely be said to rise convincingly above the continuum. Even the prominent 18.6 year "nodal tide", \( \mathcal{N} \), is indistinguishable. The dominance of the low-frequency noise is also clear from the smoothed sea-level record plotted in figure 6. Variations with a time scale of decades are principally due to winds and pressure, to changes in sea temperature, to global changes in sea level associated with the melting of glaciers and ice caps, and to the up-and-down movement of continental blocks.

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Figure 4: Honolulu tide spectra at 1 cpy resolution for the semi-diurnal spherical harmonic \( \mathcal{P}_2 \).

Figure 5: Honolulu tide spectra at 1 cpy resolution for the lowest frequencies in spherical harmonic \( \mathcal{P}_2 \). The "DOODSON" numbers in the upper panel refer to \( \mathcal{P}_2, \mathcal{Q}_2, \mathcal{P}_4, \mathcal{Q}_4 \) in equation (2.3). The lower panel shows sea energy for the periods 1955-14-1942, 14 and 1920-86-1957, 86 respectively. A plot of mean sea level covering both periods is shown in figure 6.

(*) As a matter of minor interest, the line at (0 0 1 -1 0) is not included in DOODSON's 1921 schedule, though above his threshold level. This is the only such case we have found.
d) The spectrum at nodal resolution

Figure 5 was derived from two overlapping 37-year records, and represents the highest spectral resolution we attempted (*). We show only the results for species 9. Each column has one degree of freedom, as opposed to 19 in figures 2, 3 and 4, and 247 in figure 1. The noise level at the low frequencies is too great for any of the tidal lines to stand out except the (mainly radiational) annual and semi-annual constituents. Three adjoining lines centered at 0.85 cpy are probably associated with the 14 month "pole-tide" due to the wobble of the Earth (HAUBRICH and MUNN, 1959), but their energy can scarcely be said to rise convincingly above the continuum. Even the prominent 18.6 year "nodal tide" N, is indistinguishable. The dominance of the low-frequency noise is also clear from the smoothed sea-level record plotted in figure 6. Variations with a time scale of decades are principally due to wind and pressure, to changes in sea temperature, to global changes in sea level associated with the melting of glaciers and ice caps, and to the up-and-down movement of continental blocks.

Figure 4: Honolulu tide spectra at 1 cpy resolution for the semi-diurnal spherical harmonic $P_2^o$.

2/2
e) The spectrum for gravitational harmonics of degree 3

Spectra and admittance points in figure 7 have been obtained by sequential analysis, admittance curves by lumped analysis. The energy of $P_3^r$ is generally four orders of magnitude below that of $P_2^r$, but there are significant third degree contributions to groups (1 0) and (2 1), and no predominant contributions to the ter-diurnal species. Apart from the low frequency harmonics there was sufficient coherent energy to yield plausible estimates of admittance. It is interesting to note that these admittances are quite different in magnitude and general shape from the second degree admittances. Therefore, there is no justification in the customary procedure to allow third degree harmonics by combining them with second degree harmonics in the same proportion and phase as they occur in the equilibrium tide.

One is tempted to ascribe the large non-coherent energy in group (3 1) to shallow-water interaction between diurnal and semi-diurnal tides. But such an effect must be dominated by the $M_2$ line (due to interaction between $K_1$ and $M_2$); yet the sea-level spectra at 1 cpy resolution (not reproduced here) show the energy of $M_2$ to be about the same as its neighbors. At any rate, a quite trivial amount of energy is involved. Sea-level energy hardly rises in the 4 cpy zone. All these results confirm that non-linear interaction at Honolulu is unimportant.

f) Radiational tides

Upon subtraction of all gravitational inputs of degrees 2 and 3 we are left with residuals that can be examined for radiational effects. For input functions we use $a_{n0}^{(r)}(1) U_n^r(0, \lambda)$ were $a_{n0}^{(r)}(1)$ is the fractional variation in radiation (Appendices A). The results of such an analysis in the solar groups (0 0), (1 0) and (2 2) is shown in figure 8. The radiational analysis is complicated by the existence of harmonics of degree 1 (these are absent from the gravitational potential). It is seen from the top panel that the radiational inputs of degree 1 are relatively strong for $S_3$ and $S_3^r$; in comparison, the gravitational input into $S_3$ and $S_3^r$ is relatively weak. Radiational spectra of degree 2 resemble the solar gravitational spectra.

Radiational harmonics of degrees 1 and 2 contain precisely the same frequencies (of the form $a_{n0}^{(r)}(1) + a_{n0}(1)$). The two admittances have been separated by the method of multiple inputs assuming constant admittances within groups (0 0) and (1 1). Values are given in the bottom panel. Radiational effects are seen to account for significant amounts of energy at 1 and 2 cpy. Results in the diurnal range are confused, possibly because of inaccuracies in the subtraction of the gravitational effects, but the coherent energy is considerable, particularly at 1 cpy where the gravitational input is trivial.

Figure 6: Honolulu 'mean' sea level, with frequencies above 1 cpy attenuated.

Figure 7: Honolulu tide spectra at 1 cpm resolution for spherical harmonics of degree 3. The display is analogous to that in figure 1, except that degree 2 indices have been removed from the sea energy.
e) The spectrum for gravitational harmonics of degree 3

Spectra and admittance points in figure 7 have been obtained by sequential analysis, admittance curves by lumped analysis. The energy of $P_3^o$ is generally four orders of magnitude below that of $P_1^o$, but there are significant third degree contributions to groups (1 0) and (2 1), and predominantly contributions to the ter-diurnal species. Apart from the low frequency harmonics there was sufficient coherent energy to yield plausible estimates of admittance. It is interesting to note that these admittances are quite different in magnitude and general shape from the second degree admittances. Therefore, there is no justification in the customary procedure to allow for third degree harmonics by combining them with second degree harmonics in the same proportion and phase as they occur in the equilibrium tide.

One is tempted to ascribe the large non-coherent energy in group (3 1) to shallow-water interaction between diurnal and semi-diurnal tides. But such an effect must be dominated by the $MK_2$ line (due to interaction between $K_1$ and $M_2$); yet the sea-level spectra at 1 cpy resolution (not reproduced here) show the energy of $MK_2$ to be about the same as its neighbors. At any rate, a quite trivial amount of energy is involved. Sea-level energy hardly rises in the 4 cpy zone. All these results confirm that non-linear interaction at Honolulu is unimportant.

f) Radiational tides

Upon subtraction of all gravitational inputs of degrees 2 and 3 we are left with residues that can be examined for radiational effects. For input functions we use $c^o_k(1) U^o_0(0, \lambda)$ were $c^o_k(1)$ is the fractional variation in radiation (Appendix A). The results of such an analysis in the solar groups (0 0), (1 1) and (2 2) is shown in figure 8. The radiational analysis is complicated by the existence of harmonics of degree 1 (these are absent from the gravitational potential). It is seen from the top panel that the radiational inputs of degree 1 are relatively strong for $S_2$ and $S_3$; in comparison, the gravitational input into $S_2$ and $S_3$ is relatively weak. Radiational spectra of degree 2 resemble the solar gravitational spectra.

Radiational harmonics of degrees 1 and 2 contain precisely the same frequencies (of the form $a_k \text{ cpy} = a_k \text{ cpy}$). The two admittances have been separated by the method of multiple inputs assuming constant admittances within groups (0 0) and (1 1). Values are given in the bottom panel. Radiational effects are seen to account for significant amounts of energy at 1 and 2 cpy. Results in the diurnal range are confused, possibly because of inaccuracies in the subtraction of the gravitational effects, but the coherent energy is considerable, particularly at 1 cpy where the gravitational input is trivial.
Individual admittances were computed for a radiational input $P_1^R$. There is no input of degree 1, and the only other possible input is a radiational $P_4^R$, which appears to be negligible (though not so insignificant as the gravitational $P_4^R$). Admittances for four constituents are plotted, but some may be unreliable, and a constant admittance derived for $S_4$ would probably be a fair representation.

The radiational sea level energy at 2 c.p.d. may be represented as a wave of 1.8 cm amplitude with maximum occurring 5 h 25 m after the Sun's transit, (i.e., a phase lead of 2.93 radians). The $S_4$ atmospheric pressure tide in the tropics has a maximum close to 10 h, and a minimum 4 h after transit. Therefore, the radiational ocean tide at Honolulu lags the inverse barometer effect of the atmospheric tide by 2 h 25 m, or 1.26 radians. This is reasonably close to the observed phase lag of 1.38 radians for the $S_4$ gravitational tide, suggesting that the ocean responds to the atmospheric pressure wave in much the same way. At the same time there must be direct radiational effects on the oceans, for the pressure amplitude in the tropics is 1.2 mb, and the gravitational admittance is only 0.62. The response to the atmospheric $S_4$ tide must not be confused with the response of sea level to random pressure variations at 2 c.p.d., for these have much larger spatial wave numbers, and little spatial coherence. According to MUNK and BULLARD (1963) the sea level response to the random variations is quasi-static.

VI - PREDICTION VARIANCE

a) Tidal predictions

Table 1 summarizes the observed, predicted, and residual variances of Honolulu sea level $C(t)$. Convolutions of species 0 were computed from a 19 years 12-hourly series derived by low-passing $C(t)$ with 0, 5 c.p.d. cut-off. Other species refer to the 19 x 355 days of 3-hourly series mentioned previously. The first row for each species shows the results for a convolution scheme with a maximum number of input variables; subsequent rows are for optimum predictions when certain variables are successively dropped. Weights corresponding to the top line for each species in Table 1 are listed in Table 2. The lumped analysis leading to the smooth admittance curves refer to these weights.

In species 0, the radiational potentials marked with a prime are

$$\phi^0_1(t) = (365, 242/260) \left[ \phi^1_1(t + 14/360) - \phi^1_1(t - 14/360) \right]$$

and

$$\phi^0_2(t) = (365, 242/400) \left[ \phi^1_2(t + 14/720) - \phi^1_2(t - 14/720) \right]$$

(6.1)

Since $\phi^1$ and $\phi^2$ are almost periodic functions at 1 c.p.d. and 2 c.p.d. respectively (see their spectra in figure 8), the functions defined in (6.1) are approximately their time-derivatives. They are included to accommodate fairly large phase lags arising from thermal inertia, which, unlike gravitational inertia, is important even at such low frequencies. We could have used instead terms like $\phi^1(t - t^1)$ but such a large time lag is awkward to handle.

In the frequency band 0 to 4 c.p.m. (strictly 1/19 c.p.d. to 4 c.p.m.) we expect a large prediction error from the high noise level. In fact, the residual (observed minus predicted) variance is 56% of the observed variance, Without any gravitational inputs the residual is 63%, and so from a point of view of practical prediction the gravitational terms are hardly worth keeping.

The constants used for harmonic prediction were taken from International Hydrographic Bureau Special Publication no. 649. They were derived from four years of sea level records except for $S_4$ and $S_6$, which were derived from thirty years of monthly mean levels. Harmonic
Individual admittances were computed for a radiational input \( P_1^R \). There is no input of degree 1, and the only other possible input is a radiational \( P_1^R \) which appears to be negligible (though not so insignificant as the gravitational \( P_1^G \)). Admittances for four constituents are plotted, but some may be unreliable, and a constant admittance derived for \( S_2 \) would probably be a fair representation.

The radiational sea level energy at 2 cpy may be represented as a wave of 1.8 cm amplitude with maximum occurring 6 h 25 m after the Sun’s transit. (i.e., a phase lead of 2.93 radians). The \( S_2 \) atmospheric pressure tide in the tropics has a maximum close to 10 h, and a minimum 4 h after transit. Therefore, the radiational ocean tide at Honolulu lags the inverse barometer effect of the atmospheric tide by 2 h 25 m, or 1.26 radians. This is reasonably close to the observed phase lag of 1.38 radians for the \( S_2 \) gravitational tide, suggesting that the ocean responds to the atmospheric pressure wave in much the same way. At the same time there must be direct radiational effects on the oceans, for the pressure amplitude in the tropics is 1.2 mb, and the gravitational admittance is only 0.62. The response to the atmospheric \( S_2 \) tide must not be confused with the response of sea level to random pressure variations at 2 cpy, for these have much larger spatial wave numbers, and little spatial coherence. According to MUNK and BULLARD (1963) the sea level response to the random variations is quasi-static.

VI - PREDICTION VARIANCE

a) Tidal predictions

Table 1 summarizes the observed, predicted, and residual variances of Honolulu sea level \( C(t) \). Convolutions of species 0 were computed from a 19 years 12-hourly series derived by low-passing \( C(t) \) with 0.6 cpy cut-off. Other species refer to the 19 x 355 days of 3-hourly series mentioned previously. The first row for each species shows the results for the convolution scheme with a maximum number of input variables; subsequent rows are for optimum predictions when certain variables are successively dropped. Weights corresponding to the top line for each species in table 1 are listed in table 2. The lumped analysis leading to the smooth admittance curves refer to these weights.

In species 0, the radiational potentials marked with a prime are

\[
\sigma_0^+(t) = (365, 242/26) \left[ a_0^+(t + 14/2) - a_0^+(t - 14/2) \right]
\]

and

\[
\sigma_0^+(t) = (365, 242/46) \left[ a_0^+(t + 14/2) - a_0^+(t - 14/2) \right]
\]

Since \( a_0^+ \) and \( a_0^- \) are almost periodic functions at 1 cpy and 2 cpy respectively (see their spectra in figure 8), the functions defined in (6.1) are approximately their time-derivatives. They are included to accommodate fairly large phase lags arising from thermal inertia, which, unlike gravitational inertia, is important even at such low frequencies. We could have used instead terms like \( a_0^+(t - 1/2) \) but such a large time lag is awkward to handle.

In the frequency band 0 to 4 cpm (strictly 1/19 cpy to 4 cpy) we expect a large prediction error from the high noise level. In fact, the residual (observed minus predicted) variance is 56% of the observed variance. Without any gravitational inputs the residual is 63%, and so from a point of view of practical prediction the gravitational terms are hardly worth keeping.

The constants used for harmonic prediction were taken form International Hydrographic Bureau Special Publication no. 649. They were derived from four years of sea level records except for \( S_4 \) and \( S_8 \), which were derived from thirty years of monthly mean levels.
Table 1
Honolulu prediction variances for 1938-1957

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<th>Variances (cm²)</th>
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<tr>
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<td>c14^2</td>
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(*) For example, c11^2 t designates the 6 variables a1^2 t - a1^2 t, a1^2 t - a1^2 t, b1^2 t - b1^2 t, b1^2 t - b1^2 t, b1^2 t - b1^2 t, and the variances without parentheses were used with undagreed t only. c11^2 t refers to the radiational potential of a1^2 t.

(**) The "observed variances" are the energies of c11^2 t in the frequency ranges 0 - 4 cpm, 5 cpm - 6 cpm, and 7 cpm - 8 cpm, respectively. The "total" observed variance is the overall variance of the lowpassed series c11^2 t, including inter-tidal noise.

Table 2
Honolulu harmonic variances

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<td>0.000004</td>
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</tbody>
</table>

The harmonic variances are listed in the order of the period as defined by G.W. Gershberg, who finds some harmonic variances to be very similar.

(*) This harmonic analysis was performed even though we used the program (Vardex) developed by G.W. Gershberg, which permits some of the harmonic constants not normally employed in harmonic constants, but not necessarily employed in Table 1.
### Table 1

<table>
<thead>
<tr>
<th>No of Harmonic</th>
<th>Harmonic Method</th>
<th>Variables (1)</th>
<th>Minimum number of variables listed above</th>
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<td>10</td>
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### Table 2

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<th>v_b</th>
<th>Variable s</th>
<th>u_a</th>
<th>v_b</th>
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<td>0.00314</td>
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</tbody>
</table>

(*) For the zonal radiation harmonics, \( y \) refers to \( a^2_1 \), \( c^2_1 \), equation (5.1).

The convolution method using \( S \) and \( S_{a} \) (hence four station constants \( (*) \) gives a residual error of 14.4 cm. The convolution method with the same number of station constants gives 13.5 cm; with 7 constants it gives 13.0 cm and with 2 constants 14.8 cm. So the convolution prediction does about as well as harmonic prediction with half the station constants, but neither method does very well.

In species 1 and 2 the advantage of the convolution method is more pronounced, but now both methods do very well. For species 1 the harmonic method with 20 station constants has a residual of 0.4 cm as compared to the observed variance 154.8 cm; the convolution method with half the station constants has half the residual variance. (**)
The variables $\gamma^*_{0,1}$ are non-linear terms to study the interaction between the leading gravitational potential and low frequency (< 0.5 cpd) sea level (*). In this manner we had hoped to account for the cusp energies of order 0.1 cm$^2$ at $L_1$ and 1 cm$^2$ at $M_4$ (Figures 3 and 4). In fact the cusp energies were not significantly reduced (see section 9 for further discussion). The input $\gamma^*_{0,1}$ could serve also to predict the result of line-line interactions such as $Sea-M_1$, but these are negligible anyway.

The reduction in $\gamma^*_{0,1}$ when a term is excluded is not necessarily a good test of its importance. The weights derived for the radiational potential $\gamma^*_{0,1}$ ($\eta_2 = -1.88$, $\eta_3 = -1.37$), when multiplied by the appropriate latitude factor, agree very well with the radiational energy of about 1.6 cm$^2$ deduced from the sequential analysis (section 5). The fact that its omission causes $\gamma^*_{0,1}$ to fall by as little as 0.03 cm$^2$ is due to a re-adjustment of the weights of the gravitational terms to compensate for the absence of $\gamma^*_{0,1}$. In effect the estimate of gravitational admittance becomes distorted in the solar frequency group to allow partially for the apparent anomaly. This self-compensation for faulty (or incomplete) input functions does not apply to cusps since these occupy largely non-tidal parts of the spectrum.

The convolutions for species 3 are ineffective in prediction because the energy of the semi-diurnal tides is very small as compared with the continuum energy (which itself is very low). No semi-diurnal harmonic constants are given in the published list.

In assessing the overall prediction variances at the bottom of Table 1, we have to remember that the total variance of $\zeta(t)$ contains also the energy between the bands of tidal frequency, especially in the range between species 0 and 1. Also, the efficiency of the diurnal and semi-diurnal predictions are to some extent marred by the inefficiency at species 0. The noise level at Honolulu is less than in places which experience stronger weather, yet compared to the exceptionally low tides the noise level is rather high, so that the residual ratio, about 9% for the convolution, 10% for harmonic predictions, is higher than the corresponding ratio for typical ports.

**b) Self-Prediction**

We now consider the variation in sea level after removal of all tidal effects. The variance of the residual "sea level" in the frequency band 0-0.5 cpd is 20.8 cm$^2$. Imperfections in the removal of gravitational and radiational components are negligible as compared to this residual, and any significant improvement in prediction must depend on some means of predicting the residual.

Two procedures (and their combination) suggest themselves: (i) to include some meteorological variables $\zeta(t)$ as additional input functions into the prediction formulae (see section 10); and (ii) to make use of the past tide record itself as an input.

Figures 9 and 10 show the power spectrum and auto-covariance of the residual sea level at Honolulu. Nearly all the contribution to the variance comes from frequencies below 12 cpd and, accordingly, the auto-covariance remains large for lags up to one month. With this degree of persistence one expects useful self-prediction for periods up to one month.

For a prediction $\gamma_3$ days in advance we may write

$$\hat{\zeta}(t) = \sum_{\gamma=1}^{\gamma_3} \omega_\gamma \zeta(t - \gamma)$$

with the prediction weights, $\omega_\gamma$, to be determined by the condition that the residual variance be a minimum. The problem is closely analogous to prediction using the tide potential as input function, except that in the present case the input is the (real) process $\zeta(t)$ itself, and that only its past values are permissible.

(*) For the purpose of computing tide prediction tables, the use of recorded sea level $\zeta(t)$ among the input functions would of course be ruled out.
The results are shown in figure 9. The series analyzed consisted of a tidal series sampled at half-day intervals over a period of 18 years. For any given prediction time of \( t \) days, we chose the lags \( \tau \), \( 1.25 \tau, \) \( 1.50 \tau \) in one model, and a single lag \( \tau \) in a second model. The improvement of the triple lag model over the single lag model is surprisingly small. (Prediction with \( \tau, \) \( 1.50 \tau, \) \( 2 \tau \) led to the same conclusion). The result is reminiscent of a Markovian sequence for which optimum prediction is based only on the most recent observed value, a knowledge of values at earlier times being of no use whatsoever in improving the prediction.

We have also attempted to fit the observed spectrum and co-variance by a class of functions which includes the Markovian process as a special case. The best fit was obtained for the Markovian case. Apparently the prediction of residual sea level, as so many geophysical processes, is possible only in this limited sense.

For the single lag model the predicted sea level is

\[
\hat{z}(t) = w_\tau \cdot \zeta(t - \tau)
\]

where

\[
w_\tau = \frac{C(\tau)}{C(0)}
\]

is the auto-correlation. The associated variances are:

<table>
<thead>
<tr>
<th>observed</th>
<th>predicted</th>
<th>residual</th>
<th>residual/observed</th>
</tr>
</thead>
</table>
| \( c^2 \) | \( w_\tau^2 \cdot c^2 \) | \( 1 - w_\tau^2 \) | \( 1 - w_\tau^2 \) \( c^2 \) \( (6,4) \)

Self-prediction 14 days in advance reduces the variance by 20%, but this value.

For one month the reduction is small, from 20%, to 20% - 1.6% - 18%, and for one year the reduction is utterly negligible. Self-prediction is then of no use in improving published Honolulu tide tables. But it is conceivable that records from digital tide gauges can be removed on (say) 24 January, promptly analyzed to give "improved February tide" predictions, and be distributed prior to 1 February to interested parties. Whether this is worthwhile is an operational question.

So far we have dealt with the prediction of instantaneous sea level \( \tau \) days in advance. A prediction of the mean monthly or mean annual sea level can of course be given with greater precision, because smoothing reduces the high-frequency components of the spectrum and extends the covariance to larger lags. Smoothing times and prediction times are independent variables; in the special case that these are equal (e.g., monthly means predicted one month ahead) we have, for a smoothed Markov process,

\[
\bar{w}_\tau = \frac{1}{\gamma + \gamma + \gamma} \cdot \bar{w}_\tau = 1 - \frac{1}{\gamma + \gamma + \gamma}
\]

as compared to \( p_\tau = \bar{w}_\tau \) for the unsmoothed process (MUNK, 1960). The prediction time is lengthened by only 50% for the smoothed process.

VII - NON-LINEAR PERTURBATIONS

It is well known that non-linear effects, particularly those associated with shallow water, can lead to significant distortions of the tidal profile. The departure from linearity varies greatly from place to place, from a small perturbation at San Francisco to a near-bore condition at Avonmouth. Accordingly we may divide ports into three classes, depending on whether

(i) tides are virtually linear,

(ii) tides are analyzable by perturbations up to second or third order,

(iii) perturbations are divergent or otherwise unmanageable.

Honolulu typifies (i); we now consider category (ii), with Newlyn as example. Category (iii) is outside the scope of this paper, but we shall comment in section 10.

Non-linear interactions are accommodated in harmonic prediction by allowing constituents at sums and differences of the frequencies of the main linear constituents. However, the number of additional constituents so required increases rapidly with increasing non-linearity (allowing for "disguised" non-linear constituents; e.g., those whose frequencies coincide with linear constituents), and the \( \mu \) modulating scheme becomes inaccurate. Perturbations to the convolution scheme are better manageable.

The bilinear prediction can be written

\[
\hat{z}(t) = \sum \sum w_{mn} c_m(t - \tau_m) c_n(t - \tau_n)
\]

(7,1)

where \( c_m(t) \) represents the complex linear input functions for any \( m, n \), whether gravitational, radiational, or otherwise. In the double summation, the product \( c_m(t - \tau_m) c_n(t - \tau_n) \) can be of various types:

(i) first order predictions:

\[
\hat{z}(t - \tau) \hat{z}(t)
\]

(ii) various linear inputs:

\[
c_m(t - \tau_m) c_n(t - \tau_n)
\]

(iii) recorded and linear inputs:

\[
\zeta(t - \tau_m) c_n(t - \tau_n)
\]

In procedure (i) we first evaluate \( w_{mn} \) to obtain the linear prediction \( \hat{z}(t) \) according to (6,17). In a second step, \( \hat{z}(t) \) serves as input function into (7,1), and the linear weight \( w_{mn} \) are re-estimated together with \( w_{mn} \), in a combined linear-linear matrix inversion (3). This procedure has the advantage that linear modifications of the tides are already allowed for in the bilinear input functions, and accordingly the biadmittances can be expected to be smoother than for (ii), and require fewer lags. For (ii) the biweights have to absorb both linear and quadratic effects, but there is the advantage that linear and non-linear weights are evaluated in a single step. (iii) is useful for research on non-linear interactions; as a predictor it is limited to the smallest value of \( \tau \).

Whichever of the foregoing schemes is adopted, the bilinear input can be written as a (complex) product of \( c_m(t) \) and \( c_n(t) \), representing various complex (linear) functions belonging to species \( m \) and \( m' \), with \( m \times m' \). We use

\[
c_m c_n^* + c_{m'} c_{n'}^* \quad c_m c_{n'}^* + c_{m'} c_n^*
\]

(7,2)

for a shorthand notation to designate overtones of species \( m \times m' \). The associated biadmittances are given by the two-dimensional Fourier transforms

\[
Z(\omega, \omega') = \sum_{m,n} s_{mn} \exp(-2\pi i(\omega \tau_m + \omega' \tau_n))
\]

(7,3)

At the discrete frequencies \( \omega = \pi/355 \) cpd, \( \omega' = \pi r/355 \) cpd, the foregoing expression equals precisely the estimate

\[
\sum_{m,n} s_{mn} \exp(-2\pi i(\omega \tau_m + \omega' \tau_n))
\]

(7,3)
The results are shown in figure 9. The series analyzed consisted of residual \( C(t) \) sampled at half-day intervals over a period of 19 years. For any given prediction time of \( t \) days, we chose the lags \( \tau_n \), 1.25 \( \tau_n \), 1.50 \( \tau_n \) in one model, and a single lag \( \tau_n \) in a second model. The improvement of the triple lag model over the single lag model is surprisingly small. (Prediction with \( \tau_n \), 1.50 \( \tau_n \), 2 \( \tau_n \), led to the same conclusion). The result is reminiscent of a Markov's sequence for which optimum prediction is based only on the most recent observed value, a knowledge of values at earlier times being of no use whatsoever in improving the prediction.

We have attempted to fit the observed spectrum and co-variance by a class of functions which includes the Markov process as a special case. The best fit was obtained for the Markovian case. Apparently the prediction of residual sea level, as so many geophysical processes, is possible only in this limited sense.

For the single lag model the predicted sea level is

\[
\hat{\zeta}(t) = \xi(t - \tau_n)
\]

where

\[
\xi_n = \frac{<\zeta(t)\zeta(t - \tau_n)>}{<\zeta^2(t)>}
\]

is the auto-correlation. The associated variances are:

<table>
<thead>
<tr>
<th>observed</th>
<th>predicted</th>
<th>residual</th>
<th>residual/observed</th>
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<td>( \zeta^2(t) )</td>
<td>( w_n ) ( \zeta^2(t) )</td>
<td>1 - ( w_n ) ( \zeta^2(t) )</td>
<td>1 - ( w_n ) ( \zeta^2(t) )</td>
</tr>
</tbody>
</table>

(6,4)

Self-prediction 14 days in advance reduces the variance from 26.6 cm² to half this value. For one month the reduction is small, from 26.6 cm² to 26.6 - 7.6 - 19 cm², and for one year the reduction is utterly negligible. Self-prediction is then of no use in improving published Honolulu tide tables. But it is conceivable that records from digital tide gauges can be removed on (say) 24 January, promptly analyzed to give "improved February tide" predictions, and be distributed prior to 1 February to interested parties. Whether this is worthwhile is an operational question.

So far we have dealt with the prediction of instantaneous sea level \( \tau_n \) days in advance. A prediction of the mean monthly or mean annual sea level can of course be given with greater precision, because smoothing reduces the high-frequency components of the spectrum and extends the covariance to larger lags. Smoothing times and prediction times are independent variables; in the special case that these are equal (e.g., monthly means predicted one month ahead) we have, for a smoothed Markov process,

\[
\tilde{w}_n = \frac{1}{1 + \frac{1}{n}}
\]

as compared to \( p_n = \sigma_m \) for the unsmoothed process (MUNK, 1960). The prediction time in lengthened by only 50% for the smoothed process.

VII - NON-LINEAR PERTURBATIONS

It is well known that non-linear effects, particularly those associated with shallow water, can lead to significant distortions of the tidal profile. The departure from linearity varies greatly from place to place, from a small perturbation at San Francisco to a near-bore condition at Avenmouth. Accordingly we may divide ports into three classes, depending on whether

(1) tides are virtually linear,
(2) tides are analyzable by perturbations up to second or third order,
(3) perturbations are divergent or otherwise unmanageable.

H Kundt analyses (1); we now consider category (2), with Newlin as example. Category (3) is outside the scope of this paper, but we shall comment in section 10.

Non-linear interactions are accommodated in harmonic prediction by allowing constituents at sums and differences of the frequencies of the main linear constituents. However, the number of additional constituents so required increases rapidly with increasing non-linearity, (allowing for "disguised" non-linear constituents; e.g., those whose frequencies coincide with linear constituents), and the \( \mu \) modulating scheme becomes inaccurate. Perturbations to the convolution scheme are better manageable.

The bilinear prediction can be written

\[
\hat{\zeta}(t) = \sum_m \sum_n w_{mn} \zeta_m(t - \tau_n) + \sum_m \sum_n w_{mn} \zeta_m(t - \tau_n) \zeta_n(t - \tau_m)
\]

(7,1)

where \( \zeta_m(t) \) represents the complex linear input functions for any \( m, n \), whether gravitational, radiational, or otherwise. In the double summation, the product \( \zeta_m(t - \tau_n) \zeta_n(t - \tau_m) \) can be of various types:

(1) first order predictions : \( \hat{\zeta}(t - \tau_n) \zeta(t - \tau_m) \)
(2) various linear inputs : \( \zeta(t - \tau_n) \zeta_n(t - \tau_m) \)
(3) recorded and linear inputs : \( \zeta(t - \tau_n) \zeta_n(t - \tau_m) \)

In procedure (1) we first evaluate \( w_{mn} \) to obtain the linear prediction \( \hat{\zeta}(t) \) according to (4,17). In a second step, \( \hat{\zeta}(t) \) serves as input function into (7,1), and the linear weight \( w_{mn} \) are re-evaluated together with \( w_{mn} \) in a combined linear-non-linear matrix inversion (4). This procedure has the advantage that linear modifications of the tides are already allowed for in the bilinear input functions, and accordingly the biadmittances can be expected to be smoother than for (ii), and require fewer blights. For (ii) the biweights have to absorb both linear and quadratic effects, but there is the advantage that linear and non-linear weights are evaluated in a single step. (iii) is useful for research on non-linear interactions; as a predictor it is limited to the smallest value of \( \tau_n \).

Whichever of the foregoing schemes is adopted, the bilinear input can be written as a (complex) product of \( \zeta_m(t) \) and \( \zeta_n(t) \), representing various (complex) linear functions belonging to species \( m \) and \( m' \), with \( m > m' \). We use

\[
\zeta_m^* = \zeta_m^* \zeta_n^* \zeta_n^* = \zeta_m^* \zeta_n^*
\]

(7,2)

for a shorthand notation to designate overtones of species \( m \equiv m' \). The associated biadmittances are given by the two-dimensional Fourier transforms

\[
Z(t, f') = \sum_m \sum_n \zeta_m^* \zeta_n^* \exp(-2\pi i (f' \tau_n - f' \tau_m))
\]

(7,3)

At the discrete frequencies \( f = \tau/355 \text{ cph} \), \( f' = \tau'/355 \text{ cph} \), the foregoing expression equals precisely the estimate

\[\text{------}\]

(*) The matrix formalism in section (4) can readily be adopted to include non-linear terms.
derived from the one-dimensional Fourier transforms \( \mathcal{H}_{\mathcal{U}}, G, \mathcal{G}_{\mathcal{U}}, \mathcal{G} \), of \( c^{\mathcal{U}} \), \( c, c^{*} \) respectively.

Trilinear convolutions can be dealt with by an extension of the foregoing analysis.

VIII - RESULTS FOR NEWLYN

a) Procedure and choice of variables

Newlyn \( \left( \phi = 39^\circ 34', \lambda = 5^\circ 34' \right) \) is a typical example of non-linear category (ii). While better exposed to the North Atlantic Ocean than most European ports, it is on a continental shelf some 200 km wide and typically 100 m deep, and this causes an appreciable but not excessive amount of shallow-water distortion.

We analyzed precisely the same 10 x 355 day period as for Honolulu, but in a somewhat different way. The sequential analysis adopted for Honolulu is less effective here because of the many disguised non-linear constituents, (Section 4c, multiple input ii). We therefore carried through only the lumped (convolution) analysis, including bilinear terms. Choice of bilinear variables is simplified by the strong dominance of semi-diurnal tides at Newlyn. Products with \( m = m', n = 1 \) or \( 3 \) can be ignored, so the only bilinear inputs we need consider are \( c_4^* \) and \( c_6^* \) for species 0, 2, and 4, respectively. The procedure of solving the weight matrices was as follows. First a smoothed series with 4 cpd cut-off, 8 values per day, was produced as for Honolulu, and all 10 x 355 days processed to give weights for species 1, 2, and 3. The convolution on \( c_2^* \) (with other variables eliminated) gave the first order prediction \( \hat{c}_2 \) for species 2, which served as bilinear inputs \( \left( \hat{c}_2 \right)^{\mathcal{U}} \) to species 0 and 4. In the subsequent bilinear analysis for species 0, we used 19 years of a twice-daily smoothed series with 0.5 cpd cut-off; for species 4 we used the original hourly readings of \( c \), but for only three 355 days periods, at 6 yearly intervals.

The only published list of harmonic constants for Newlyn at present is I.H.B. sheet no. 1, which is based on only 6 months' analysis of data in 1915. We consider it an unfair test of the harmonic prediction to use this list, and therefore computed our own set of harmonic constants (table 5) from a 710-day record with central data 1948 April 13. Sa and Ss were computed from the whole 19 years of data.

b) Prediction variances

Tables 3 and 4 summarize the results of the lumped analysis for Newlyn. For species 0 we experimented with types (i) and (ii) of bilinear inputs (section 7); the former give \( \hat{c}_2 \) and are based on first order prediction, the latter give \( a_{2}^* \) arising from the linear input functions.

The two types of bilinear input do about equally well. Here it is remarkable how small the difference-frequency is, i.e. order 1 cm\(^2\), as compared to 100 cm\(^2\) for the sum-frequencies producing species 4. As another measure of this disparity we may compare the amplitudes of the harmonic constituents MSF and MS, namely 1.4 and 7.4 cm respectively (*). The residual variance in species 0 is much higher than at Honolulu because of the more severe weather at the higher latitude.

For species 1 we experimented with types (ii) and (iii) of bilinear inputs, \( c_{1}^* \) and \( \hat{c}_{1} \), where \( \hat{c}_{1} \) is the unlagged recorded sea level low-passed to 0.5 cpd. Type (iii) gives somewhat better predictions.

(*) Actually, as cpy resolution MSF is scarcely distinguishable from continuous noise; its amplitude is probably less than 1.4 cm.

---

Table 3

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<th>Species</th>
<th>Number of</th>
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<th>Convolution</th>
<th>Harmonic</th>
<th>Harmonic</th>
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Table 4

<table>
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<th>Variables (x)</th>
<th>Convolution</th>
<th>Harmonic</th>
<th>Harmonic</th>
<th>Harmonic</th>
<th>Harmonic</th>
<th>Harmonic</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>m</td>
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<tr>
<td></td>
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<td>60</td>
</tr>
</tbody>
</table>

---

(*) The symbols in the same as in table 1, except for bilinear terms, \( a_{2}^* \), \( c_{3}^* \), \( c_{4}^* \), \( c_{5}^* \), \( c_{6}^* \). \( \hat{c}_{1} \) represents the three variables \( c_{1}^*, c_{2}^*, c_{3}^* \), respectively.
Table 3

<table>
<thead>
<tr>
<th>Species</th>
<th>Method</th>
<th>Variables (I)</th>
<th>No. of Station constants</th>
<th>Variance (cm²)</th>
<th>Predicted</th>
<th>Residual</th>
<th>Ratio</th>
</tr>
</thead>
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<tr>
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</tr>
<tr>
<td>0</td>
<td>Convolution</td>
<td>(a_0^0, 0, \pm 1), (a_1^0, 0, \pm 1), (a_2^0, 0, \pm 1), (s_1^0, 0, \pm 1), (s_2^0, 0, \pm 1)</td>
<td>10</td>
<td>155</td>
<td>20</td>
<td>135</td>
<td>0.871</td>
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<td></td>
<td>Harmonic</td>
<td>Na, N, Ms, Na</td>
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<td>7</td>
<td>18</td>
<td>136</td>
<td>0.887</td>
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<tr>
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<td>Convolution</td>
<td>(c_0^0, 0, \pm 2, \pm 3), (c_1^0, 0, \pm 2, \pm 3), (c_2^0, 0, \pm 2, \pm 3), (c_3^0, 0, \pm 2, \pm 3)</td>
<td>26</td>
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<td>Harmonic</td>
<td>Na, N, Ms, Ms</td>
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<td>37</td>
<td>37</td>
<td>0.795</td>
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<td>2</td>
<td>Convolution</td>
<td>(c_0^0, 0, \pm 1), (c_1^0, 0, \pm 1), (c_2^0, 0, \pm 1)</td>
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<td>17060</td>
<td>17030</td>
<td>30</td>
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<tr>
<td></td>
<td>Harmonic</td>
<td>Na, N, Ms, Ms</td>
<td></td>
<td>22</td>
<td>17028</td>
<td>17020</td>
<td>0.009</td>
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<tr>
<td>3</td>
<td>Convolution</td>
<td>(c_0^0, 0, \pm 1), (c_1^0(0, 0), (0, -1), (1, 0), (0, 1))</td>
<td>16</td>
<td>1.2</td>
<td>0.7</td>
<td>0.5</td>
<td>0.42</td>
</tr>
<tr>
<td></td>
<td>Harmonic</td>
<td>Na, N, SK</td>
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<td>6</td>
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<td>0.9</td>
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<td>4</td>
<td>Convolution</td>
<td>(c_0^0(0, 0), (0, 0), (-1, 1), (1, 1), (0, 0), (1, 1))</td>
<td>12</td>
<td>103</td>
<td>102</td>
<td>1</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>Harmonic</td>
<td>Na, N, Ms, Ms, Na, S</td>
<td></td>
<td>2</td>
<td>101</td>
<td>100</td>
<td>0.99</td>
</tr>
<tr>
<td>Total</td>
<td>Convolution</td>
<td>All variables listed above</td>
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<td>94</td>
<td>17400</td>
<td>17191</td>
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<td>Harmonic</td>
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<td></td>
<td>70</td>
<td>17209</td>
<td>191</td>
<td>0.011</td>
</tr>
</tbody>
</table>

(*) The symbolism is the same as in Table 1, except for bilinear terms. \(a_0^0(0, 0, \pm 1)\) designates the three variables \(a_0^0(t, \pm 1), a_0^0(t, \pm 1), a_0^0(t, -1)\). Without parenthesis designates \(a_0^0(t, 0)\). \(c_{0,1}^{(2)}\) represents the difference frequencies of the unlagged complex linear predictions for species 2.
Table 4
Newlyn weights

<table>
<thead>
<tr>
<th>Variable</th>
<th>$u_k$</th>
<th>$v_k$</th>
<th>Variable</th>
<th>$u_k$</th>
<th>$v_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_k$</td>
<td>0.1779</td>
<td>-</td>
<td>0</td>
<td>2.5876</td>
<td>3.5872</td>
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<tr>
<td>$b_k$</td>
<td>0.0011</td>
<td>-</td>
<td>1.87964</td>
<td>2.0716</td>
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</tr>
<tr>
<td>$c_k$</td>
<td>0.1916</td>
<td>-</td>
<td>23.6192</td>
<td>17.2928</td>
<td></td>
</tr>
<tr>
<td>$d_k$</td>
<td>10.1900</td>
<td>1</td>
<td>0.004874</td>
<td>34.24955</td>
<td></td>
</tr>
<tr>
<td>$e_k$</td>
<td>10.2773</td>
<td>-</td>
<td>2.51777</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>$(c^2)^{1.2}$</td>
<td>0.46</td>
<td>-</td>
<td>2.42526</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>$(c^2)^{1.3}$</td>
<td>0.25529</td>
<td>0.23613</td>
<td>0.11587</td>
<td>4.14509</td>
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</tr>
<tr>
<td>$(c^2)^{1.4}$</td>
<td>0.47527</td>
<td>0.37177</td>
<td>0.07286</td>
<td>3.22524</td>
<td></td>
</tr>
<tr>
<td>$(c^2)^{1.5}$</td>
<td>0.67590</td>
<td>0.55670</td>
<td>2.42123</td>
<td>0.20819</td>
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</tr>
<tr>
<td>$(c^2)^{1.6}$</td>
<td>0.17575</td>
<td>0.47904</td>
<td>0.080092</td>
<td>0.18129</td>
<td></td>
</tr>
<tr>
<td>$(c^2)^{1.7}$</td>
<td>0.05493</td>
<td>1.22221</td>
<td>0.000159</td>
<td>0.00428</td>
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<tr>
<td>$(c^2)^{1.8}$</td>
<td>0.23117</td>
<td>0.07900</td>
<td>0.00089</td>
<td>0.00288</td>
<td></td>
</tr>
<tr>
<td>$(c^2)^{1.9}$</td>
<td>7.1996</td>
<td>4.56025</td>
<td>0.92303</td>
<td>0.73636</td>
<td></td>
</tr>
<tr>
<td>$(c^2)^{2.0}$</td>
<td>0.0006</td>
<td>0.00012</td>
<td>0.64951</td>
<td>1.31029</td>
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<tr>
<td>$(c^2)^{2.1}$</td>
<td>0.00355</td>
<td>0.00328</td>
<td>0.91930</td>
<td>0.18372</td>
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<tr>
<td>$(c^2)^{2.2}$</td>
<td>0.0153</td>
<td>0.0041</td>
<td>0.00005</td>
<td>0.00029</td>
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</tr>
<tr>
<td>$(c^2)^{2.3}$</td>
<td>0.00047</td>
<td>0.0006</td>
<td>0.000025</td>
<td>0.000013</td>
<td></td>
</tr>
<tr>
<td>$(c^2)^{2.4}$</td>
<td>0.000017</td>
<td>0.00029</td>
<td>0.00005</td>
<td>0.000016</td>
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</tr>
<tr>
<td>$(c^2)^{2.5}$</td>
<td>0.00109</td>
<td>0.00053</td>
<td>0.000004</td>
<td>0.000022</td>
<td></td>
</tr>
<tr>
<td>$(c^2)^{2.6}$</td>
<td>0.0061</td>
<td>1.0989</td>
<td>368.102</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For species 2 the predicted sea level in the binlar input $c^2$ has been smoothed for negative $t$ only, so $c^2$ was effectively $c(t - 3)$. The time lag of three days causes no particular difficulties. The interaction $c^2$ produces cosines of the order of 1 cm², but these account for only a small fraction of the total square of energy. Other types of binlar input are inadequate, since $c^2$, $d^2$, . . . contribute so little to the low frequency sea level.

Prediction variance for species 2 is reduced by 10 cm² by neglecting $\chi^2$. This is not a true measure of the semi-diurnal radiation because the gravitational convolution adapts itself somewhat to the missing input. The weights associated with $c^2$ are seen to be very high, and in fact represent a 2 cph constituent of 33 cm amplitude lagging the Sun’s transit by 9 h 57 min (6.21 radians). The equivalent lag on the inverse atmospheric tide is 5 h 57 min 5.11 radians, and compares favorably with the gravitational phase lag of 2.55 radians corresponding to the $c^2$ convolution, 33 cm may seem to be remarkably large for a non-gravitational tide, but having in mind that the admittance to the $F_2$ spherical harmonic is of order 15 at Newlyn, compared with 0.6 at Honolulu (where we got 1.8 cm radialional $B_2$), we see that it is quite reasonable.

Table 5
Harmonic constants used for Newlyn

<table>
<thead>
<tr>
<th>Constituent</th>
<th>$H$</th>
<th>$\nu$</th>
<th>Constituent</th>
<th>$H$</th>
<th>$\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_a$</td>
<td>5.48</td>
<td>223.4</td>
<td>$2Q_1$</td>
<td>0.43</td>
<td>282.1</td>
</tr>
<tr>
<td>$S_{ma}$</td>
<td>1.57</td>
<td>111.4</td>
<td>$c_1$</td>
<td>0.36</td>
<td>268.1</td>
</tr>
<tr>
<td>$M_{se}$</td>
<td>1.44</td>
<td>158.1</td>
<td>$Q_1$</td>
<td>0.01</td>
<td>291.8</td>
</tr>
<tr>
<td>$M_{fe}$</td>
<td>1.35</td>
<td>223.3</td>
<td>$\tau_1$</td>
<td>0.45</td>
<td>307.1</td>
</tr>
<tr>
<td>$M_0$</td>
<td>0.07</td>
<td>0.060</td>
<td>$O_1$</td>
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<td>339.1</td>
</tr>
<tr>
<td>$M_1$</td>
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<td>274.2</td>
<td>$M_3$</td>
<td>0.33</td>
<td>319.0</td>
</tr>
<tr>
<td>$S_{ka}$</td>
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<td>07.0</td>
<td>$\tau_1$</td>
<td>0.33</td>
<td>099.5</td>
</tr>
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<td>$M_{na}$</td>
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<td>117.4</td>
<td>$P_1$</td>
<td>0.35</td>
<td>079.0</td>
</tr>
<tr>
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<td>145.1</td>
<td>$S_1$</td>
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<td>014.4</td>
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<td>$J_1$</td>
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</tr>
<tr>
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<td>1.03</td>
<td>259.7</td>
<td>$2M_3$</td>
<td>0.90</td>
<td>007.6</td>
</tr>
</tbody>
</table>

$H$ (cm) and $\nu$ (degrees) are in the conventional notation.

For species 4 we note that a binlar input $(c^2)^{2.5}$ gives almost as large a predicted variance with unlagged $t$ as $(c^2)^{1.2}$ with 6 lag-pairs, and does considerably better than $(c^2)^{1.0}$ alone. This is because the linear admittance for species 2 is rather irregular, and so its product cannot be well simulated with just one lag pair.

In all species except 2, the convolution predictions are slightly better than the harmonic predictions, even with a lesser number of constants. In species 2 the harmonic method is superior. The reason seems to be that the harmonic method allows for triple interactions, explicitly as in the case of the trilinear constituents $M_2$, $S_2$, and $2M_2$, and implicitly as in the case of $2S_2$, which is ignored under $\nu_2$. We see from table 5 that the neighboring constituents $2N_2$ and $\nu_2$ differ in phase by $\nu^2$, presumably due to the multiple inputs $2N_2$ and $2M_2$; $\nu_2$ and $2M_2$, but a "jumping" admittance (section 5c) may be partly responsible. Since $2N_2$ and $\nu_2$ differ in frequency by only 2 cpy, our smoothed admittances cannot adapt to such rapid changes, and so they become inaccurate for both constituents.

With regard to the overall picture, we note that at Newlyn the energy of the low frequency residual is five times that at Honolulu, and the tidal energy fifty times as large. As a result the overall residual ratio is less at Newlyn than Honolulu, 1% as compared to 10%. The harmonic prediction does slightly better because of its advantage in species 2 as discussed; the precision is nearly the same with 70 harmonic constants than it is with 39 convolution weights. With triple interactions included, particularly the variety $(2 + 2 + 2)$, we estimate convolution residual of about 180 cm² as compared to 191 cm² for the harmonic residuals.

On subtracting low frequency tidal effects from the lowpassed $c(t)$ with cut-off at 0.5 cph we obtained a residual series with variance 148 cm². Autocovariances and self-prediction variances for this series are shown in figure 11. The persistence is less at Newlyn because of the larger contribution by "weather" to the continuum at 1 or 2 cycles per week (compare figure 12, panel 5 with figure 1, panel 3). Consequently, the self-prediction variance for Newlyn falls off much more rapidly with prediction time and is negligible for a 10 day prediction.
Table 4

<table>
<thead>
<tr>
<th>Variable</th>
<th>$u_4$</th>
<th>$v_4$</th>
<th>$u_3$</th>
<th>$v_3$</th>
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<tr>
<td>$a_1^2$</td>
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<td>0.23613</td>
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</tr>
<tr>
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<td>0.47527</td>
<td>0.37177</td>
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<td>0.55067</td>
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</tr>
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<td>0.07906</td>
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<td>4.56255</td>
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</tr>
<tr>
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</tr>
<tr>
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<td>0.00053</td>
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</tr>
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</table>

Table 5

Harmonic constants used for Newlyn

<table>
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<tr>
<th>Constituent</th>
<th>$H$</th>
<th>$K$</th>
<th>$H$</th>
<th>$K$</th>
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<tr>
<td>Sa</td>
<td>5.482</td>
<td>223.4</td>
<td>0.432</td>
<td>282.1</td>
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<td>1.541</td>
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<td>0.011</td>
<td>291.8</td>
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<tr>
<td>MF</td>
<td>1.352</td>
<td>223.3</td>
<td>0.452</td>
<td>307.1</td>
</tr>
<tr>
<td>M1</td>
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<td>5.292</td>
<td>339.1</td>
</tr>
<tr>
<td>MK1</td>
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<td>274.2</td>
<td>0.332</td>
<td>319.0</td>
</tr>
<tr>
<td>SK1</td>
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<td>070.0</td>
<td>0.332</td>
<td>095.9</td>
</tr>
<tr>
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<td>117.4</td>
<td>2.392</td>
<td>097.0</td>
</tr>
<tr>
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<td>0.232</td>
<td>014.4</td>
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<tr>
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<td>Sn</td>
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<td>259.7</td>
<td>-</td>
<td>-</td>
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</tbody>
</table>

H (cm) and $K$ (degrees) are in the conventional notation.

For species $2$ we note that a binocular input $1^{(2+2)}$ gives almost as large a predicted variance with unlagged $t$ as $c_2^{12}$ with $6$ lag-pairs, and does considerably better than $c_2^{12}$ (0.0) alone. This is because the linear admittance for species $2$ is rather irregular, and so its product cannot be well simulated with just one lag pair.

In all species except $2$, the convolution predictions are slightly better than the harmonic predictions, even with a lesser number of constants. In species $2$ the harmonic method is superior. The reason seems to be that the harmonic method allows for triple interactions, explicitly as in the case of the nonlinear admittance $MNS_2$ and $2M_3$, and implicitly as in the case of $2M_3$, which is disregarded under $i_2$. We see from table 5 that the neighboring constants $2N_2$ and $i_2$ differ in phase by $2^*$, presumably due to the multiple inputs $2N_2$ and $2M_3$, and $i_2$ in frequency by only $2$ cpy. Our smoothed admittances cannot adapt to such rapid changes, and so they become inaccurate for both constants.

With regard to the overall picture, we note that at Newlyn the energy of the low frequency residual is five times that at Honoholu, and the tidal energy fifty times as large. As a result the overall residual ratios are less at Newlyn than Honolulu, 1% as compared to 10%. The harmonic prediction does slightly better because of its advantage in species $2$ as discussed; the precision is nearly the same with $70$ harmonic constants than it is with $39$ convolution constants. With triple interactions included, particularly the variety $(2 + 2 + 2)$, we estimate convolution residuals of about $180$ cm$^2$ as compared to $191$ cm$^2$ for the harmonic residuals.

On subtracting low frequency tidal effects from the low-passed $c(t)$ with cut-off at $0.5$ cpy we obtained a residual series with variance $148$ cm$^2$. Autocovariances and self-prediction variances for this series are shown in figure 11. The persistence is less at Newlyn because of the larger contribution by "weather" to the continuum at 1 or 2 cycles per week (compare figure 12, panel 5 with figure 1, panel 3). Consequently, the self-predicted variance for Newlyn falls off much more rapidly with prediction time and is negligible for a 10 day prediction.
c) Spectral composition and admittances

Figure 12 is the spectral representation of the convolution referred to in tables 3 and 4. The spectrum of three linear inputs was multiplied by the squared admittance corresponding to the convolution weights in table 4 to give the output spectra in the top three panels. Dispectra were computed and multiplied by the appropriate biadmittance, and the mean energies plotted in the fourth panel. The fifth panel is very similar to the third panels of figures 1 and 7, except that the residual energy is the mean energy of \( \zeta(t) \) after subtraction of the vector sum of all four components shown above. All ensemble averages are over 19 years and 13 adjacent cppy harmonics.

Cusp-type residuals are conspicuous at species 2 and 4. The inaccuracies due to neglect of triple interactions can be seen as occasional bumps in the lesser groups of species 2. The largest of these bumps is at (2-2) which contains the constituents \( \mu^2 \) and \( 2N_2 \) mentioned in (b). If it were removed by including triple interaction, the residual variance in table 3 would be reduced by about 15 cm². Another large bump of 18 cm² in the \( M_4 \) group (2-0) could probably be removed by allowing triple interaction.

All the linear admittances to gravitational inputs are plotted in the bottom diagram. \( F_1^3 \) was allowed only unlagged \( \tau \), so the equivalent constant admittance is printed. The admittance curves for \( F_3 \) and \( F_5 \) are noticeably more wiggly than the corresponding curves for Honolulu. The steep decrease of phase for \( F_3^3 \), about 34 radians/cpd, is remarkable. It implies that a smoother admittance would be obtained by using \( c_{1}^3 (1 - 5\tau) \) instead of \( c_{1}^3 (\tau) \).

Some irregularities in the \( F_2^2 \) curve are result of attempted compensation for the triple interactions, especially at the edges of the band. The isolated peak at (2.1) followed by a trough at the solar group (2-2) is reflected in the unusually large \( L_2 \) constituent and the fact that the radiational term accounts for a fair proportion of the \( S_2 \) constituent. (The true gravitational \( S_2 \) has amplitude 78 cm and phase -146°). It might be suspected that the trough at (2-2) is an analytical defect, allowing \( F_2^2 \) to contribute too much to the group. That this is not so was proved by estimating the gravitational admittance directly from the purely lunar lines such as (2 2 0 0 1), which were also used for Honolulu. The resulting admittance
e) Spectral composition and admittances

Figure 12 is the spectral representation of the convolution referred to in the previous section. The spectrum of three linear inputs was multiplied by the squared admittance corresponding to the convolution weights in Table 4 to give the output spectra in the top three panels. Bispctra were computed and multiplied by the appropriate biadmittance, and the mean energy plotted in the fourth panel. The fifth panel is very similar to the third panels of Figures 1 and 7, except that the residual energy is the mean energy of $\xi(t)$ after subtraction of the vector sum of all four components shown above. All ensemble averages are over 10 years and 13 adjacent cpy harmonics.

Cusp-type residuals are conspicuous at species 2 and 4. The inaccuracies due to neglect of triple interactions can be seen as occasional bumps in the lesser groups of species 2. The largest of these bumps is at (2-2) which contains the constituents $p^1$ and $2N_1$ mentioned in (b). If it were removed by including triple interaction, the residual variance in Table 3 would be reduced by about 15 cm$^2$. Another large bump of 10 cm$^2$ in the $M_2$ group (2 0) could probably be removed by allowing triple interaction.

All the linear admittances to gravitational inputs are plotted in the bottom diagram. $P_2^0$ was allowed only unlagged t, so the equivalent constant admittance is printed. The admittance curves for $P_2^1$ and $P_2^2$ are noticeably wiggly than the corresponding curves for Honolulu. The steep decrease of phase for $P_2^1$, about 34 radians/cpd, is remarkable. It implies that a smoother admittance would be obtained by using $c_2^1(t - 8^9)$ instead of $c_2^1(t)$.

Some irregularities in the $P_2^2$ curve are a result of attempted compensation for the triple interactions, especially at the edges of the band. The isolated peak at (2 1) followed by a trough at the solar group (2 0) is reflected in the unusually large $L_2$ constituent and the fact that the radiational term accounts for a fair proportion of the $S_2$ constituent. (The true gravitational $S_2$ has amplitude 70 cm and phase -146°). It might be expected that the trough at (2 0) is an analytical defect, allowing $P_2^2$ to contribute too much to the group. That this is not so was proved by estimating the gravitational admittance directly from the purely linear lines such as (2 2 0 0 1), which were also used for Honolulu. The resulting admittance...
\[ X = -12.3, \ Y = -2.9, \ |Z| = 12.9, \ \text{arg}(Z) = -3.23 \]

comparing favorably with the values
\[ X = -13.0, \ Y = -5.0, \ |Z| = 14.4, \ \text{arg}(Z) = -3.23 \]

from the admittance curve in figure 12 at the frequency of (2 2 0).

It is interesting to note that the admittances to \( P_1^2 \) and \( P_4^2 \) are, like the admittances to \( P_4 \), of order 10 in magnitude, even though their resulting energies are still very small.

IX - CUSPS AND JITTER

At Honolulu and Newlyn the continuum spectrum was found to rise sharply in the vicinity of the strong lines. The simplest explanation is the one proposed by MUNKE, ZETTLER and GROVES (1965), that the cusps represent the saddlebends due to the modulation of a carrier (the tides) by a low-frequency noise (the continuum at very low frequencies). Accordingly we expected significant bilinear coherences for the noise-line interaction \( \zeta_c \). The computed bilinear coherences were insignificant. We have considered and rejected three possible explanations,

(i) The cusp is merely a result of some numerical defect in the computing procedure. We generated an artificial series of 350 days, consisting of a harmonic tide prediction plus a random "noise" with the appropriate low frequency enhancement. All dimensions corresponded roughly to the sea spectrum at Honolulu. The artificial record was analyzed in the same manner as the Honolulu record. The non-coherent energy is virtually uniform in the neighborhood of the strong tidal lines,

(ii) The cusp energy is due to non-linearity in the tide gauge itself, as a result of clock errors or octopus tenacles in the orifice, etc. A year's record from a precision bottom-placed pressure recorder at La Jolla (SNODGRASS, 1964) yields a cusp centered on group (2 0) of about 1 cm² energy, very much as at Honolulu,

(iii) The cusp is due to global, rather than local, interaction, so that there is no relation between the cusps and the locally recorded continuum near zero frequency. But then there should still be relationships between cusps at two adjoining stations, and between the two sides of a cusp at a single station. We isolated the cusp spectra at cpy resolution from 7 years of simultaneous tide records at Honolulu and Kahaluu (an island port about 100 km south-east of Honolulu), and compared cross-spectra and coherences. The energy of the cusp at Kahaluu is about half that at Honolulu, although the tidal amplitudes are about the same. The cross-spectra at cpy resolution bear no uniform relation, and the coherence from ensemble averaging over 7 record years and 19 neighboring cpy harmonics centered on \( M_4 \) is practically zero. Since the separation of the ports is a small fraction of a tidal wave length, we should expect high coherence under hypothesis (iii). In another test we examined separately the left and right sides of the cusp about \( M_4 \) at Hawaii. Upon demodulation, each the side bands was expected to carry approximately the same information. The test was negative.

As a result of these tests we have no model of the noise-line interaction that presumably produces the cusps. There is also the question as to why the bilinear line-line interactions at Newlyn are more effective in producing sum frequencies than difference frequencies. This may simply be a case of selective amplification at higher frequencies, and the computed admittances (figure 12) generally support this. But the sum/difference discrepancy may also be the result of the non-linear processes themselves.

The derivation of the port admittances now provides a challenge for interpreting these in terms of tidal dynamics and offshore topography. For Newlyn the admittances are surprisingly complex. At Honolulu the admittances are reasonably smooth, particularly when viewed at cpy resolution. But even there appears to be a significant "jitter" between values separated by 2 cpy. This perhaps suggests that declinational splitting is not properly taken into account in the input functions. We have considered and rejected the following possible expla-

\[ X - \text{FURTHER REMARKS CONCERNING PREDICTION} \]

Ultimately the prediction scheme for a given port must depend upon the relative contribution from "noisy" processes (processes with a continuum spectrum) and from non-linear processes.

For noisy processes one is limited to short range predictions, depending on persistence. The prediction times might be of the following order:

- storm tides, 1 day
- changes in level due to river runoff, 1 week
- "mean" sea level, 1 month

The appropriate formalism is

\[ \tilde{h}(t) = \sum \sum w_{is} c_i(t - \tau_{is}) \]  

(1)

where the input processes \( c_i(t) \) may include atmospheric pressure and wind at strategic locations, sea level at various ports (including the port in question, see section 6b on self prediction), etc. The weights \( w_{is} \) are determined from post records by matrix inversion (section 4g). The problem is complicated by the fact that the input functions will in general be partially coherent with one another. Thus, the optimum weights \( w_{is} \) associated with any particular input process will depend on what other processes are included. But all this can be sorted out. The principal task for a good short range prediction of storm tides will be the establishment of a communications network to supply the appropriate input functions into a computer for "real time" processing and prediction.

With regard to nonlinearity we have considered the cases of

(i) Honolulu, linear prediction adequate,

(ii) Newlyn, prediction should allow for bilinear (and trilinear) interactions; it is not practical to go to higher order interactions. For the case of

(iii) wholly non-linear stations,

we are considering a method of predicting for the "equivalent deep water port", a flotilla station so defined as to have tides identical with the actual port if non-linear processes were lacking. The transformation from the predicted equivalent tide \( \tilde{h}(t) \) to the predicted actual tide \( \tilde{h}(t) \) is afforded by introducing delay and distortion terms:

\[ \tilde{h}(t) = \tilde{h}(t - \tau) \]  

(2)

\[ \tilde{h}(t) = h(t) + \tilde{h}(t - \tau) \]  

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with \( \eta \) and \( \tau \) functions not of frequency, but of elevation \( \zeta(t) \) (as in the method of characteristics). The proposed development has some analogy to our suggestion for replacing the har-

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(1) This hypothesis was tested by subtracting the tidal effect at (0 2 2) cpy with the admittance derived at 0 0, then examining the coherence of the residual energy with the appropriate product of the \( M_4 \) and \( P_4 \) admittances; the result was insignificant.
X = 12.3, Y = -3.9, |Z| = 12.9, arg (Z) = -3.83

comparatively with the values

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from the admittance curve in Figure 12 at the frequency of (2 2 0).

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IX - CUSPS AND JITTER

At Honolulu and Newlyn the continuum spectrum was found to rise sharply in the vicinity of the strong slines. The simplest explanation is the one proposed by MUNK, ZETLIER and GROVES (1963), that the cusps represent the sidebands due to the modulation of a carrier (the tides) by a low-frequency noise (the continuum at very low frequencies). Accordingly we expected significant bilinear coherences for the noise-tide interaction Z, c. The computed bilinear coherences were insignificant, We have considered and rejected three possible explanations,

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- neglect of higher order terms in the lunar orbit theory;
- neglect of spherical harmonics of degree 3;
- neglect of the Earth's oblateness;
- statistical sampling;
- interaction effects (e.g., M2 and P1 could interact to produce a line 2 cpy above 0/1).

X - FURTHER REMARKS CONCERNING PREDICTION

Ultimately the prediction scheme for a given port must depend upon the relative contribution from "noisy" processes (processes with a continuous spectrum) and from non-linear processes.

For noisy processes one is limited to short range predictions, depending on persistence. The prediction times might be of the following order:

- storm tides, 1 day
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- "mean" sea level, 1 month

The appropriate formalism is

\[ \tilde{\xi}(t) = \sum_{i} \sum_{j} w_{ij} c_{ij}(t - r) \]  

(1)

where the input processes \( c_{ij}(t) \) may include atmospheric pressure and wind at strategic locations, sea level at various ports (including the port in question, see section 6B on self prediction), etc. The weights \( w_{ij} \) are determined from post records by matrix inversion (section 6G). The problem is complicated by the fact that the input functions will in general be partially coherent with one another. Thus, the optimum weights \( w_{ij} \) associated with any particular input process will depend on what other processes are included. But all this can be sorted out. The principal task for a good short range prediction of storm tides will be the establishment of a communications network to supply the appropriate input functions into a computer for "real time" processing and prediction.

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we are considering a method of predicting for the "equivalent deep water port", a flotillas station so defined as to have tides identical with the actual port if non-linear processes were lacking. The transformation from the predicted equivalent tide \( \tilde{\xi}_{e}(t) \) to the predicted actual tide \( \tilde{\xi}(t) \) is afforded by introducing delay and distortion terms:

\[ \tilde{\xi}(t) = \tilde{\xi}_{e}(t - \tau) \]  

(2)

\[ A \quad \text{with } a \text{ and } \tau \text{ functions not of frequency, but of elevation } \xi_e(t) \text{ (as in the method of characteristics).} \]

The proposed development has some analogy to our suggestion for replacing the har-

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This hypothesis was tested by subtracting the tidal effect at \( \theta = 2 \text{ cpy} \) with the admittance derived at \( \theta \), then examining the coherence of the residual energy with the appropriate product of the \( M_2 \) and \( P_1 \) harmonics; the result was insignificant.
monic by a convolution scheme. This avoids considerations of the splitting of each species into a spectral cluster. Here we propose to avoid even the decomposition into species as far as the non-linear effects are concerned, with the assumption that the propagation of a tide across a shallow shelf depends only on the instantaneous elevation of water level at the outer edge.

Severe nosiness and non-linearity tend to go together in shallow water ports, and one may concomitantly a combination of the schemes sketched in this section. But it is by no means clear whether a perturbation scheme can be devised which converges to a separation of linear from non-linear effects. One may have to resort to off-shore measurements of tides, predict these, and allow for non-linearity by direct comparison of off-shore and on-shore tides. Concerning such efforts, it seems appropriate to close with the remarks of Hilaire Belloc "When they pontificate on the tides it does no great harm, for the salarman cares nothing for their theories, but goes by real knowledge".

This study has been generously supported by the U.S. COAST and Geodetic SURVEY, and by the National Science Foundation.

APPENDIX A. DERIVATION OF INPUT FUNCTIONS

a) Gravitational potential

The gravitational potential resulting from a mass M (K or @) at a test point P(r, θ, λ) is

\[ V = \frac{GM}{\rho} \]

where G is the gravitational constant, and ρ the distance PM. Expanding in terms of the parallax \( \xi = \frac{\rho - R}{R} \), we have (figure 13):

\[ V = \frac{GM}{\rho} \left( 1 - 2\xi \mu + \xi^2 \right) = \frac{GM}{R} \left( 1 + \xi \mu + \frac{1}{2} \xi^2 \right) P'_1(\mu) \]  

(1)

Figure 13: a is the zenith angle of M (Moon or Sun) at a point P on the Earth's surface, Z is the polar angle and L the terrestrial east longitude of M, θ, λ are polar angle (colatitude) and longitude of P, r = OP is the distance from the Earth's center to P, and R = GM the distance to M, \( \rho = P\theta \) the distance from P to M.

\[ P_\nu (\mu) = \frac{1}{2^\nu (\nu)!} \frac{d^\nu}{d\xi^\nu} \left( \mu^2 - 1 \right) \]

are the Legendre polynomials:

\[ P_0 = 1, \quad P_1 = \mu, \quad P_2 = \frac{3}{2} \mu^2 - \frac{1}{2}, \quad P_3 = \frac{5}{2} \mu^3 - \frac{3}{2} \mu, \ldots \]

The first term in (1) is a constant of no interest. The second term, \( (GM/R^3)\cos \alpha \), represents a uniform force \( GM/R^2 \) in the direction \( M \to P \) on all points of the Earth; this enters in the Keplerian equations for orbital motion and is not part of the tidal effect.

Now write \( G = ga^2/M \) where \( a \) is the Earth radius and \( M \) its mass. The equilibrium tide can be written

\[ \frac{\nu}{g} = \sum_{\nu=0,2,4,\ldots} \frac{\nu^{\nu+1}}{\nu + 1} P_\nu (\mu) = \sum_{\nu=0,2,4,\ldots} K_\nu \frac{R^\nu}{g} \mu^\nu P_\nu (\mu), \quad K_0 = \frac{a}{R} \]

(2)

where \( \mu = a/R \). For the Moon and Sun, \( K_0 = 35,785 \) cm and 16,427 cm, respectively (*). This is the usual formulation (SHREMB, 1941, p. 119, DOODSON (**), 1949, p. 307).

b) Radiational function

We define a radiational function

\[ R = S \left( \frac{R}{g} \right) \cos \alpha \text{ in day time, } 0 < \alpha < \frac{1}{2} \pi \]

\[ 0 \text{ in night time, } \frac{1}{2} \pi < \alpha < \pi \]

where \( S = 1.946 \text{ cal cm}^{-2} \text{ min}^{-1} \) is the "solar constant". Using the expansion for \( 1/\mu \) as in equation (1), the day time value can be written

\[ R = \sum_{\nu=0}^{\infty} \frac{\nu^{\nu+1}}{\nu + 1} P_\nu (\mu) = S \left( \frac{R}{g} \right) \mu (1 + \xi \mu + \ldots), \quad 1 > \mu > 0 \]

plus terms of order \( \xi^2 \) which are negligible (\( \xi^2 \approx 1/23,455 \)). Expanding \( R \) in spherical harmonics,

\[ R = S \left( \frac{R}{g} \right) \sum_{\nu=0}^{\infty} \mu^\nu P_\nu (\mu) \]

we find

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(*) These figures are derived from the estimates quoted by ALLISON(1963): \( M_0/M_\odot = 81.33 \pm .09 \), \( M_M = 33270 \) \( M_\odot \), \( a = 6371,0 \) km. The Moon's and Sun's mean equatorial horizontal parallaxes are \( \xi = 342252 \) and \( 61944 \), respectively. The ellipticity of the Earth is ignored in this derivation.

(**) DOODSON uses \( 3/2 \xi \).
monic by a convolution scheme. This avoids considerations of the splitting of each species into a spectral cluster. Here we propose to avoid even the decomposition into species as far as the non-linear effects are concerned, with the assumption that the propagation of a tide across a shallow shelf depends only on the instantaneous elevation of water level at the outer edge.

Severe noisiness and nonlinearity tend to go together in shallow water ports, and one may contemplate a combination of the schemes sketched in this section. But it is by no means clear whether a perturbation scheme can be devised which converges to a separation of linear from non-linear effects. One may have to resort to off-shore measurements of tides, predict these, and allow for non-linearity by direct comparison of off-shore and on-shore tides. Concerning such efforts, it seems appropriate to close with the remarks of Hilaire Belloc "When they pontificate on the tides it does no great harm, for the sailorman cares nothing for their theories, but goes by real knowledge".

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$$
V = \frac{GM}{\rho} \left(1 - 2\xi + \xi^2\right) = \frac{GM}{R} \left[1 + \xi + \frac{\xi^2}{2} \right]
$$

(1)

Figure 13: $\alpha$ is the zenith angle of $M$ (Moon or Sun) at a point $P$ on the Earth's surface, $Z$ is the polar angle and $L$ the terrestrial east longitude of $M$, $\theta, \lambda$ are polar angle (colatitude) and longitude of $P$, $r = OP$ is the distance from the Earth's center to $P$, and $R = GM$ the distance from $P$ to $M$.

where $\mu = \cos \alpha$, and

$$
P_\ell (\mu) = \frac{1}{2\ell!} \frac{d^\ell}{d\mu^\ell} \left(\mu^2 - 1\right)^\ell
$$

are the Legendre polynomials:

$$
P_0 = 1, \quad P_1 = \mu, \quad P_2 = \frac{3}{2} \mu^2 - \frac{1}{2}, \quad P_3 = \frac{5}{2} \mu^3 - \frac{3}{2} \mu, \ldots
$$

The first term in (1) is a constant of no interest. The second term, $GM/R^2 \cos \alpha$, represents a uniform force $GM/R^2$ in the direction $M_{\beta}$ to $M$ on all points of the Earth; this enters in the Keplerian equations for orbital motion and is not part of the tidal effect.

Now write $G = ga^2/M_a$ where $a$ is the Earth radius and $M_a$ its mass. The equilibrium tide can be written

$$
\frac{V}{a^2} = \frac{M}{a \sum \frac{a}{R_a}} \sum \frac{\xi^{\ell+1}}{\ell+1} P_\ell (\mu) = \sum \frac{R}{a} \sum \frac{1}{a \cdot a} K_\ell P_\ell (\mu), \; K_\ell = \frac{M}{a \sum \frac{a}{R_a}} \xi^{\ell+1}
$$

(2)

where $\xi = (a/R)$. For the Moon and Sun, $K_2 = 35,785$ cm and 16,427 cm, respectively (*). This is the usual formulation (SHURMAN, 1941, p. 119, DOODSON (**), 1949, p. 307).

b) Radial function

We define a radial function

$$
R = S \left(\frac{R}{a}\right) \cos \alpha \quad \text{in day time, } 0 < \alpha < \frac{1}{2} \pi
$$

$$
R = S \left(\frac{R}{a}\right) \cos \alpha \quad \text{in night time, } \frac{1}{2} \pi < \alpha < \pi
$$

where $S = 1,946$ cal cm$^{-2}$ min$^{-1}$ is the "solar constant". Using the expansion for $1/\mu$ as in equation (1), the day time value can be written

$$
S \left(\frac{R}{a}\right) \mu \sum_{\ell=0}^\infty P_\ell (\mu) = S \left(\frac{R}{a}\right) \mu (1 + \xi \mu + \ldots), \quad 1 > \mu > 0
$$

plus terms of order $\xi^2$ which are negligible ($\xi^2 = 1/32, 455$). Expanding $R$ in spherical harmonics,

$$
R = S \left(\frac{R}{a}\right) \sum_{\ell=0}^\infty \mu \sum_{\mu} P_\ell (\mu)
$$

we find

(*): These figures are derived from the estimates quoted by ALLISON (1935): $M_a/M_M = 81,33 \pm 0.9, M_a = 34,276 M_M$.

(**) DOODSON uses (2/3)M_a.
\( \kappa_0 = \frac{3}{2} \zeta_0^2 (\mu + \zeta \mu^2 + \ldots) \delta \mu_0 = \frac{1}{2} \frac{3}{2} \zeta + \ldots \)
\( \kappa_1 = \frac{3}{2} \zeta^2 (\mu + \zeta \mu^2 + \ldots) \mu_0 = \frac{1}{2} \frac{3}{2} \zeta \)
\( \kappa_2 = \frac{3}{2} \zeta^3 (\mu + \zeta \mu^2 + \ldots) \left( \frac{3}{2} \mu^2 - \frac{1}{2} \right) \delta \mu_2 = \frac{5}{16} \frac{1}{2} \zeta + \ldots \)

e. For subsequent odd coefficients, the leading terms can be written
\( \kappa_n = \frac{2n + 1}{2} \left( \frac{1}{2} \frac{1}{2} \mu + \sum_{\mu=1}^{\infty} \kappa_n \mu_0 \right) \)
and these are of order \( \zeta \), can again be ignored. For the even terms
\( \mathcal{R} = \sum \left( \frac{-G}{R^3} \right) \left( \frac{1}{2} + \frac{1}{2} \mu + \sum_{\mu=1}^{\infty} \kappa_n \mu_0 \right) \)
\[ (4) \]

The first term in (4) represents the mean radiation, \( \mathcal{R}_0 = \frac{1}{4} \mathcal{S} \). For the entire surface this equals \( \frac{1}{4} \mathcal{S} \), the cross section times the mean radiation. The term is balanced by infrared radiation and enters into the overall heat balance of the planet. The first term in (4) plays a role similar to the term \( \zeta_0 \mu_0 \) in the expression (1) for the gravitational potential. These terms represent mean radiation and mean forces, respectively, and both can be ignored in the study of tides. An essential distinction is that the radiational function commences with \( \mu_1 \), \( \mu_2 \), the gravitational potential with \( \mu_1 \).

c) Expansion in Greenwich coordinates

The expressions (2) and (4) for the gravitational and radiational functions, respectively, are of the form \( \sum f_n \mu, \delta, R \) as \( \mu, \delta \) are functions of \( R \). We need to express \( \mu, \delta \) as a function of the station coordinates \( \theta, \lambda \), and of the Moon's (Sun's) angular coordinates \( Z, L \).

Let
\[ V_n^0 = \Psi_n^0 = \Psi_n^0 = (-1)^n \left( \frac{2n + 1}{4n} \right)^{1/2} \frac{1}{\sqrt{n}} P_n^0(\cos \theta) \sin \phi \]
\[ (5) \]
designate the complex spherical harmonic so normalized (BACKUS, 1958) that
\[ \int |V_n^0|^2 \sin \theta d\theta d\phi = 1 \]
and
\[ P_n^0(\mu) = \left( \frac{2n + 1}{2} \right) \left( \frac{n!}{(n-m)!} \right)^{1/2} (\mu^2 - 1)^m, \quad n > 0, \quad |\mu| \leq n \]
is the associated Legendre function (1)\(^{(*)} \)

\( (*) \) This is the normalization used by MORSE and FRIEBACH (1945) and \( n!/(n-m)! \) times the normalization by JEFFREYS and JEFFREYS (1950).

\[ P_n^0 = \cos \theta \]
\[ P_n^0 = \cos \theta \]
\[ P_n^0 = \sin \theta \]
\[ P_n^0 = \sin \theta \]
\[ P_n^0 = \sin \theta \]
\[ P_n^0 = \sin \theta \]
\[ P_n^0 = \sin \theta \]

\[ \sum = \sum \Psi_n^0(\theta, \lambda) \Psi_n^0(\theta, \lambda) \]

depends only on the distance between the two points, and is the same regardless of where the coordinate system is centered. First, center the coordinate system at the station \( P (i = 1) \) with zero longitude through the sub-inna (sub-solar) point \( Q (i = 2) \):

\[ \sum = \sum \Psi_n^0(\theta, \lambda) \Psi_n^0(\theta, \lambda) \]

Next, orient the coordinates in accordance with the Greenwich system

\[ \sum = \sum \Psi_n^0(\theta, \lambda) \Psi_n^0(\theta, \lambda) \]

Equating the two expressions for \( \sum \) yields

\[ P_n^0(\mu) = \frac{4 \pi}{2n + 1} \left( \frac{1}{2} \mu \right)^n P_n^0(\cos \theta) \sin \phi \]
\[ (6) \]

where \( Y = Y + i Y \); in equation (6) we have avoided negative values of \( m \) by use of the identity \( Y_n^m = (-1)^m Y_n^{-m} \). With this substitution for \( P_n(\mu) \) the gravitational function (2) or the radiational function (4) can be written in the form

\[ \Psi_n^0(Z, L) \Theta_n^0(\theta, \lambda) = \Psi_n^0(Z, L) \Theta_n^0(\theta, \lambda) \]
\[ (7) \]

d) Solar orbital constants

We require the polar angle \( Z \), the terrestrial longitude \( L \) and the distance \( R \) of Sun or Moon in terms of the orbital parameters. From triangle \( \bar{Z} = \sin \bar{Z} \tan \bar{Z} \)

\[ \cos \bar{Z} = \sin \bar{Z} \tan \bar{Z} \]
\[ \tan \frac{1}{2} \bar{Z} = \frac{\sin \frac{1}{2} (90^\circ + \omega)}{\sin \frac{1}{2} (90^\circ - \omega)} \tan \frac{1}{2} \bar{Z} \]

where \( \omega = 23^\circ, 452 \) is the obliquity of the ecliptic, and \( \bar{Z} \) is the instantaneous longitude (in the ecliptic). By KEPLER'S laws,
\[ n_0 = \frac{1}{2} \left( \mu + \xi \mu^2 + \ldots \right) d_0 = \frac{1}{2} \frac{1}{3} \xi + \ldots \]
\[ n_1 = \frac{3}{2} \left( \mu + \mu^2 \xi + \ldots \right) \mu d_1 = \frac{1}{2} \frac{3}{2} \xi \]
\[ n_2 = \frac{5}{2} \left( \mu + \mu^2 \xi + \ldots \right) \left( \frac{3}{2} \xi \mu^2 + \frac{1}{2} \xi \right) d_2 = \frac{5}{2} \frac{1}{3} \xi + \ldots \]

etc. For subsequent odd coefficients, the leading terms can be written
\[ n_k = \frac{2n+1}{2} \left( \frac{1}{4} + \frac{1}{2} \mu + \sum_{n=0}^{\infty} n_k \mu^n \right) \]
and these are of order \( \xi \), can again be ignored. For the even terms

\[ R_s = \sum_{\mu} \frac{Y_{\mu}^* Y_{\mu}}{R_s} \left( \frac{1}{4} + \frac{1}{2} \mu + \sum_{n=0}^{\infty} n_s \mu^n \right) \]
(4)

The first term in (4) represents the mean radiation, \( R_s = \frac{1}{4} S \). For the entire surface this equals \( \frac{1}{4} S \), \( 4 \pi a^2 \), the cross section times the mean radiation. The term is balanced by infrared radiation and enters into the overall heat balance of the planet. The first term in (4) plays a role similar to the term \( \xi \mu \) in the expression (1) for the gravitational potential. These terms represent mean radiation and mean forces, respectively, and both can be ignored in the study of tides. An essential distinction is that the radiational function commences with \( P_1(\mu) = \mu \), the gravitational potential with \( P_1(\mu) \).

c) Expansion in Greenwich coordinates

The expressions (2) and (4) for the gravitationa and radiational functions, respectively, are of the form \( \sum_{\mu} f_{\mu}(\mu, \lambda) \) where \( \mu, \lambda \) are functions of time. We need to express \( \mu(\lambda) \) as a function of the station coordinates \( \theta, \lambda \), and of the Moon's (Sun's) angular coordinates \( \phi(\theta, \lambda) \).

Let
\[ U^* = U^* = \left( -1 \right)^{\frac{n+1}{4}} \left( \frac{2n+1}{4} \right)^\frac{1}{2} \frac{n!}{(n+m)!} \frac{1}{n+m+1} P_n^*(\cos \theta) \rho^m \]
(5)
designate the complex spherical harmonics so normalized (BACKUS, 1958) that
\[ \int |U^*|^2 \sin \theta \ d\theta = 1 \]
and
\[ P_n^*(\mu) = \left( 1 + \mu^2 \right)^{\frac{n+1}{2}} \frac{1}{(n-m)!} \frac{1}{2n+1} (\mu^2 - 1)^m \]
for \( n > 0, |m| < n \)
is the associated Legendre function (f)

(f) This is the normalization used by MORSE and FISCHBACH (1953) and \( 1/(n-m)! \) times the normalization by JEFFREYS and JEFFREYS (1950).

\[ P_n^* = 1 \quad P_n^* = \cos \theta \quad P_n^* = \sin \theta \]
\[ P_n^* = \cos \theta - \frac{1}{2} \quad P_{n-1}^* = 3 \sin \theta \cos \theta \quad P_{n-2}^* = 3 \sin^2 \theta \]
\[ P_n^* = \frac{1}{2} \cos \theta - \frac{1}{2} \quad P_{n-1}^* = 5 \cos \theta \sin \theta - 1 \quad P_{n-2}^* = 15 \sin^2 \theta \sin \theta \quad P_{n-2}^* = 15 \sin^2 \theta \]

For any two points on a sphere, \( \theta, \lambda \), \( i = 1, 2 \), the expression

\[ \sum = \sum_{\mu} Y_{\mu}^* i Y_{\mu}^* i \]

depends only on the distance between the two points, and is the same regardless of where the coordinate system is centered. First, center the coordinate system at the station \( P = (1) \) with zero longitude through the sub-inner (sub-solar) point \( Q = (2) \) :

\[ \sum = \sum_{\mu} Y_{\mu}^* (0, 0) Y_{\mu}^* (\eta, \lambda) = Y_{\mu}^* (0, 0) Y_{\mu}^* (\eta, \lambda) - \frac{2n+1}{4} \int_{\lambda} \phi(\lambda) \]

Next, orient the coordinates in accordance with the Greenwich system

\[ \sum = \sum_{\mu} Y_{\mu}^* (0, \lambda) Y_{\mu}^* (0, \lambda) \]

Equating the two expressions for \( \sum \) yields

\[ P_\mu(\lambda) = \frac{4x}{2n+1} \int_{\lambda} Y_{\mu}^* (Z, L) Y_{\mu}^* (\eta, \lambda) \]
(6)

\[ = \frac{4x}{2n+1} \left( U_{\mu}(Z, L) U_{\mu}(\eta, \lambda) + \frac{2}{3} \int_{\mu} U_{\mu}^* (Z, L) U_{\mu}^* (0, \lambda) + U_{\mu}^* (Z, L) U_{\mu}^* (0, \lambda) \right) \]

where \( Y = Y^* \); in equation (6) we have avoided negative values of \( m \) by use of the identity \( Y_{\mu}^* = (-1)^m Y_{\mu}^* \). With this substitution for \( R_\mu(\lambda) \) the gravitational function (2) or the radiational function (4) can be written in the form

\[ Y_{\mu}^* (Z, L) = \mu_{\eta}^* (Z, L) Y_{\mu}^* (\eta, \lambda) + \mu_{\mu}^* (Z, L) Y_{\mu}^* (0, \lambda) \]
(7)
d) Solar orbital constants

We require the polar angle \( Z \), the terrestrial longitude \( L \) and the distance \( R \) of Sun or Moon in terms of the orbital parameters. From triangle \( \triangle Z_0 \Theta_0 \) figure 14, we have

\[ \cos \Theta_0 = \sin \lambda \sin \mu_0 \]
(8)
\[ \tan \frac{1}{2} \frac{Z_0}{\sin \lambda} = \frac{\mu_0}{\sin \lambda} \tan \frac{1}{2} \left( \frac{90^\circ + \mu_0}{\sin \lambda} \right) \]
(9)

where \( \mu_0 = 23^\circ.5452 \) is the obliquity of the ecliptic, and \( \lambda \) is the instantaneous longitude (in the ecliptic), By KEPLER'S laws.

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respectively, The eccentricity is taken by DOODSON as constant, e_0 = 0.0187504. A somewhat more accurate expression is

\[ e_0 = 0.0167504 - 0.0000418 T - 0.00000126 T^2. \]  

(16)

c) Lunar orbital constants

Here the situation is more complicated. The mean longitude \( \nu_0(T) \), longitude of perigee \( \nu_0(T) \) (all along the ecliptic) are given by

\[ \nu_0 = 4.7200088 + 8389.792743 T + 0.0000346 T^2 \]

\[ \nu_0 = 5.8351526 + 71.0180412 T + 0.0001801 T^2 \]

\[ \nu_0 = 4.3236016 - 0.3798463 T + 0.0000363 T^2. \]  

(17)

Solving the triangle \( \Delta AD \) with sides \( v, AD, A \), we have

\[ \cos \nu_0 = \cos \nu_0 \cos \varphi - \sin \nu_0 \sin \varphi \cos \nu_0. \]

\[ \sin v = \sin \nu_0 \sin \varphi \sin \nu_0 \cos \varphi. \]

\[ \sin AD = \sin \varphi \sin \nu_0 \sin \nu_0 \cos \varphi. \]

\[ \cos AD = \cos \varphi \cos \nu_0 \cos v + \cos \varphi \sin \nu_0 \sin \nu_0 \sin v. \]

\[ \tan \frac{1}{2} AD = \sin AD / (1 - \cos AD). \]

Figure 15: \( T \) is vernal equinox, \( \Delta G \) is longitude along ecliptic of ascending node reckoned from equinox, \( \nu_0 = 23^\circ 452 \) is obliquity of ecliptic, \( \nu_0 \) is longitude of Moon \( \nu_0 \) along its orbit. \( \Delta G \) is polar angle, \( \Delta G \) is terrestrial east longitude, and \( T \) is the Moon's right ascension.

The mean longitude of the Moon in its orbit is

\[ \varphi = \nu_0 - \nu + \Delta G. \]  

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respectively. The eccentricity is taken by DOODSON as constant, \(e_0 = 0.0187504\). A somewhat more accurate expression is
\[ e_0 = 0.0187504 - 0.00000180 T - 0.000000126 T^2. \]

(16)

e) Lunar orbital constants

Here the situation is more complicated. The mean longitude \(h(T)\), longitude of perigee \(\lambda(T)\) and of the node \(\Omega(T)\), (all along the ecliptic) are given by
\[ h = 4.7200089 + 8399.792745 T + 0.0000345 T^2 \]
\[ \lambda = 5.851526 + 71.0180412 T + 0.0001801 T^2 \]
\[ \Omega = 4.5226016 - 33.7971463 T + 0.000363 T^2. \]

(17)

Solving the triangle \(\Delta A\) with sides \(\nu, A\), \(\nu\) we have
\[ \cos \nu = \cos \omega_0 \cos i - \sin \omega_0 \sin i \cos \nu_0 \]
\[ \sin \nu = \sin i \sin \nu_0 \sin w, \]
\[ \sin A_0 = \sin \nu_0 \sin \omega_0 \sin \nu_0 \]
\[ \cos A_0 = \cos \nu_0 \cos \nu + \cos \omega_0 \sin \nu_0 \sin \nu. \]
\[ \tan \frac{1}{2} A_0 = \sin A_0/(1 + \cos A_0). \]

Finally, the distance \(R_0\) between Sun and Earth is related to the eccentricity \(e_0\) and the mean anomaly \(h_0\) by KREPLINSKY's laws:
\[ \frac{R_0}{R_0} = 1 + e_0 \cos (h_0 - \nu_0) + e_0^2 \cos 2 (h_0 - \nu_0). \]

(13)

where \(R_0 \propto 1/\text{mean equatorial parallax}\), not the Sun's mean distance. Equations (6), (12) and (13) determine \(Z_0\), \(l_0\) and \(R_0\) in terms of \(h_0, \nu_0\) and \(e_0\). When these are substituted into (7) and (2), and the resulting expressions expanded in powers of small parameters (the ellipticity, obliquity and parallax), we obtain the classical line spectrum of frequencies \(f_1\) (equation 2.2).

It remains to express \(h_0, \nu_0\) and \(e_0\) as functions of time. Let \(t\) designate Greenwich time in hours since 1900 January 1 at 0 hours GCT. Then
\[ T = \frac{t + 12}{(24) (36525)}. \]

(14)

is the time in Julian centuries since 1900 January 0.5, as used in most astronomical texts. The Sun's mean longitude, and the longitude of perigee (both in radians) are given by
\[ h_0 = 4.8816280 + 626.3319509 T + 0.0000052 T^2. \]
\[ \nu_0 = 4.9082295 + 0.030053 T + 0.0000729 T^2. \]

(15)
and the (instantaneous) longitude \( L_\xi \) in its orbit equals (4)

\[
L_\xi = \xi + 2e\xi \sin (0\xi - p\xi) + \frac{5}{2} \varepsilon_\xi \sin 2(0\xi - p\xi)
+ \frac{17}{8} m^2 e\xi \sin (30\xi - 2h\xi - p\xi) + \frac{77}{10} m^2 \varepsilon_\xi \sin (2h\xi - 3h\xi + p\xi)
\]

where \( e\xi = 0.054900 \) is the eccentricity of the Moon's orbit, and \( m = 0.074804 \) the ratio of mean motions of \( \theta \) to \( \zeta \). The Moon's distance is given by (4)

\[
\frac{N_\xi}{N_\zeta} = 1 + \left[ 1 + \left( \frac{1}{16} \right)^2 \right] \left[ e\xi \cos (0\xi - p\xi) + \varepsilon_\xi \cos 2(0\xi - p\xi)
+ \frac{15}{8} \varepsilon_\xi \cos (0\xi - p\xi) + \frac{80}{16} m \sin (0\xi - p\xi) + \frac{75}{16} \varepsilon_\xi \cos (20\xi - 2h\xi - p\xi)
+ \frac{77}{10} m^2 e\xi \cos (30\xi - 2h\xi + p\xi) + \frac{77}{10} m^2 \varepsilon_\xi \cos (2h\xi - 3h\xi + p\xi)\right]
\]

The polar distance \( Z\xi \) is given by

\[
\cos Z\xi = \sin L_\xi \sin \omega \xi \quad 0 < Z\xi < x
\]

Also,

\[
\tan \frac{1}{2} Z\xi = \sin \frac{1}{2} (90\eta + \omega \xi) \tan \frac{1}{2} (90\eta - \omega \xi) + Z\xi - 90\eta
\]

with \( 0 < \frac{1}{2} \eta < x \). The Moon's right ascension in then \( L\xi = \nu + \gamma \xi \) and its longitude east of Greenwich is

\[
L\xi = \nu + \gamma \xi - \gamma \zeta
\]

APPENDIX B. SAMPLING DISTRIBUTIONS AND CONFIDENCE LIMITS

We refer to the complex spectrum estimators \( G_x, H_x \) (section 4b) at a particular frequency (\( \nu/355 \)) c, p, d, and consider ensemble averages of their product from analysis of \( p \) independent segments of the time series. From equation (4, 8) we obtain as an estimate of the admittance \( Z \),

\[
\hat{Z} = \hat{X} + i\hat{Y} \quad <G_x H_x> = <G_x G_x^*> = Z + \epsilon
\]

where the sampling error is

\[
\epsilon = \frac{\sqrt{\beta G_x}}{1 + \frac{\sigma^2}{\sigma_x^2} (2\pi)^2 H_x \gamma \frac{1}{1 + \frac{\sigma^2}{\sigma_x^2}}}
\]

The input energy \( \beta G_x \) is by definition noise-free and approximately constant from sample to sample, and the relative phase \( \gamma \) has uniform expectation in (0, 2\pi). Therefore the real and imaginary parts of (1) have approximately normal independent probability distributions, each with mean value zero, and variance given by

\[
\sigma^2 = (2\pi)^2 \frac{\sigma_x^2}{\sigma_x^2} (2\pi)^2 H_x \gamma \frac{1}{1 + \frac{\sigma^2}{\sigma_x^2}} - 1
\]

(3)

where \( \sigma^2 \) and \( \sigma_x^2 \) are the long-term average variances of noise and input signal respectively, \( H_x = \psi^2 + \phi^2 - 2\psi \phi \gamma \) and \( \gamma \) is the true coherence. This result differs from the more difficult case analyzed by GOODMAN (1957), in which \( \beta G_x \) has a Rayleigh distribution appropriate to a random input.

\( \hat{X} \) and \( \hat{Y} \) are thus unbiased, independent estimates of \( X \) and \( Y \), with the joint probability distribution

\[
\Phi(\hat{X}, \hat{Y}) = (2\pi)^2 \frac{\sigma_x^2}{\sigma_x^2} (2\pi)^2 H_x \gamma \frac{1}{1 + \frac{\sigma^2}{\sigma_x^2}}
\]

The distributions of \( \hat{X} = \psi \hat{X}^2 + \phi \hat{Y}^2 \) and \( \hat{Y} = \psi \hat{X} \hat{Y} + \phi \hat{Y} \hat{X} \) are derived by transforming (3) to the variables \( \hat{R}, \hat{\theta} \) and integrating with respect to \( \hat{R} \) and \( \hat{\theta} \) respectively. With normalised variables,

\[
\rho = \frac{\hat{R}}{\hat{R}_0}, \quad \theta = \frac{\hat{\theta}}{\hat{\theta}_0} \quad \text{true phase lead}, \quad \phi = \frac{\phi}{\hat{\phi}_0}
\]

the results are

\[
\rho(\phi) = |\rho| \phi \exp(-\rho(1-2\phi/\phi_0)) \cdot \exp\left(\frac{2\phi}{\phi_0}\right) L_0(\rho/\phi_0)
\]

\[
\rho(\phi) = |\rho| \phi \exp(-\rho(1-2\phi/\phi_0)) \cdot \exp\left(\frac{2\phi}{\phi_0}\right) L_0(\rho/\phi_0)
\]

where \( L_0(\rho) \) is a modified Bessel Function, and

\[
F(\phi) = x^{\phi/2} \int_0^\phi e^{x/2} \sin \phi \, d\phi
\]

The functions defined by (4) and (5) have been computed, and are represented in the top half of figure 16 for \( \sigma = 0.1, 0.2, 0.4, 0.6, 0.8 \) and 1.0. For values of \( \sigma > 1.0 \) the distributions are too broad to be of any practical use. For values of \( \sigma < 0.1 \), both \( \rho(\phi) \) and \( \rho(\phi) \) approximate to normal distributions with variance \( \sigma^2 \).

It is seen that the distribution of \( \hat{\theta} \) is always symmetrical about \( \hat{\theta} = 0 \), i.e. \( \hat{\theta} = 0 \) is an unbiased estimator of \( \theta \), and the distribution of \( \hat{\phi} \) is biased positively. (i.e. \( \hat{R} \) tends to be somewhat greater than \( \hat{R} \)). The mean or "expected" value of \( \hat{\phi} \) is found to be

\[
\left( \frac{1}{2} \right)^{1/2} \alpha \gamma F_1 \left( \frac{1}{2} ; 1 : -\alpha \right)
\]

where \( \alpha \gamma F_1 \left( a ; b ; x \right) \) is Kummer's Hypergeometric Function (tabulated in SLATER, 1960). However, its r.m.s. value is simply

\[
(1 + 2\pi)^{1/2}
\]
and the (instantaneous) longitude \( L_\xi \) in its orbit equals (*):

\[
l_\xi = \xi + 2 \xi \sin (\varpi_\xi + \varpi_\psi) + \frac{5}{8} \xi^2 \sin 2(\varpi_\psi - \varpi_\zeta)
+ m_\xi \left( \frac{15}{8} \cdot \frac{203}{16} \right) \sin (2\varpi_\psi - 2\varpi_\psi + \varpi_\zeta + m_\xi) + \frac{77}{16} \xi (\varpi_\psi - \varpi_\zeta)
+ \frac{17}{5} m_\xi \xi \sin (3\varpi_\psi - 2\varpi_\psi + \varpi_\zeta) + \frac{77}{18} m_\xi \xi \sin (2\varpi_\psi - 3\varpi_\psi + \varpi_\zeta)
\]

where \( \xi = 0.054900 \) is the eccentricity of the Moon's orbit, and \( m = 0.074804 \) is the ratio of mean motions of \( E \) to \( \zeta \). The Moon's distance is given by (*):

\[
\frac{R_\xi}{R_{\zeta}} = 1 + \left[ \frac{1 + \frac{1}{8} \xi^2}{m_\xi} \right] \left[ \xi \cos (\varpi_\psi - \varpi_\zeta) + \xi^2 \cos 2(\varpi_\psi - \varpi_\zeta)
+ m_\xi \left( \frac{15}{8} \cdot \frac{203}{16} \right) \cos (2\varpi_\psi - 2\varpi_\psi + \varpi_\zeta) + m_\xi \left( \frac{15}{8} \cdot \frac{19}{16} \right) \xi \cos (3\varpi_\psi - 2\varpi_\psi + \varpi_\zeta) + \frac{1}{8} m_\xi \xi \cos (2\varpi_\psi - 3\varpi_\psi + \varpi_\zeta) \right]
\]

(18)

The polar distance \( Z_\zeta \) is given by:

\[
\cos Z_\zeta = \sin l_\xi \sin \varpi_\zeta, \quad 0 < Z_\zeta < \pi.
\]

Also,

\[
\tan \frac{1}{2} \varpi_\zeta = \frac{\sin \frac{1}{2} (90^\circ - \omega_\xi)}{\sin \frac{1}{2} (90^\circ - \omega_\zeta)} \tan \frac{1}{2} \varpi_\xi
\]

with \( 0 < \frac{1}{2} \varpi_\zeta < \pi \). The Moon's right ascension is then \( \Gamma_\xi = \varpi + \varpi_\zeta \) and its longitude east of Greenwich is

\[
L_\xi = \varpi + \varpi_\xi - \varpi_\zeta.
\]

(21)

**APPENDIX B. SAMPLING DISTRIBUTIONS AND CONFIDENCE LIMITS**

We refer to the complex spectrum estimators \( G_\xi, H_\xi \) (section 4b) at a particular frequency \((\nu/355)\) c, p., d., and consider ensemble averages of their product from analysis of \( p \) independent segments of the time series. From equation (4.8) we obtain an estimate of the admittance \( Z \),

\[
Z = X + iY = <G_\xi H_\xi^*, G_\xi^* H_\xi> = Z + \varepsilon
\]

where the sampling error is

\[
\varepsilon = \epsilon / |N|, \quad |G_\xi| \epsilon^{1/2} \sim |G_\xi|^{1/2}.
\]

The input energy \( |G_\xi|^2 \) is by definition noise-free and approximately constant from sample to sample, and the relative phase \( \varphi \), has uniform expectation in \((0, 2\pi)\). Therefore the real and imaginary parts of (1) have approximately normal independent probability distributions, each with mean value zero, and variance given by

\[
\sigma^2 = \left( 2m_\xi \right) \frac{\sigma^2}{\sigma^4}
\]

where \( \sigma^2 \) and \( \sigma^4 \) are the long-term average variances of noise and input signal respectively, \( R^2 = X^2 + Y^2 \), and \( \gamma \) is the true coherence. This result differs from the more difficult case analyzed by GOODMAN (1959), in which \( |G_\xi| \) has a Rayleigh distribution appropriate to a random input.

\( X \) and \( Y \) are thus unbiased, independent estimates of \( X \) and \( Y \), with the joint probability distribution

\[
p(X, Y) = \left( \frac{2n \sigma^2 \pi}{\sqrt{\pi}} \right) \exp \left(-\left(\frac{X^2}{\sigma^2} + \frac{Y^2}{\sigma^2}\right)\right)
\]

(3)

The distributions of \( \Re = \sqrt{X^2 + Y^2} \) and \( \theta = \arctan(Y/X) \) are derived by transforming (3) to the variables \((\Re, \theta)\) and integrating with respect to \( \Re \) and \( \theta \) respectively. Normalized variables,

\[
\rho = \frac{\Re}{\Re_0}, \quad \theta = \theta_0 - \text{true phase lead}, \quad \sigma = \frac{\sigma^2}{\sigma^4},
\]

the results are

\[
p(\rho, \theta) = p(\rho) \exp(-\left(\rho - 1\right)^2/2\sigma^2), \quad \left( e^{\sigma^2 - \frac{1}{2}} \right)^2 = \left( e^{\sigma^2 - \frac{1}{2}} \right)^2
\]

(4)

\[
p(\theta) = \left( 2\pi \sigma^2 \right) \exp(-\left(\rho - 1\right)^2/2\sigma^2)
\]

(5)

where \( p(x) \) is a modified Bessel Function, and

\[
P(x) = xe^{x/2} \int_0^x e^{\sigma^2 - \frac{1}{2}} \sigma^2 d\sigma.
\]

The functions defined by (4) and (5) have been computed and are represented in the top half of figure 16 for \( \sigma = 0.1, 0.2, 0.4, 0.8, 0.8, 1.0. \) For values of \( \sigma > 1.0 \) the distributions are too broad to be of any practical use. For values of \( \sigma < 0.1 \), both \( p(\rho) \) and \( p(\theta) \) approximate to normal distributions with variance \( \sigma^2 \).

It is seen that the distribution of \( \theta \) is always symmetrical about \( \theta = 0 \), i.e. \( \theta = 0 \) is an unbiased estimator of \( \theta \), but the distribution of \( \rho \) is biased positively, i.e. \( \Re \) tends to be somewhat greater than \( \Re_0 \). The mean of "expected" value of \( \rho \) is found to be

\[
\left( \frac{2\pi}{\Re_0} \right) \sigma = \frac{1}{2} \sigma^2
\]

(6)

where \( \rho = \rho_0(a; b; \sigma) \) is Kummer's Hypergeometric Function (tabulated in SLATER, 1960). However, its r.m.s. value is simply

\[
(1 + 2\sigma^2)^{1/2}
\]

(7)
Both (6) and (7) are represented in the lower left hand part of figure 16. The 95% confidence limits of $\rho$ and $\theta$, defined such that 0.025 of the distribution lies to the left of the lower, and 0.025 to the right of the upper limit, computed by quadrature, are also plotted.

The sampling distribution of the coherence estimate, $\gamma^2 = |G_r H_r|^2 / |G_r| |H_r|^2$, is much more complicated, even with constant $|G_r|$. We have

$$\gamma^2 = \frac{(X + c_1)^2 + (Y + c_2)^2}{(X + c_1)^2 + (Y + c_2)^2} = 1 - \frac{|c_1|^2 + |c_2|^2}{(X + c_1 + c_2 Y + c_2 Y + c_2 Y + c_2)}.$$

where

$$c_1 + ic_2 = c, \quad and \quad c_2^2 = \frac{<|N_r|^2>}{c(G_r)|H_r|^2}.$$ (9)

With $p = 1$, (i.e., no ensemble averaging), comparison of (1) and (9) shows that

$$c_2^2 = c_1 + c_2^2,$$

so that $\gamma^2 = 1$ identically. With $p > 1$, it can be shown that $\gamma^2 < 1$, but $\gamma^2 - \gamma_r$ is still biased positively by an amount which decreases with increasing $p$. We make the simplifying approximation that the "expected" value of $\gamma^2$, say $E(\gamma^2)$, is the same as the right side of (8) with numerator and denominator each put equal to its own "expected" value. A little algebra then gives

$$E(\gamma^2) = \gamma^2 + p^{-1} (1 - \gamma^2).$$

Values of $E(\gamma^2)$ so defined compare reasonably well with exact mean values of the distributions for GOODMAN'S case of random input, computed and tabulated by AMOS and KOOPMANS (1963). An approximately unbiased estimate of the true coherence $\gamma$ is therefore

$$\frac{p(\gamma^2 - 1)}{(p - 1)} (p > 1)$$ (10)

The expression (10) is negative if $\gamma^2 < p^{-1}$, but in such a case the true coherence (fundamentally positive) is probably too small to be of interest.

To obtain the confidence limits shown in figure 1 etc., we used the unbiased estimate of $\gamma^2$ from (10) to form the normalized parameter $\sigma^2 = (2p)\gamma^2 - (\gamma^2 - 1)$, and thence the normalized confidence limits for $\rho$ and $\theta$. The limits for $\rho$ and $\theta$ are then obtained from $\bar{R}$ and $\bar{\theta}$ by dividing by or subtracting, respectively, the normalized limits. The limits for $X$ and $Y$ are simply $X = 1, 96 \sigma$, $Y = 1, 96 \sigma$, from (2).

Although the above results are derived strictly for a fixed frequency $f$, which implies $|G_r|$ is nearly constant, they may be shown to be approximately valid for ensemble averages over adjacent values of $f$ also.

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Both (6) and (7) are represented in the lower left hand part of figure (16). The 95% confidence limits of \( p \) and \( \theta \), defined such that 0.025 of the distribution lies to the left of the lower, and 0.025 to the right of the upper limit, computed by quadrature, are also plotted.

The sampling distribution of the coherence estimate, \( \hat{\gamma} \), is much more complicated, even with constant \( |G_1| \). We have

\[
\frac{1}{\left[ (X + \epsilon_1)^2 + (Y + \epsilon_2)^2 \right] (X + \epsilon_1)^2 + (Y + \epsilon_2)^2} = 1 - \frac{\epsilon_1^2}{\epsilon_1^2 + \epsilon_2^2},
\]

where

\[
\epsilon_1 + \epsilon_2 = c, \quad \text{and} \quad \frac{\epsilon_1^2}{\epsilon_1^2 + \epsilon_2^2} = \frac{1}{c^2}.
\]

With \( p = 1 \), (i.e., no ensemble averaging), comparison of (1) and (9) shows that

\[
\hat{\gamma} = 1 \quad \text{identically}, \quad \text{With} \quad p > 1, \quad \text{it can be shown that} \quad \hat{\gamma} < 1, \quad \text{but} \quad \gamma - \hat{\gamma} \quad \text{is still biased positively by an amount which decreases with increasing} \quad p. \quad \text{We make the simplifying approximation that the "expected" value of} \quad \gamma, \quad \text{say} \quad E(\gamma), \quad \text{is the same as the right side of} \quad (8) \quad \text{with numerator and denominator each put equal to its own "expected" value. A little algebra then gives}
\]

\[
E(\gamma) = \gamma + p^{-1}(1 - \gamma).
\]

Values of \( E(\gamma) \) so defined compare reasonably well with exact mean values of the distributions for GOODMAN'S case of random input, computed and tabulated by AMOS and KOOPMANS (1963). An approximately unbiased estimate of the true coherence \( \gamma \) is therefore

\[
\frac{p \left( \frac{\gamma}{p} - 1 \right)}{(p - 1)} \quad (p > 1).
\]

The expression (10) is negative if \( \gamma < p^{-1} \), but in such a case the true coherence (fundamentally positive) is probably too small to be of interest.

To obtain the confidence limits shown in figure 1 etc., we used the unbiased estimate of \( \gamma \) from (10) to form the normalized parameter \( \gamma^2 = (2p+1) \gamma - 1 \), and thence the normalized confidence limits for \( \gamma^2 \) and \( \gamma \). The limits for \( \gamma \) and \( \theta \) are then obtained from \( \hat{R} \) and \( \hat{\theta} \) by dividing, or by subtracting, respectively, the normalized limits. The limits for \( X \) and \( Y \) are simply \( X \pm 1.96 \sigma, \text{Y} \pm 1.96 \sigma \), from (2).

Although the above results are derived strictly for a fixed frequency \( r \), which implies \( |G_1| \) is nearly constant, they may be shown to be approximately valid for ensemble averages over adjacent values of \( r \) also.

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OBSERVATIONS SUR LA COMMUNICATION DU DR CARTWRIGHT

Dr. ROSITER, in congratulating Mrs. MUNK and CARTWRIGHT on this enormously valuable contribution to this symposium, I would like to emphasize the undoubted scientific importance of their work, and the probable practical importance which may ultimately result. In particular, I believe the manner in which the technique enables the tidegenerating potential to be treated in the same fashion as the radiation and meteorological potentials, will be of special value. The Honolulu results indicate that the nodal tide is not visible above the level of noise. Might this be because of the proximity of Honolulu to the nodal line of this tide, which in the equilibrium theory appears at 39° N and S?

Dr. CARTWRIGHT, at 21° latitude, the amplitude of the equilibrium nodal line at Honolulu is not much smaller than its value at other parts of the globe. However, it is possible that the actual nodal tide in the Pacific Ocean is zero at latitudes lower than 33°; in which case, the results I presented were unduly pessimistic, and the nodal tide may have better signal/noise ratio at other places.

M. EVREUX, la prédiction harmonique ne prétend pas être la seule méthode mathématique pour extrapoler dans le temps des résultats observés. Après tout, elle n'est pas parfaite parce que, pratiquement, on ne peut considérer assez de termes. C'est parce que la méthode devient très lourde (grand nombre de termes) qu'on utilise encore la méthode de LAPLACE sur les côtes de France. Je suis agréablement surpris que le principe du travail de l'auteur repose sur la comparaison d'une corde calculée avec une corde observée; c'est un principe que nous appliquons pour les courtes durées d'observation et il est, je crois, fructueux.

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